Implicit Differentiation - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q2

Question Number	Scheme	Marks
	$2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$	M1 (A1) A1
	$\frac{dy}{dx} = 0 \Rightarrow x + y = 0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of x or y.	M1
	$y^2 - 2y^2 - 3y^2 + 16 = 0$ or the same equation in x	
	$y = \pm 2$ or $x = \pm 2$	A1
	$(2, -2), (-2, 2)$	A1
		[7]
	Note: $\frac{dy}{dx} = \frac{x+y}{3y-x}$	
	Alternative	
	$3y^2 - 2xy - (x^2 + 16) = 0$	
	$y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$	
	$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3} \cdot \frac{8x}{\sqrt{(16x^2 + 192)}}$	M1 A1± A1
	$\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$	M1
	$64x^2 = 16x^2 + 192$	
	$x = \pm 2$	M1 A1
	$(2, -2), (-2, 2)$	A1
		[7]

Question 2: Jan 06 Q1

Question Number	Scheme	Marks
	Differentiates	M1
	to obtain : $6x + 8y\frac{dy}{dx} - 2,$	A1,
	......................$+(6x\frac{dy}{dx} + 6y) = 0$	+(B1)
	$\left[\frac{dy}{dx} = \frac{2-6x-6y}{6x+8y}\right]$	
	Substitutes $x = 1, y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx} = -\frac{8}{10}$	M1, A1
	Uses line equation with numerical 'gradient' $y - (-2) =$ (their gradient)$(x - 1)$ or finds c and uses $y = (\textit{their gradient})\ x + "c"$	M1
	To give $5y + 4x + 6 = 0$ (or equivalent $= 0$)	A1√ **[7]**

Question 3: June 06 Q1

Question Number	Scheme		Marks
	$\left\{\frac{dx}{dx} \times\right\}$ $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) Correct equation.	M1 A1
	$\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1√ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient' ;	M1;
	N: $7x + 2y - 2 = 0$	Correct equation in the form $'ax + by + c = 0'$, where a, b and c are integers.	A1 oe **cso** **[7]**
			7 marks

Question 4: Jan 07 Q5

Question Number	Scheme		Marks
(a)	$\sin x + \cos y = 0.5 \quad (\text{eqn} *)$		
	$\left\{\frac{dx}{dx} \times\right\} \quad \cos x - \sin y \frac{dy}{dx} = 0 \quad (\text{eqn} \#)$	Differentiates implicitly to include $\pm \sin y \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
	$\frac{dy}{dx} = \frac{\cos x}{\sin y}$	$\frac{\cos x}{\sin y}$	A1 cso
			[2]
(b)	$\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$	Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.	M1 $\sqrt{}$
	giving $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$	both $x = -\frac{\pi}{2}, \frac{\pi}{2}$ or $x = \pm 90°$ or awrt $x = \pm 1.57$ required here	A1
	When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$	Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn *	M1
	$\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	Only one of $y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ or $120°$ or $-120°$ or awrt -2.09 or awrt 2.09	A1
	In specified range $(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$	Only exact coordinates of $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ **Do not award this mark if candidate states other coordinates inside the required range.**	A1
			[5]
			7 marks

Question 5: Jan 07 Q6

Question Number	Scheme		Marks
	$y = 2^x = e^{x\ln 2}$		
(a) **Way 1**	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	M1
	Hence $\frac{dy}{dx} = \ln 2 . (2^x) = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
Aliter (a) **Way 2**	$\ln y = \ln(2^x)$ leads to $\ln y = x\ln 2$	Takes logs of both sides, then uses the power law of logarithms…	
	$\frac{1}{y}\frac{dy}{dx} = \ln 2$	… and differentiates implicitly to give $\frac{1}{y}\frac{dy}{dx} = \ln 2$	M1
	Hence $\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
(b)	$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x.\,2^{(x^2)}.\ln 2$	$Ax\ 2^{(x^2)}$	M1
		$2x.\,2^{(x^2)}.\ln 2$ or $2x.y.\ln 2$ if y is defined	A1
	When $x = 2$, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes $x = 2$ into their $\frac{dy}{dx}$ which is of the form $\pm k2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]
			6 marks

Question 6: Jan 08 Q5

Question Number	Scheme		Marks
(a)	$x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by *completing the square*.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \dfrac{\cancel{dx}}{\cancel{dx}} \times \right\} \quad 3x^2 - 8y\dfrac{dy}{dx};= \left(\underline{12y + 12x\dfrac{dy}{dx}}\right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$ Correct LHS equation; Correct application of product rule	M1 A1; (B1)
	$\left\{ \dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ *correctly* found.	dM1 A1 A1 cso
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\left\{ \dfrac{\cancel{dx}}{\cancel{dy}} \times \right\} \quad 3x^2\dfrac{dx}{dy} - 8y; = \left(\underline{12y\dfrac{dx}{dy} + 12x}\right)$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy} = ...$ Correct LHS equation Correct application of product rule	M1 A1; (B1)
	$\left\{ \dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ *correctly* found.	dM1 A1 A1 cso
			[6]

Question Number	Scheme		Marks
Aliter (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn *)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}(g(x))$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found. Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1 A1
			[6]

Question 7: June 08 Q4

Question Number	Scheme	Marks
(a)	$3x^2 - y^2 + xy = 4$ (eqn *)	
	$\left\{\frac{dy}{dx}\right\}$ $6x - 2y\frac{dy}{dx} + \left(y + x\frac{dy}{dx}\right) = 0$	M1 B1 A1
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	M1
	giving $-18x - 3y = 8x - 16y$	
	giving $13y = 26x$	M1
	Hence, $y = 2x \Rightarrow y - 2x = 0$	A1 cso (6)
(b)	At P & Q, $y = 2x$. Substituting into eqn *	
	gives $3x^2 - (2x)^2 + x(2x) = 4$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow x = \pm 2$	A1
	$y = 2x \Rightarrow y = \pm 4$, hence coordinates are $(2, 4)$ and $(-2, -4)$	A1 (3)
		(9 marks)

Question 8: Jan 09 Q1

Question Number	Scheme		Marks
(a)	$C\colon\ y^2 - 3y = x^3 + 8$		
	$\left\{\frac{dy}{dx} \times \right\}\quad 2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\frac{3x^2}{2y-3}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, *may* indicate a correct follow through.		
			[7]

Question 9: June 09 Q4

Question Number	Scheme		Marks	
Q (a)	$e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = 2 + 2y\frac{dy}{dx}$	A1 correct RHS	M1 A1	
	$\frac{d}{dx}\left(ye^{-2x}\right) = e^{-2x}\frac{dy}{dx} - 2ye^{-2x}$		B1	
	$\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2 + 2ye^{-2x}$		M1	
	$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$		A1	(5)
(b)	At P, $\frac{dy}{dx} = \frac{2 + 2e^{0}}{e^{0} - 2} = -4$		M1	
	Using $mm' = -1$			
	$m' = \frac{1}{4}$		M1	
	$y - 1 = \frac{1}{4}(x - 0)$		M1	
	$x - 4y + 4 = 0$	or any integer multiple	A1	(4)
				[9]
	Alternative for (a) differentiating implicitly with respect to y.			
	$e^{-2x} - 2ye^{-2x}\frac{dx}{dy} = 2\frac{dx}{dy} + 2y$	A1 correct RHS	M1 A1	
	$\frac{d}{dy}\left(ye^{-2x}\right) = e^{-2x} - 2ye^{-2x}\frac{dx}{dy}$		B1	
	$\left(2 + 2ye^{-2x}\right)\frac{dx}{dy} = e^{-2x} - 2y$		M1	
	$\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2ye^{-2x}}$			
	$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$		A1	(5)

Question 10: Jan 10 Q3

Question Number	Scheme		Marks	
	(a) $-2\sin 2x - 3\sin 3y\frac{dy}{dx} = 0$		M1 A1	
	$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$	Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$	A1	(3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$		M1	
	$\cos 3y = \frac{1}{2}$		A1	
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$	awrt 0.349	A1	(3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$		M1	
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$		M1	
	Leading to $6x + 9y - 2\pi = 0$		A1	(3)
				[9]

Question 11: June 10 Q3

Question Number	Scheme		Marks	
	$\frac{d}{dx}\left(2^x\right) = \ln 2.2^x$		B1	
	$\ln 2.2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$		M1 A1= A1	
	Substituting $(3, 2)$			
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$		M1	
	$\frac{dy}{dx} = 4\ln 2 - 2$	Accept exact equivalents	M1 A1	(7)
				[7]

Question 12: June 11 Q5

Question Number	Scheme		Marks	
		$\frac{1}{y}\frac{dy}{dx} = \ldots$	B1	
		$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
	At $x = 2$,	$\ln y = 2(2)\ln 2$	M1	
	leading to	$y = 16$ Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$	$\frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$	M1	
		$\frac{dy}{dx} = 16(2 + 2\ln 2)$	A1	(7)
				[7]
	Alternative			
		$y = e^{2x\ln x}$	B1	
		$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
		$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1	
	At $x = 2$,	$\frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1	
		$= 16(2 + 2\ln 2)$	A1	(7)