Name:

Total Marks:

Pure Mathematics 1

Advanced Subsidiary

Practice Paper J10

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

$$f(x) = x^2 + 4kx + (3+11k), \text{ where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k. **(3)**

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k. **(4)**

Given that $k = 1$,

(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis. **(3)**

(Total 10 marks)

Question 2

(a) Factorise completely $x^3 - 4x$ **(3)**

(b) Sketch the curve C with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the x-axis. **(3)**

The point A with x-coordinate −1 and the point B with x-coordinate 3 lie on the curve C.

(c) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants. **(5)**

(d) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found. **(2)**

(Total 13 marks)

Question 3.

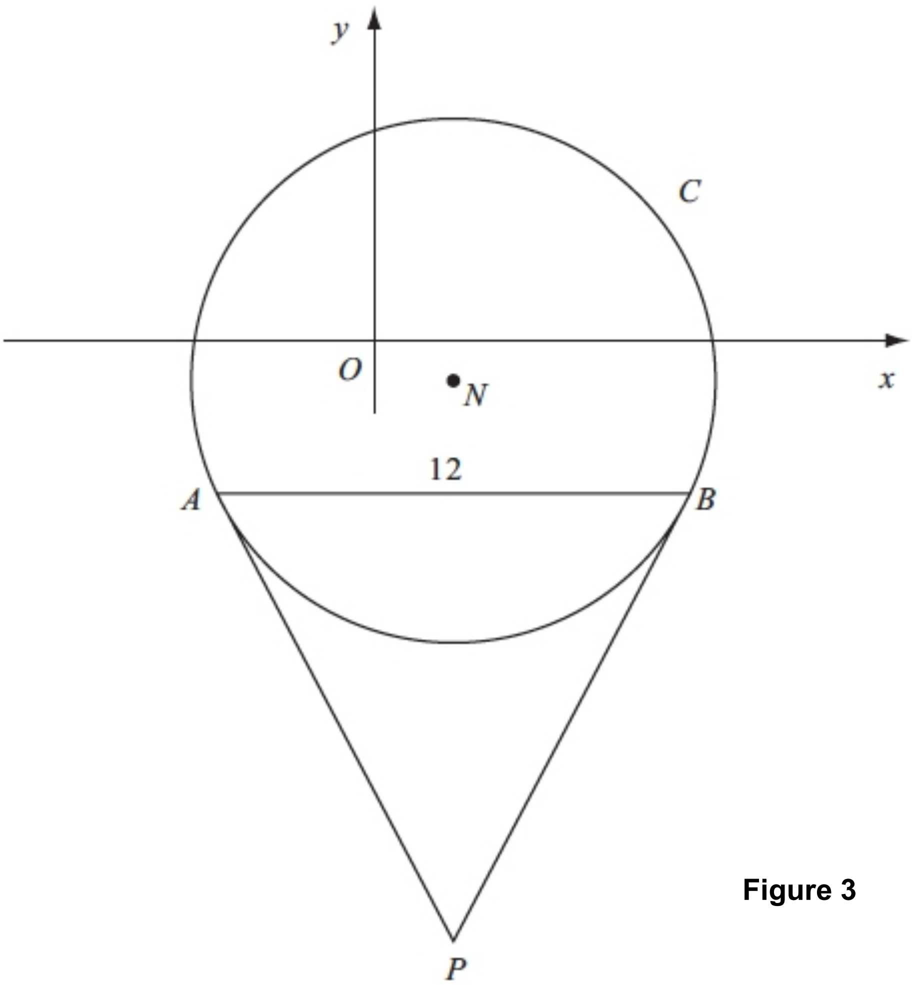

Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x - 2) + (y + 1)^2 = \frac{169}{4}$$

(a) Write down the coordinates of N. **(2)**

(b) Find the radius of C. **(1)**

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in Figure 3.

(c) Find the coordinates of A and the coordinates of B. **(5)**

(d) Show that angle ANB = 134.8°, to the nearest 0.1 of a degree. **(2)**

The tangents to C at the points A and B meet at the point P.

(e) Find the length AP, giving your answer to 3 significant figures. **(2)**

(Total 12 marks)

Question 4

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \qquad x > 0$

(a) Use calculus to find the coordinates of the turning point on C. **(7)**

(b) Find $\dfrac{d^2y}{dx^2}$. **(2)**

(c) State the nature of the turning point. **(1)**

(Total 10 marks)

Question 5

(a) Find the positive value of x such that

$$\log_x 64 = 2$$ **(2)**

(b) Solve for x

$$\log_2 (11 - 6x) = 2 \log_2 (x - 1) + 3$$ **(6)**

(Total 8 marks)

Question 6

(i) Find the exact solutions to the equations

(a) $\ln (3x - 7) = 5$ **(3)**

(b) $3^x e^{7x + 2} = 15$ **(5)**

(Total 8 marks)

Question 7

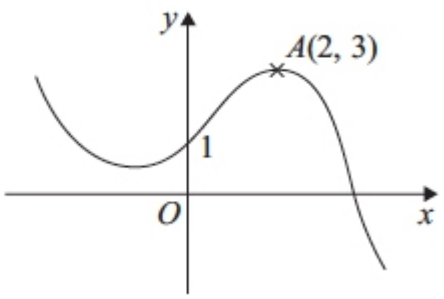

Figure 1

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = f(-x) + 1$,

(ii) $y = f(x + 2) + 3$,

(iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed. **(9)**

(Total 9 marks)

Question 8

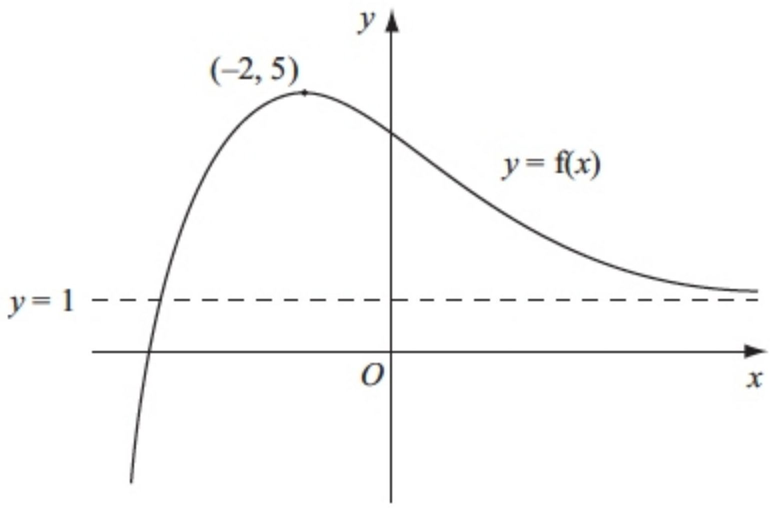

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in Figure 1.

On separate diagrams, sketch the curve with equation

(a) $y = f(x) + 2$ **(2)**

(b) $y = 4f(x)$ **(2)**

(c) $y = f(x + 1)$ **(3)**

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

(Total 7 marks)

Question 9

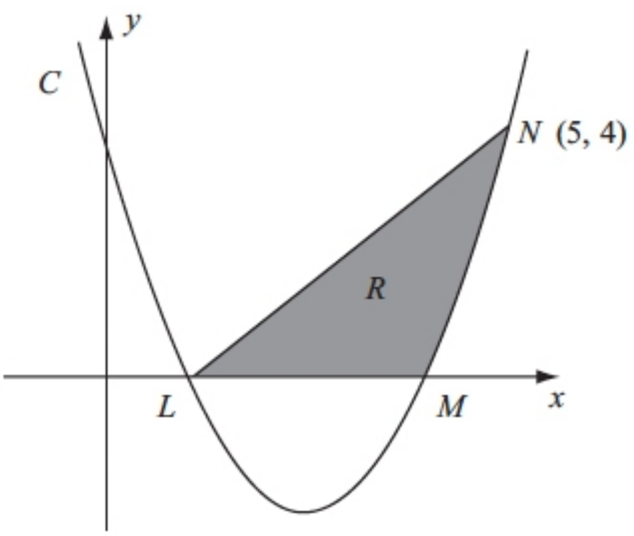

Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M. **(2)**

(b) Show that the point N (5, 4) lies on C. **(1)**

(c) Find $\int (x^2 - 5x + 4)\mathrm{d}x$.

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R. **(5)**

(Total 10 marks)

Question 10

(a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form $2 \sin^2 x + 5 \sin x - 3 = 0$ **(2)**

(b) Solve, for $0 \leqslant x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$ **(4)**

(Total 6 marks)

Question 11

In triangle ABC, $AB = (3 - x)$, $BC = (x - 4)$ and angle $ABC = 120^0$

(a) Show that $AC^2 = x^2 - 7x + 13$ **(3)**

(b) Find the value of x for which AC has a minimum value **(4)**

(Total 7 marks)

TOTAL FOR PAPER IS 100 MARKS