Pure Mathematics 1 Practice Paper J8 MARK SCHEME

Q1.

Question number	Scheme	Marks	
	(a) $m=\frac{4-(-3)}{-6-8}$ or $\frac{-3-4}{8-(-6)}$, $\quad = \frac{7}{-14}$ or $\frac{-7}{14}$ $\quad \left(=-\frac{1}{2}\right)$	M1, A1	
	Equation: $y-4=-\frac{1}{2}(x-(-6))$ or $y-(-3)=-\frac{1}{2}(x-8)$	M1	
	$x+2y-2=0$ (or equiv. with <u>integer</u> coefficients… must have '= 0')	A1	(4)
	(e.g. $14y+7x-14=0$ and $14-7x-14y=0$ are acceptable)		
	(b) $(-6-8)^2+(4-(-3))^2$	M1	
	14^2+7^2 or $(-14)^2+7^2$ or $14^2+(-7)^2$ (M1 A1 may be implied by 245)	A1	
	$AB=\sqrt{14^2+7^2}$ or $\sqrt{7^2(2^2+1^2)}$ or $\sqrt{245}$		
	$7\sqrt{5}$	A1cso	(3)
			7
	(a) 1st M: Attempt to use $m=\frac{y_2-y_1}{x_2-x_1}$ (may be implicit in an equation of L). 2nd M: Attempting straight line equation in any form, e.g $y-y_1=m(x-x_1)$, $\frac{y-y_1}{x-x_1}=m$, with any value of m (except 0 or ∞) and either (–6, 4) or (8, –3). N.B. It is also possible to use a different point which lies on the line, such as the midpoint of AB (1, 0.5). Alternatively, the 2nd M may be scored by using $y=mx+c$ with a numerical gradient and substituting (–6, 4) or (8, –3) to find the value of c. Having coords the <u>wrong way round</u>, e.g. $y-(-6)=-\frac{1}{2}(x-4)$, loses the 2nd M mark <u>unless</u> a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$. (b) M: Attempting to use $(x_2-x_1)^2+(y_2-y_1)^2$. <u>Missing bracket</u>, e.g. -14^2+7^2 implies M1 if no earlier version is seen. -14^2+7^2 with no further work would be M1 A0. -14^2+7^2 followed by 'recovery' can score full marks.		

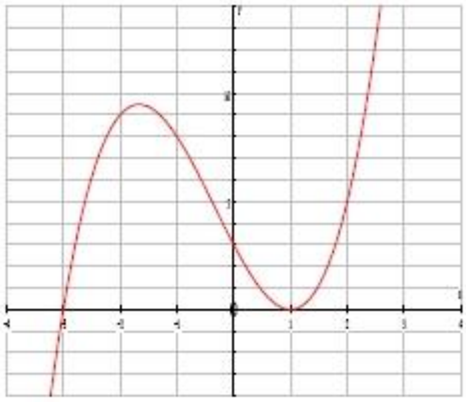
Q2.

Question Number	Scheme	Marks
(a)	$(x-6)^2+(y-4)^2=;\ 3^2$	B1; B1 (2)
(b)	Complete method for MP: $=\sqrt{(12-6)^2+(6-4)^2}$	M1
	$=\sqrt{40}$ or awrt 6.325	A1
	[These first two marks can be scored if seen as part of solution for (c)]	
	Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$ e.g. $\cos\theta=\frac{MT}{MP}=\frac{3}{candidate's\sqrt{40}}$ $(=0.4743)$ $(\theta=61.6835^{\circ})$ [If TP = 6 is used, then M0]	M1
	$\theta=1.0766$ rad **AG**	A1 (4)
(c)	Complete method for area TMP; e.g. $=\frac{1}{2}\times3\times\sqrt{40}\sin\theta$	M1
	$=\frac{3}{2}\sqrt{31}$ $(=8.3516..)$ allow awrt 8.35	A1
	Area (sector) MTQ $=0.5\times3^2\times1.0766$ $(=4.8446\ldots)$	M1
	Area TPQ = candidate's $(8.3516..-4.8446..)$	M1
	$=3.507$ awrt [Note: 3.51 is A0]	A1 (5) **[11]**
Notes	(a) Allow 9 for 3^2. (b) First M1 can be implied by $\sqrt{40}$ or $\sqrt{31}$ For second M1: May find TP = $\sqrt{(\sqrt{40})^2-3^2}=\sqrt{31}$, then either $\sin\theta=\frac{TP}{MP}=\frac{\sqrt{31}}{\sqrt{40}}$ $(=0.8803\ldots)$ *or* $\tan\theta=\frac{\sqrt{31}}{3}$ $(1.8859..)$ or cos rule **NB. Answer is given, but allow final A1 if all previous work is correct.** (c) First M1: (alternative) $\frac{1}{2}\times3\times\sqrt{40-9}$ Second M1: allow even if candidate's value of θ used. (Despite being given !)	

Q3.

Question Number	Scheme	Marks
(a)	(Total area) = $3xy + 2x^2$	B1
	(Vol:) $x^2y = 100$ $\left(y = \dfrac{100}{x^2}, xy = \dfrac{100}{x}\right)$	B1
(b)	Deriving expression for area in terms of x only	M1
	(Substitution, or clear use of, y or xy into expression for area)	
	(Area =) $\dfrac{300}{x} + 2x^2$ **AG**	A1 cso (4)
(c)	$\dfrac{dA}{dx} = -\dfrac{300}{x^2} + 4x$	M1A1
	Setting $\dfrac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1	
	[$x^3 = 75$]	
	$x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)	A1 (4)
	$\dfrac{d^2A}{dx^2} = \dfrac{600}{x^3} + 4$ = positive , > 0; therefore minimum	M1;A1 (2)
(d)	Substituting found value of x into (a)	M1
	(Or finding y for found x and substituting both in $3xy + 2x^2$)	
	$[y = \dfrac{100}{4.2172^2} = 5.6228]$	
	Area = 106.707 awrt 107	A1 (2)
		[12]
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (b) First M1: At least one power of x decreased by 1, and no "c" term. (c) For M1: Find $\dfrac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive" A1: Candidate's $\dfrac{d^2A}{dx^2}$ must be correct for their $\dfrac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found) Alternative: M1: Find value of $\dfrac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign A1: Indicate sign change of negative to positive for $\dfrac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with"107" A1: Both values greater than "$x = 107$" and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused. Throughout , allow confused notation, such as dy/dx for dA/dx.	

Q4.

Question number	Scheme	Marks	
	(a) 		
	Shape (drawn anywhere)	B1	
	Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1	
	$(-3, 0)$ (or -3 shown on $-$ve x-axis)	B1	
	(0, 3) (or 3 shown on +ve y-axis)	B1	(4)
	N.B. The max. can be anywhere.		
	(b) $y = (x+3)(x^2 - 2x + 1)$	M1	
	$= x^3 + x^2 - 5x + 3$ $\quad (k = 3)$ $\quad$ Marks can be awarded if this is seen in part (a)	A1cso	(2)
	(c) $\frac{dy}{dx} = 3x^2 + 2x - 5$	M1 A1	
	$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$	M1	
	$(3x - 4)(x + 2) = 0 \quad x = \ldots$	M1	
	$x = \frac{4}{3}$ (or exact equiv.) , $x = -2$	A1, A1	(6)
			12

(a) The individual marks are independent, <u>but</u> the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted.

B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch.

(b) M: Attempt to multiply out $(x-1)^2$ and write as a product with $(x+3)$, or attempt to multiply out $(x+3)(x-1)$ and write as a product with $(x-1)$, or attempt to expand $(x+3)(x-1)(x-1)$ directly (at least 7 terms).

The $(x-1)^2$ or $(x+3)(x-1)$ expansion must have 3 (or 4) terms, so should not, for example, be just $x^2 + 1$.

A: It is not necessary to state explicitly '$k = 3$'.
Condone missing brackets if the intention seems clear and a fully correct expansion is seen.

(c) 1st M: Attempt to differentiate (correct power of x in at least one term).

2nd M: Setting their derivative equal to 3.

3rd M: Attempt to solve a 3-term quadratic based on their derivative.

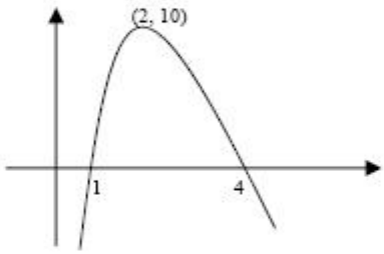
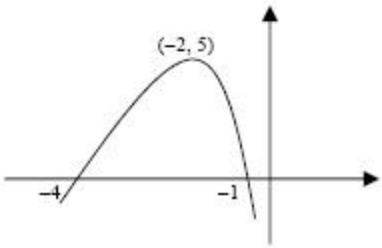
The equation <u>could</u> come from $\frac{dy}{dx} = 0$.

N.B. After an incorrect k value in (b), full marks are still possible in (c).

Q5.

Question Number	Scheme	Marks
	Method 1 (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of $3b$ for a (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on **correctly derived** $\log_3 p = q$ e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{½}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	Method 2 (Working with two equations in $\log_3 a$ and $\log_3 b$)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1½$, $\log_3 b = ½$]	M1
	Using base correctly to find a or b	M1
	Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1 **[6]**
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on **Method 1**, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of $a = 3b$, (iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$ *Three examples of applying first 4 marks in Method 1:* (i) $\log_3 3b + \log_3 b = 2$ gains second M1 $\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1 $(2\log_3 b = 1,\ \log_3 b = ½)$ no mark yet $b = 3^{½}$ gains third M1, and if correct A1 (ii) $\log_3 (ab) = 2$ gains first M1 $ab = 3^2$ gains third M1 $3b^2 = 3^2$ gains second M1 (iii) $\log_3 3b^2 = 2$ has gained first 2 M marks $\Rightarrow 2\log_3 3b = 2$ or similar type of error $\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly	

Q6.

Question number	Scheme		Marks
	(a) 	Shape: Max in 1st quadrant and 2 intersections on positive x-axis	B1
		1 and 4 labelled (in correct place) or clearly stated as coordinates	B1
		(2, 10) labelled or clearly stated	B1 (3)
	(b) 	Shape: Max in 2nd quadrant and 2 intersections on negative x-axis	B1
		−1 and −4 labelled (in correct place) or clearly stated as coordinates	B1
		(−2, 5) labelled or clearly stated	B1 (3)
	(c) $(a=)\ 2$	May be implicit, i.e. $\mathrm{f}(x+2)$	B1 (1)
	Beware: The answer to part (c) may be seen on the first page.		7
	(a) and (b): 1st B: 'Shape' is generous, providing the conditions are satisfied. 2nd and 3rd B marks are dependent upon a sketch having been drawn. 2nd B marks: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> the sketch is correct. Points must be labelled correctly and be in appropriate place (e.g. (−2, 5) in the first quadrant is B0). (b) <u>Special case</u>: If the graph is reflected in the x-axis (instead of the y-axis), B1 B0 B0 can be scored. This requires shape and coordinates to be <u>fully correct</u>, i.e. Shape:  Minimum in 4th quadrant and 2 intersections on positive x-axis, 1 and 4 labelled (in correct place) or clearly stated as coordinates, (2, −5) labelled or clearly stated.		

Q7.

Question Number	Scheme	Marks
(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$)	B1 (1)
	or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	
(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = ...	M1
	$x = 4$ (and $x = 0$)	A1
	Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	A1 (3)
(c)	(Area =) $\int_{(0)}^{(4)} (6x - x^2)\,dx$ Limits not required	M1
	Correct integration $3x^2 - \frac{x^3}{3}$ (+ c)	A1
	Correct use of correct limits on their result above (see notes on limits)	M1
	["$3x^2 - \frac{x^3}{3}$"]^4 − ["$3x^2 - \frac{x^3}{3}$"]$_0$ with limits substituted [$= 48 - 21\frac{1}{3} = 26\frac{2}{3}$]	
	Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1)	A1
	Shaded area = ± (area under curve − area of triangle) applied correctly	M1
	$(= 26\frac{2}{3} - 16)$ $= 10\frac{2}{3}$ (awrt 10.7)	A1 (6)**[10]**

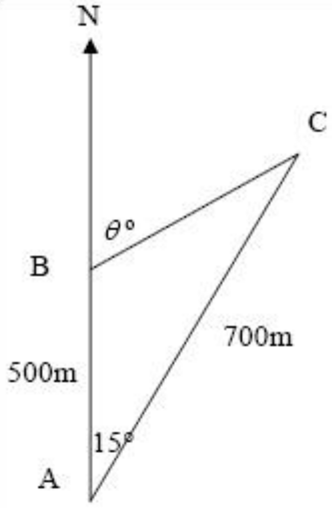
Notes	(b) In scheme first A1: need only give $x = 4$ If *verifying approach* used: Verifying (4,8) satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, (0,0) needs to be mentioned ; accept " clear from diagram" (c) **Alternative** Using Area $= \pm \int_{(0)}^{(4)} \{(6x - x^2); \quad -2x\}\,dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with $= \pm \int_{(0)}^{(4)} (4x - x^2)\,dx$, **first** M mark and **third** M mark $= (\pm)\,[\,2x^2 - \frac{x^3}{3}\ (+c)\ ]$ A1, Correct use of correct limits on their result **second** M1, Totally correct, unsimplified ± expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given - $10\frac{2}{3}$, they correct it] M1 *for correct use of correct limits*: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$ If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. **Final M1: limits for area under curve and triangle must be the same.** S.C.(1) $\int_0^6 (6x - x^2)dx - \int_0^6 2x\,dx = \left[3x^2 - \frac{x^3}{3}\right]_0^6 - \left[x^2\right]_0^6 = ...$ award M1A1 MO(limits)AO(triangle)M1(bod)A0 (2) If, having found ± correct answer, thinks this is not complete strategy and does more, do not award final 2 A marks Use of trapezium rule: M0A0MA0possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

Q8.

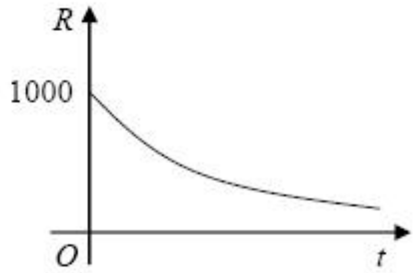
Question Number	Scheme	Marks
(a)	$3\sin^2\theta - 2\cos^2\theta = 1$	
	$3\sin^2\theta - 2(1 - \sin^2\theta) = 1$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$)	M1
	$3\sin^2\theta - 2 + 2\sin^2\theta = 1$	
	$5\sin^2\theta = 3$ cso AG	A1 (2)
(b)	$\sin^2\theta = \frac{3}{5}$, so $\sin\theta = (\pm)\sqrt{0.6}$	M1
	Attempt to solve both $\sin\theta$ = +.. and $\sin\theta$ = - ...(may be implied by later work)	M1
	$\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first M1 only)	A1
	θ (= 180° – 50.7685$_c$°); = 129.23...° awrt 129.2°	M1; A1 √
	[f.t. dependent on first M and 3rd M]	
	$\sin\theta = -\sqrt{0.6}$	
	$\theta = 230.785°$ and $309.23152°$ awrt 230.8°, 309.2° (both)	M1A1 (7)
		[9]

Notes:	(a) N.B: **AG**; need to see at least one line of working after substituting $\cos^2\theta$ (b) First M1: Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or θ [Allow such results as $\sin\theta = \frac{3}{5}$, $\sin\theta = \frac{\sqrt{3}}{5}$ .....for M1] Second M1: Considering the – value for $\sin\theta$. (usually later) First A1: Given for awrt 50.8°. **Not** dependent on **second** M. Third M1: For (180 – **candidate' s** 50.8)°, need not see written down Final M1: **Dependent** on **second** M (but **may be implied by answers)** For (180 + candidate' s 50.8)° **or** (360 – candidate' s 50.8)° **or equiv** Final A1: Requires both values. (**no follow through**) [Finds $\cos^2\theta = k$ (k = 2/5) and so $\cos\theta = (\pm)$...M1, then mark equivalently] **NB** Candidates who **only consider positive value for sin** θ can score max of 4 marks: M1M0A1M1A1M0A0 – Very common. Candidates **who score first M1 but have wrong sin** θ can score maximum M1M1A0M1A√ M1A0 **SC** Candidates who obtain one value from each set, e.g 50.8 and 309.2 M1M1(bod)A1M0A0M1(bod)A0 Extra values out of range – no penalty **Any very tricky or " outside scheme methods" , send to TL**	

Q9.

Question Number	Scheme	Marks
		
(a)	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ $(= \quad 63851.92\ldots)$ $BC = 253$ awrt	M1 A1 A1 (3)
(b)	$\dfrac{\sin B}{700} = \dfrac{\sin 15}{\text{candidate's } BC}$ $\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep on 1st M $\theta = 180° -$ candidate's angle B (Dep. on first M only, B can be acute) $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9)	M1 M1 M1 A1 (4) [7]
Notes:	(a) If use cos 15° = ..., then A1 not scored until written as BC^2 = ... correctly *Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC* Finding value for BX and CX and using Pythagoras M1 $BC^2 = (500\sin 15°)^2 + (700 - 500\cos 15°)^2$ A1 $BC = 253$ awrt A1 (b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)) (i) $\cos B = \dfrac{500^2 + \text{candidate's} BC^2 - 700^2}{2 \text{x} 500 \text{x} \text{candidate's} BC}$ or $700^2 = 500^2 + BC_c{}^2 - 2x500xBC_c$ M1 Finding angle B M1 dep., then M1 as above (ii) 2 triangle approach, as defined in notes for (a) $\tan CBX = \dfrac{700 - valueforAX}{valueforBX}$ M1 Finding value for $\angle CBX$ (≈ 59 °) dep M1 $\theta = [180° - (75° + candidate's\ \angle CBX)]$ M1 (iii) Using sine rule (or cos rule) to find C first: Correct use of sine or cos rule for C M1, Finding value for C M1 Either $B = 180° - (15° +$ candidate's $C)$ or $\theta = (15° +$ candidate's $C)$ M1 (iv) $700\cos 15° = 500 + BC\cos\theta$ M2 {first two Ms earned in this case} Solving for θ; $\theta = 45.8$ (allow 46 or5.7, 45.8, 45.9) M1;A1 Note: S.C. In main scheme, if θ used in place of B, third M gained immediately; Other two marks likely to be earned, too, for correct value of θ stated.	

Q10.

Question Number	Scheme		Marks	
	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	(2) **[9]**

Q11.

Q	Scheme	Marks
	States or implies the formula for differentiation from first principles. $\mathrm{f}(x) = 3x^3$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	
	Correctly applies the formula to the specific formula and expands and simplifies the formula. $\mathrm{f}'(x) = \lim_{h \to 0} \frac{3(x+h)^3 - 3x^3}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{3\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - 3x^3}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{9x^2h + 9xh^2 + 3h^3}{h}$	**M1**
	Factorises the '*h*' out of the numerator and then divides by *h* to simplify. $\mathrm{f}'(x) = \lim_{h \to 0} \frac{h\left(9x^2 + 9xh + 3h^2\right)}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} \left(9x^2 + 9xh + 3h^2\right)$	**A1**
	States that as $h \to 0$, $9x^2 + 9xh + 3h^2 \to 9x^2$ o.e. so derivative = $9x^2$ *	**A1***

Q12.

Q	Scheme	Marks
(a)	Gradient = $\frac{4.55-2.3}{12-.0} = 0.1875$ Creating log *P* = log *a*+ *t* log *b* Sub 2.3 and 0.1875 in above equation Equation of line: log P = 0.1875 t + 2.3 (accept 0.19 as gradient)	 M1 A1
(b)	$\log a = 2.3$ $a = 10^{2.3}$ = 199.526… = 200 (3 S.F) $\log b = 0.1875$ $b = 10^{0.1875}$ = 1.5399… = 1.54 (3 S.F)	M1 A1 M1 A1
(c)	The initial population of bacteria	A1
(d)	Sub $t = 20, a = 200$, $b = 1.54$ in $P = ab^t$ $= 200(1.54)^{20}$ = 1125756.368 = 1126000 (nearest 1000)	M1 A1