

# Pure Mathematics 1 Practice Paper J9 **MARK SCHEME**

## Question 1

Question Number	Scheme	Marks
(a)	$b^2 - 4ac > 0 \Rightarrow 16 - 4k(5 - k) > 0$ or equiv., e.g. $16 > 4k(5 - k)$ So $k^2 - 5k + 4 > 0$ (Allow any order of terms, e.g. $4 - 5k + k^2 > 0$ ) (*)	M1A1 A1cso (3)
(b)	<u>Critical Values</u> $(k - 4)(k - 1) = 0$ $k = \dots$ $k = 1$ or $4$ Choosing "outside" region $k < 1$ or $k > 4$	M1 A1 M1 A1 (4) [7]
For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.		
(a)	M1 for attempting to use the discriminant of the initial equation ( $> 0$ not required, but substitution of $a$ , $b$ and $c$ in the correct formula is required). If the formula $b^2 - 4ac$ is seen, at least 2 of $a$ , $b$ and $c$ must be correct. If the formula $b^2 - 4ac$ is <u>not</u> seen, all 3 ( $a$ , $b$ and $c$ ) must be correct. This mark can still be scored if substitution in $b^2 - 4ac$ is within the quadratic formula. This mark can also be scored by comparing $b^2$ and $4ac$ (with substitution). However, use of $b^2 + 4ac$ is M0. 1 <sup>st</sup> A1 for fully correct expression, possibly unsimplified, with $>$ symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2 - 4ac > 0$ or 'discriminant positive'. Condone a bracketing slip, e.g. $16 - 4 \times k \times 5 - k$ if subsequent work is correct and convincing. 2 <sup>nd</sup> A1 for a fully correct derivation with no incorrect working seen. Condone a bracketing slip if otherwise correct and convincing. Using $\sqrt{b^2 - 4ac} > 0$ : Only available mark is the first M1 (unless recovery is seen).	
(b)	1 <sup>st</sup> M1 for attempt to solve an appropriate 3TQ 1 <sup>st</sup> A1 for both $k = 1$ and $4$ (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$ ). ** 2 <sup>nd</sup> M1 for choosing the "outside" region. A diagram or table alone is not sufficient. Follow through their values of $k$ . The set of values must be 'narrowed down' to score this M mark... listing everything $k < 1$ , $1 < k < 4$ , $k > 4$ is M0. 2 <sup>nd</sup> A1 for correct answer only, condone " $k < 1$ , $k > 4$ " and even " $k < 1$ and $k > 4$ ", but " $1 > k > 4$ " is A0. ** Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks. Seeing 1 and 4 used as critical values gives the first M1 A1 by implication. In part (b), condone working with $x$ 's except for the final mark, where the set of values must be a set of values of $k$ (i.e. 3 marks out of 4). Use of $\leq$ (or $\geq$ ) in the final answer loses the final mark.	

## Question 2.

Question Number	Scheme	Marks
(a)	$2x^{3/2}$ or $p = \frac{3}{2}$ (Not $2x\sqrt{x}$ )	B1
(b)	$-x$ or $-x^1$ or $q = 1$ $\left(\frac{dy}{dx} = \right) 20x^3 + 2 \times \frac{3}{2} x^{1/2} - 1$ $= 20x^3 + 3x^{1/2} - 1$	B1 (2) M1 A1A1ftA1ft (4) [6]
(a)	1 <sup>st</sup> B1 for $p = 1.5$ or exact equivalent 2 <sup>nd</sup> B1 for $q = 1$	
(b)	M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ (for any of the 4 terms) 1 <sup>st</sup> A1 for $20x^3$ (the $-3$ must 'disappear') 2 <sup>nd</sup> A1ft for $3x^{1/2}$ or $3\sqrt{x}$ . Follow through their $p$ but they must be differentiating $2x^p$ , where $p$ is a <u>fraction</u> , and the coefficient must be simplified if necessary. 3 <sup>rd</sup> A1ft for $-1$ (not the unsimplified $-x^0$ ), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of $x^q$ is $-1$ ). If ft is applied, the coefficient must be simplified if necessary.  'Simplified' coefficient means $\frac{a}{b}$ where $a$ and $b$ are integers with no common factors. Only a single $+$ or $-$ sign is allowed (e.g. $--$ must be replaced by $+$ ).  If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b).  <u>Multiplying by <math>\sqrt{x}</math></u> : (assuming this is a restart) e.g. $y = 5x^4\sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$ $\left(\frac{dy}{dx} = \right) \frac{45}{2}x^{7/2} - \frac{3}{2}x^{-1/2} + 4x - \frac{3}{2}x^{1/2}$ scores M1 A0 A0 ( $p$ not a fraction) A1ft.  <u>Extra term included</u> : This invalidates the final mark. e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$ $\left(\frac{dy}{dx} = \right) 20x^3 + 4x - \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ scores M1 A1 A0 ( $p$ not a fraction) A0.  <u>Numerator and denominator differentiated separately</u> : For this, neither of the last two (ft) marks should be awarded.  <u>Quotient/product rule</u> : Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)	



### Question 3.

Question Number	Scheme	Marks
(a)	$PQ: m_1 = \frac{10-2}{9-(-3)} (= \frac{2}{3})$ and $QR: m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1: \frac{8}{12} \times \frac{6}{9-a} = -1 \quad a = 13 \quad (*)$	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds all three of the following $(9-(-3))^2 + (10-2)^2$ , (i.e.208) , $(9-a)^2 + (10-4)^2$ , $(a-(-3))^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation Solve (or verify) for $a$ , $a = 13 (*)$	M1 A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =) (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =) (13-(-3))^2 + (4-2)^2$ $(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown Eliminates second unknown	M1 M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, r^2 = 65$	A1, A1, B1cao (5) [8]

#### Notes

- (a) M1-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method)  
M1 Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round )  
A1 Obtains  $a = 13$  with no errors by solution or verification. Verification can score 3/3.
- (b) Geometrical method: B1 for coordinates of centre – can be implied by use in part (b)  
M1 for attempt to find  $r^2, d^2, r$  or  $d$  ( allow one slip in a bracket).  
A1 cao. These two marks may be gained implicitly from circle equation  
M1 for  $(x \pm 5)^2 + (y \pm 3)^2 = k^2$  or  $(x \pm 3)^2 + (y \pm 5)^2 = k^2$  ft their (5,3) Allow  $k^2$  non numerical.  
A1 cao for whole equation and rhs must be 65 or  $(\sqrt{65})^2$ , (similarly B1 must be 65 or  $(\sqrt{65})^2$ , in alternative method for (b))

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is -3/2 This is M1 They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ M1 (may still have x in this equation rather than a and there may be a small slip) They then complete to give (a)= 13 A1	M1 M1 A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ M1 (perpendicular from centre to chord bisects chord) Then using $c (= 5)$ to find a is M1 Finally $a = 13$ A1	M1 M1 M1 A1
	(iii) Vector Method: States PQ. QR = 0, with vectors stated $12i + 8j$ and $(9-a)i + 6j$ is M1 Evaluates scalar product so $108 - 12a + 48 = 0$ (M1) solves to give $a = 13$ (A1)	M1 M1 A1



# Question 4.

Question Number	Scheme	Marks
(a)	$y - 5 = -\frac{1}{2}(x - 2)$ or equivalent, e.g. $\frac{y - 5}{x - 2} = -\frac{1}{2}$ , $y = -\frac{1}{2}x + 6$	M1A1, A1cao (3)
(b)	$x = -2 \Rightarrow y = -\frac{1}{2}(-2) + 6 = 7$ (therefore $B$ lies on the line) (or equivalent verification methods)	B1 (1)
(c)	$(AB^2 =) (2 - -2)^2 + (7 - 5)^2$ , $= 16 + 4 = 20$ , $AB = \sqrt{20} = 2\sqrt{5}$	M1, A1, A1 (3)
(d)	$C$ is $(p, -\frac{1}{2}p + 6)$ , so $AC^2 = (p - 2)^2 + \left(-\frac{1}{2}p + 6 - 5\right)^2$ Therefore $25 = p^2 - 4p + 4 + \frac{1}{4}p^2 - p + 1$ $25 = 1.25p^2 - 5p + 5$ or $100 = 5p^2 - 20p + 20$ (or better, RHS simplified to 3 terms) Leading to: $0 = p^2 - 4p - 16$ (*)	M1 M1 A1 A1cso (4) [11]
(a)	M1 A1 The version in the scheme above can be written down directly (for 2 marks), and M1 A0 can be allowed if there is just one slip (sign or number). If the 5 and 2 are the wrong way round the M mark can still be given if a correct formula (e.g. $y - y_1 = m(x - x_1)$ ) is seen, otherwise M0. If $(2, 5)$ is substituted into $y = mx + c$ to find $c$ , the M mark is for attempting this and the 1 <sup>st</sup> A mark is for $c = 6$ . Correct answer without working or from a sketch scores full marks.	
(b)	A conclusion/comment is not required, except when the method used is to establish that the line through $(-2, 7)$ with gradient $-\frac{1}{2}$ has the same eqn. as found in part (a), or to establish that the line through $(-2, 7)$ and $(2, 5)$ has gradient $-\frac{1}{2}$ . In these cases a comment 'same equation' or 'same gradient' or 'therefore on same line' is sufficient.	
(c)	M1 for attempting $AB^2$ or $AB$ . Allow one slip (sign or number) <u>inside</u> a bracket, i.e. do <u>not</u> allow $(2 - -2)^2 - (7 - 5)^2$ . 1 <sup>st</sup> A1 for 20 (condone bracketing slips such as $-2^2 = 4$ ) 2 <sup>nd</sup> A1 for $2\sqrt{5}$ or $k = 2$ (Ignore $\pm$ here).	
(d)	1 <sup>st</sup> M1 for $(p - 2)^2 + (\text{linear function of } p)^2$ . The linear function may be unsimplified but must be equivalent to $ap + b$ , $a \neq 0$ , $b \neq 0$ . 2 <sup>nd</sup> M1 (dependent on 1 <sup>st</sup> M) for forming an equation in $p$ (using 25 or 5) and attempting (perhaps not very well) to multiply out both brackets. 1 <sup>st</sup> A1 for collecting like $p$ terms and having a correct expression. 2 <sup>nd</sup> A1 for correct work leading to printed answer. <u>Alternative, using the result:</u> Solve the quadratic $(p = 2 \pm 2\sqrt{5})$ and use one or both of the two solutions to find the length of $AC^2$ or $C_1C_2^2$ : e.g. $AC^2 = (2 + 2\sqrt{5} - 2)^2 + (5 - \sqrt{5} - 5)^2$ scores 1 <sup>st</sup> M1, and 1 <sup>st</sup> A1 if fully correct. Finding the length of $AC$ or $AC^2$ for both values of $p$ , or finding $C_1C_2$ with some evidence of halving (or intending to halve) scores the 2 <sup>nd</sup> M1. Getting $AC = 5$ for both values of $p$ , or showing $\frac{1}{2}C_1C_2 = 5$ scores the 2 <sup>nd</sup> A1 (cso).	

# Question 5.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}\right) = -4 + 8x^{-2}$ (4 or $8x^{-2}$ for M1... sign can be wrong) $x = 2 \Rightarrow m = -4 + 2 = -2$ $y = 9 - 8 - \frac{8}{2} = -3$ The first 4 marks <u>could</u> be earned in part (b) Equation of tangent is: $y + 3 = -2(x - 2) \rightarrow y = 1 - 2x$ (*)	M1A1 M1 B1 M1 A1cso (6)
(b)	Gradient of normal = $\frac{1}{2}$ Equation is: $\frac{y+3}{x-2} = \frac{1}{2}$ or better equivalent, e.g. $y = \frac{1}{2}x - 4$	B1ft M1A1
(c)	(A:) $\frac{1}{2}$ , (B:) 8 Area of triangle is: $\frac{1}{2}(x_B \pm x_A) \times y_P$ with values for all of $x_B, x_A$ and $y_P$ $\frac{1}{2}\left(8 - \frac{1}{2}\right) \times 3 = \frac{45}{4}$ or 11.25	(3) B1, B1 M1 A1 (4) [13]
(a)	1 <sup>st</sup> M1 for 4 or $8x^{-2}$ (ignore the signs). 1 <sup>st</sup> A1 for both terms correct (including signs). 2 <sup>nd</sup> M1 for substituting $x = 2$ into their $\frac{dy}{dx}$ (must be different from their $y$ ) B1 for $y_P = -3$ , but not if clearly found from the given equation of the <u>tangent</u> . 3 <sup>rd</sup> M1 for attempt to find the equation of tangent at $P$ , follow through their $m$ and $y_P$ . Apply general principles for straight line equations (see end of scheme). <u>NO DIFFERENTIATION ATTEMPTED</u> : Just assuming $m = -2$ at this stage is M0 2 <sup>nd</sup> A1cso for correct work leading to printed answer (allow equivalents with $2x, y$ , and 1 terms... such as $2x + y - 1 = 0$ ).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their $m$ , but if $m \neq -2$ there must be clear evidence that the $m$ is thought to be the gradient of the tangent. M1 for an attempt to find normal at $P$ using their changed gradient and their $y_P$ . Apply general principles for straight line equations (see end of scheme). A1 for any correct form as specified above (correct answer only).	
(c)	1 <sup>st</sup> B1 for $\frac{1}{2}$ and 2 <sup>nd</sup> B1 for 8. M1 for a full method for the area of triangle $ABP$ . Follow through their $x_A, x_B$ and their $y_P$ , but the mark is to be awarded 'generously', condoning sign errors.. The final answer must be positive for A1, with negatives in the working condoned. Determinant: Area = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \dots$ (Attempt to multiply out required for M1) Alternative: $AP = \sqrt{(2 - 0.5)^2 + (-3)^2}$ , $BP = \sqrt{(2 - 8)^2 + (-3)^2}$ , Area = $\frac{1}{2} AP \times BP = \dots$ M1 Intersections with $y$ -axis instead of $x$ -axis: Only the M mark is available B0 B0 M1 A0.	





# Question 6.

Question Number	Scheme	Marks
(a)	$2\pi rh + 2\pi r^2 = 800$ $h = \frac{400 - \pi r^2}{\pi r}, \quad V = \pi r^2 \left( \frac{400 - \pi r^2}{\pi r} \right) = 400r - \pi r^3 \quad (*)$	B1 M1, M1 A1 (4)
(b)	$\frac{dV}{dr} = 400 - 3\pi r^2$ $400 - 3\pi r^2 = 0 \quad r^2 = \dots, \quad r = \sqrt{\frac{400}{3\pi}} \quad (= 6.5 \text{ (2 s.f.)})$ $V = 400r - \pi r^3 = 1737 = \frac{800}{3} \sqrt{\frac{400}{3\pi}} \text{ (cm}^3\text{)}$ <p>(accept awrt 1737 or exact answer)</p>	M1 A1 M1 A1 (6)
(c)	$\frac{d^2V}{dr^2} = -6\pi r, \text{ Negative, } \therefore \text{maximum}$ <p>(Parts (b) and (c) should be considered together when marking)</p>	M1 A1 (2) [12]
Other methods for part (c):	<p><u>Either</u> M: Find <u>value</u> of <math>\frac{dV}{dr}</math> on each side of "<math>r = \sqrt{\frac{400}{3\pi}}</math>" and consider sign.</p> <p>A: Indicate sign change of positive to negative for <math>\frac{dV}{dr}</math>, and conclude max.</p> <p><u>Or</u> M: Find <u>value</u> of <math>V</math> on each side of "<math>r = \sqrt{\frac{400}{3\pi}}</math>" and compare with "1737".</p> <p>A: Indicate that both values are less than 1737 or 1737.25, and conclude max.</p>	
Notes	<p>(a) B1: For any correct form of this equation (may be unsimplified, may be implied by 1<sup>st</sup> M1)</p> <p>M1: Making <math>h</math> the subject of their three or four term formula</p> <p>M1: Substituting expression for <math>h</math> into <math>\pi r^2 h</math> (independent mark) Must now be expression in <math>r</math> only.</p> <p>A1: cso</p> <p>(b) M1: At least one power of <math>r</math> decreased by 1 A1: cao</p> <p>M1: Setting <math>\frac{dV}{dr} = 0</math> and finding a value for correct power of <math>r</math> for candidate</p> <p>A1: This mark may be credited if the value of <math>V</math> is correct. Otherwise answers should round to 6.5 (allow <math>\pm 6.5</math>) or be exact answer</p> <p>M1: Substitute a positive value of <math>r</math> to give <math>V</math> A1: 1737 or 1737.25..... or exact answer</p>	
Alternative for (a)	<p>(c) M1: needs complete method e.g.attempts differentiation (power reduced) of their first derivative and considers its sign</p> <p>A1(first method) should be <math>-6\pi r</math> (do not need to substitute <math>r</math> and can condone wrong <math>r</math> if found in (b))</p> <p>Need to conclude maximum or indicate by a tick that it is maximum.</p> <p>Throughout allow confused notation such as <math>dy/dx</math> for <math>dV/dr</math></p> <p><math>A = 2\pi r^2 + 2\pi rh, \quad \frac{A}{2} \times r = \pi r^3 + \pi r^2 h</math> is M1 Equate to <math>400r</math> B1</p> <p>Then <math>V = 400r - \pi r^3</math> is M1 A1</p>	

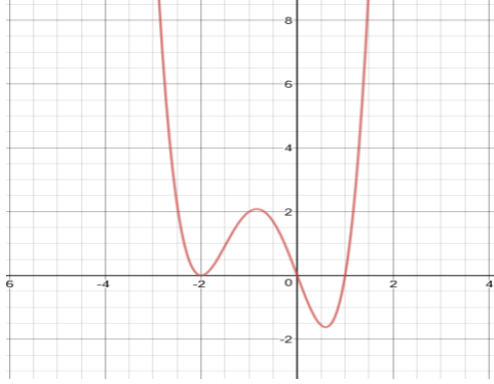


### Question 7.

Question Number	Scheme	Marks
	$2\log_5 x = \log_5(x^2), \quad \log_5(4-x) - \log_5(x^2) = \log_5 \frac{4-x}{x^2}$ $\log\left(\frac{4-x}{x^2}\right) = \log 5 \quad 5x^2 + x - 4 = 0 \text{ or } 5x^2 + x = 4 \text{ o.e.}$ $(5x-4)(x+1) = 0 \quad x = \frac{4}{5} \quad (x = -1)$	B1, M1  M1 A1  dM1 A1 <b>(6)</b> <b>[6]</b>
Notes	<p><b>B1</b> is awarded for <math>2\log x = \log x^2</math> anywhere.</p> <p><b>M1</b> for correct use of <math>\log A - \log B = \log \frac{A}{B}</math></p> <p><b>M1</b> for replacing 1 by <math>\log_k k</math>. <b>A1</b> for correct quadratic</p> <p><math>(\log(4-x) - \log x^2 = \log 5 \Rightarrow 4-x-x^2 = 5 \text{ is B1M0M1A0 M0A0})</math></p> <p><b>dM1</b> for attempt to solve quadratic with usual conventions. (Only award if previous two <b>M</b> marks have been awarded)</p> <p><b>A1</b> for 4/5 or 0.8 or equivalent (Ignore extra answer).</p>	
Alternative 1	$\log_5(4-x) - 1 = 2\log_5 x \text{ so } \log_5(4-x) - \log_5 5 = 2\log_5 x$ $\log_5 \frac{4-x}{5} = 2\log_5 x$ then could complete solution with $2\log_5 x = \log_5(x^2)$ $\left(\frac{4-x}{5}\right) = x^2 \quad 5x^2 + x - 4 = 0$ Then as in first method $(5x-4)(x+1) = 0 \quad x = \frac{4}{5} \quad (x = -1)$	M1  M1  B1  A1  dM1 A1 <b>(6)</b> <b>[6]</b>
Special cases	<p>Complete trial and error yielding 0.8 is <b>M3</b> and <b>B1</b> for 0.8</p> <p><b>A1, A1</b> awarded for each of two tries evaluated. i.e. 6/6</p> <p>Incomplete trial and error with wrong or no solution is 0/6</p> <p>Just answer 0.8 with no working is <b>B1</b></p> <p>If log base 10 or base e used throughout - can score <b>B1M1M1A0M1A0</b></p>	



### Question 8.

Q	Scheme	Marks
(a)	 <p>B1 for correct shape</p> <p>B1 for both (-2,0) and (1,0) marked on x-axis</p> <p>B1 for (0,0) seen</p>	(3)
(b)	<p>a = 1</p> <p>a = 3</p> <p>a = 4</p>	<p>A1</p> <p>A1</p> <p>A1</p> <p>(3)</p>

### Question 9.

Question Number	Scheme	Marks
	$(f(x) = \frac{3x^3}{3} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 7x(+c)$ $= x^3 - 2x^{\frac{3}{2}} - 7x(+c)$ $f(4) = 22 \Rightarrow 22 = 64 - 16 - 28 + c$ $c = 2$	<p>M1</p> <p>A1A1</p> <p>M1</p> <p>A1cso (5)</p> <p>[5]</p>
	<p>1<sup>st</sup> M1 for an attempt to integrate (<math>x^3</math> or <math>x^{\frac{3}{2}}</math> seen). The <math>x</math> term is insufficient for this mark and similarly the <math>+c</math> is insufficient.</p> <p>1<sup>st</sup> A1 for <math>\frac{3}{3}x^3</math> or <math>-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}</math> (An unsimplified or simplified correct form)</p> <p>2<sup>nd</sup> A1 for all three <math>x</math> terms correct and simplified... (the simplification may be seen later). The <math>+c</math> is not required for this mark.</p> <p>Allow <math>-7x^1</math>, but <u>not</u> <math>-\frac{7x^1}{1}</math>.</p> <p>2<sup>nd</sup> M1 for an attempt to use <math>x = 4</math> and <math>y = 22</math> in a changed function (even if differentiated) to form an equation in <math>c</math>.</p> <p>3<sup>rd</sup> A1 for <math>c = 2</math> with no earlier incorrect work (a final expression for <math>f(x)</math> is not required).</p>	



# Question 10.

Question Number	Scheme	Marks
	$y = (1 + x)(4 - x) = 4 + 3x - x^2$ M: Expand, giving 3 (or 4) terms $\int (4 + 3x - x^2) dx = 4x + \frac{3x^2}{2} - \frac{x^3}{3}$ M: Attempt to integrate $= [\dots\dots\dots]_{-1}^4 = \left(16 + 24 - \frac{64}{3}\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right) = \frac{125}{6} \quad \left(= 20\frac{5}{6}\right)$	M1 M1 A1 M1 A1      (5) [5]
Notes	<p>M1 needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. <math>1 \times 4 = 5</math>, but there needs to be a 'constant' an 'x term' and an '<math>x^2</math> term'. The x terms do not need to be collected. (Need not be seen if next line correct)</p> <p>Attempt to integrate means that <math>x^n \rightarrow x^{n+1}</math> for at least one of the terms, then M1 is awarded ( even 4 becoming <math>4x</math> is sufficient) – one correct power sufficient.</p> <p>A1 is for correct answer only, not follow through. But allow <math>2x^2 - \frac{1}{2}x^2</math> or any correct equivalent. Allow + c, and even allow an evaluated extra constant term.</p> <p>M1: Substitute limit 4 and limit -1 into a changed function (must be -1) and indicate subtraction (either way round).</p> <p>A1 must be exact, not 20.83 or similar. If recurring indicated can have the mark.  Negative area, even if subsequently positive loses the A mark.</p>	
Special cases	<p>(i) Uses calculator method: M1 for expansion (if seen) M1 for limits if answer correct, so 0, 1 or 2 marks out of 5 is possible (Most likely M0 M0 A0 M1 A0 )</p> <p>(ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark M1 gained.</p> <p>(iii) Using original method, but then change all signs after expansion is likely to lead to: M1 M1 A0, M1 A0 i.e. 3/5</p>	



# Question 11.

Question Number	Scheme	Marks
(a)	$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$	M1 A1 (2)
(b)	$4 \cos^2 x - 9 \cos x + 2 = 0 (*)$ $(4 \cos x - 1)(\cos x - 2) = 0$ $\cos x = \dots, \frac{1}{4}$ $x = 75.5$ $(\alpha)$ $360 - \alpha, 360 + \alpha$ or $720 - \alpha$ $284.5, 435.5, 644.5$	M1 A1 B1 M1, M1 A1 (6) [8]
(a)	<b>M1:</b> Uses $\sin^2 x = 1 - \cos^2 x$ (may omit bracket) <b>not</b> $\sin^2 x = \cos^2 x - 1$ <b>A1:</b> Obtains the printed answer without error – <b>must have</b> $= 0$	
(b)	<b>M1:</b> Solves the quadratic with usual conventions <b>A1:</b> Obtains $\frac{1}{4}$ accurately- ignore extra answer 2 but penalise e.g. -2. <b>B1:</b> allow answers which round to 75.5 <b>M1:</b> $360 - \alpha$ ft their value, <b>M1:</b> $360 + \alpha$ ft their value or $720 - \alpha$ ft <b>A1:</b> Three <b>and only three</b> correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic $(4 \cos x - 1)(\cos x + 2)$ Could yield, M1A0B1M1M1A1 losing one mark for the error Works in radians: Complete work in radians :Obtains 1.3 B0. Then allow M1 M1 for $2\pi - \alpha, 2\pi + \alpha$ or $4\pi - \alpha$ Then gets 5.0, 7.6, 11.3 A0 so 2/4 Mixed answer 1.3, $360 - 1.3, 360 + 1.3, 720 - 1.3$ still gets B0M1M1A0	



## Question 12

Q	Scheme	Marks
(a)	<p>Starting proof by using the expansion of <math>(a + b)^2</math>  <math>(a + b)^2</math> can also be written as <math>(a - b)^2 + 4ab</math>  since <math>(a + b)^2 = a^2 + 2ab + b^2</math>  <math>= (a - b)^2 + 4ab</math></p> <p>Since <math>(a - b)^2 \geq 0</math> ,  Therefore <math>(a + b)^2 \geq 4ab</math> (since both <math>a</math> and <math>b</math> are positive)</p> <p>Take square root on both sides <math>(a + b) &gt; \sqrt{4ab}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p><b>(3)</b></p>
(b)	<p>If <math>a = -1</math> and <math>b = -1</math> this will give <math>-2 &gt; \sqrt{4}</math> which is not true</p> <p>M1 for using 2 negative values  A1 for showing their values make the inequality false</p>	<p>M1A1</p> <p><b>(2)</b></p>





### Question 13.

Q	Scheme	Marks
(a)	$H(t) = 0$ $15.25 + 17.8t - 4.5t^2 = 0$ $t = \frac{-17.8 \pm \sqrt{17.8^2 - 4(-4.5)(15.25)}}{2(-4.5)}$ $t = -0.72 \text{ or } 4.68$ $t = 4.68s$	<p>M1A1</p> <p>A1</p> <p><b>(3)</b></p>
(b)	$H(t) = -4.5 \left[ t^2 - \frac{17.8}{4.5}t - \frac{15.25}{4.5} \right]$ $H(t) = -4.5 \left[ \left( t - \frac{17.8}{9.0} \right)^2 - \left( \frac{17.8}{9.0} \right)^2 - \frac{15.25}{4.5} \right]$ $H(t) = \frac{29567}{900} - 4.5 \left( t - \frac{89}{45} \right)^2$ $A = \frac{29567}{900} = 32.85$ $B = 4.5$ $C = \frac{89}{45} = 1.98$	<p>M1</p> <p>A1</p> <p>A1</p> <p><b>(3)</b></p> <p>1<sup>st</sup> A1 for 32.85</p> <p>2<sup>nd</sup> A1 for both 4.5 and 1.98</p>
(c)	<p>Max height = 32.85</p> <p>Time = <math>\frac{89}{45}</math> or 1.98s</p>	<p>A1</p> <p>A1</p> <p><b>(2)</b></p>