Pure Mathematics 1

Advanced Subsidiary

Practice Paper M10

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 13 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

(a) Show that $x^2 + 6x + 11$ can be written as $(x+p)^2 + q$ where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

(Total 6 marks)

Question 2

(a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

(Total 7 marks)

Question 3

(a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4-x) \quad \text{and} \quad y = x^2(7-x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)

y

x
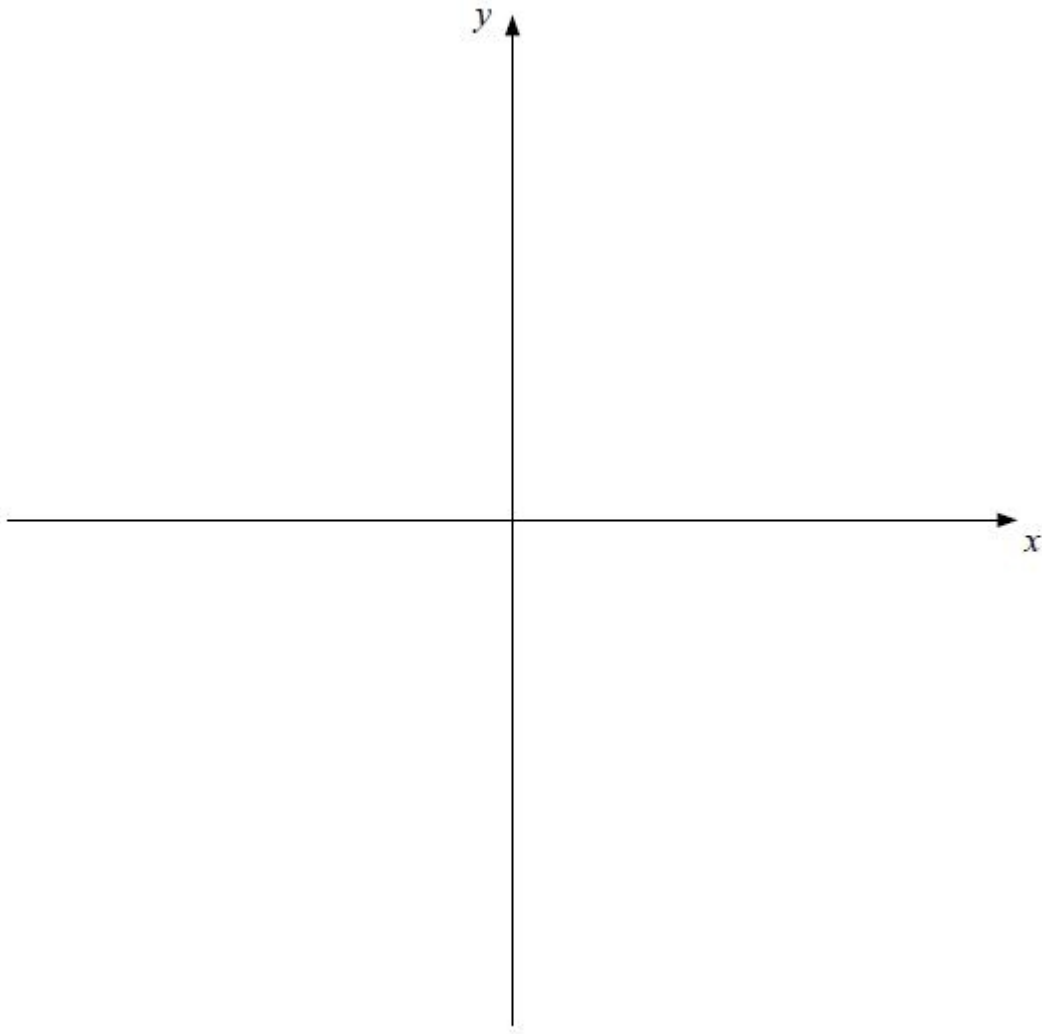
(Total 15 marks)

Question 4

(a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers. **(3)**

(b) Find the length of AB, leaving your answer in surd form. **(2)**

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t. **(1)**

(d) Find the area of triangle ABC. **(2)**

(Total 8 marks)

Question 5

The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C. **(4)**

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1. **(4)**

The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form. **(3)**

(Total 11 marks)

Question 6

The curve C has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) f(x), **(5)**

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(Total 9 marks)

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$$y = x^2 - k\sqrt{x},$$ where k is a constant.

(a) Find $\dfrac{dy}{dx}$. **(2)**

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k. **(2)**
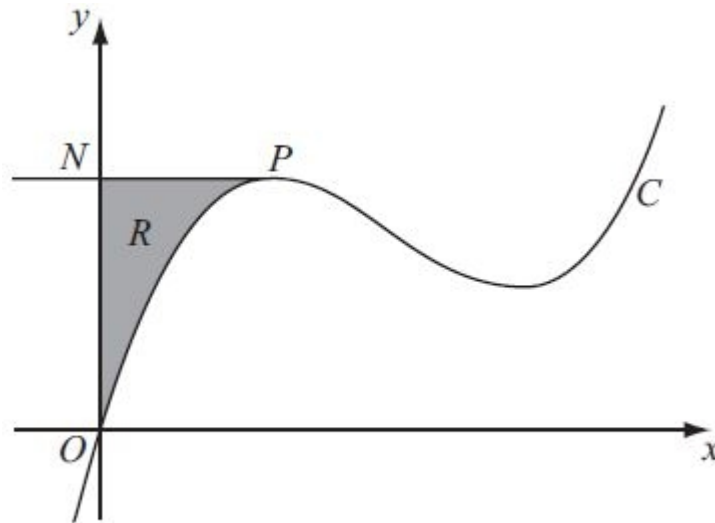
(Total 4 marks)

Question 8

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$. **(3)**

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R. **(6)**

(Total 9 marks)

Question 9

(a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$. **(5)**

(b) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1.$ **(2)**

(Total 7 marks)

Question 10

(a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$. **(1)**

(b) Solve, for $0 \le x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place. **(5)**

(Total 6 marks)

Question 11

Given that $|4\boldsymbol{i} - 2k\boldsymbol{j}| = |3k\boldsymbol{i} - 2\boldsymbol{j}|$, find the exact value of the positive constant k. **(4)**

(Total 4 marks)

Question 12

Prove that the sum of two rational numbers is also a rational number **(3)**

(Total 3 marks)

The curve with equation $y = \frac{1}{5}x(10 - x)$ is used to model the arc of a bridge over a road, where x and y are distances in metres, with the origin as shown in Fig. 12.1. The x −axis represents the road surface.
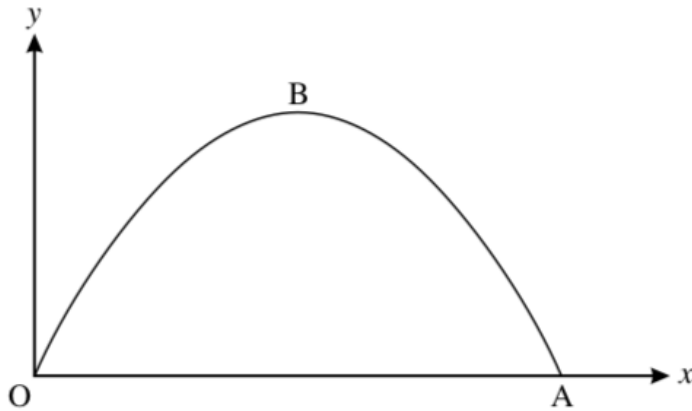

Fig. 12.1

(i)   State the value of x at A, where the arc meets the road.   **(1)**

(ii)   Using symmetry, or otherwise, state the value of x at the maximum point B on the graph. Hence find the height of the arch.   **(2)**

(iii)   Fig. 12.2 shows a lorry which is 4 m high and 3 m wide, with its cross-section modelled as a rectangle. Find the value of d when the lorry is in the centre of the road. Hence show that the lorry can pass through this arch.   **(3)**
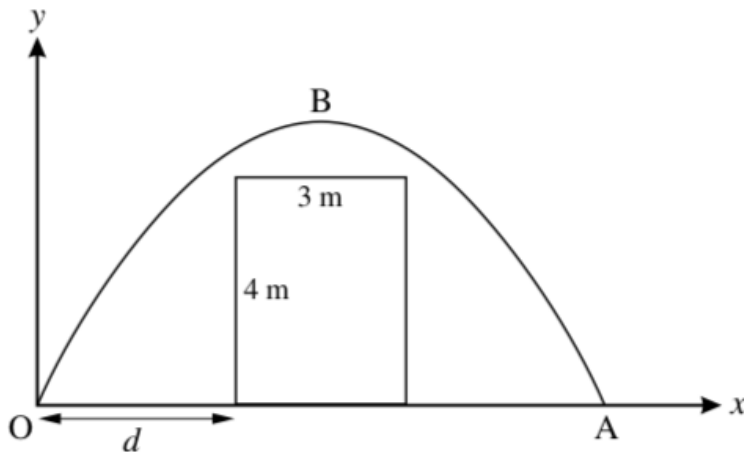

Fig. 12.2

(iv)   Another lorry, also modelled as having a rectangular cross-section, has height 4.5 m and just touches the arch when it is in the centre of the road. Find the width of this lorry, giving your answer in surd form   **(5)**

(Total 11 marks)

TOTAL FOR PAPER IS 100 MARKS