Solving Differential Equations - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q8

Question Number	Scheme	Marks
	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
	$-kV$: k is constant of proportionality and the negative shows decrease (or loss) giving $\frac{dV}{dt} = 20 - kV$ ✱ These Bs are to be awarded independently	B1
		(2)
	(b) $\int \frac{1}{20-kV}\,dV = \int 1\,dt$ separating variables	M1
	$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
	Using $V=0, t=0$ to evaluate the constant of integration	M1
	$c = -\frac{1}{k}\ln 20$	
	$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
	Obtaining answer in the form $V = A + Be^{-kt}$	M1
	$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}(1-e^{-kt})$	A1 (6)
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
	$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
	At $t=10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1 (5)
		[13]
	Alternative to (b)	
	Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
	Substituting into differential equation	
	$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
	$A = \frac{20}{k}$	M1 A1
	Using $V=0, t=0$ in printed answer to obtain $A+B=0$	M1
	$B = -\frac{20}{k}$	A1 (6)

Question 2: Jan 06 Q7

(a)	$\frac{dV}{dr} = 4\pi r^2$	B1	(1)
(b)	Uses $\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$ in any form, $= \frac{1000}{4\pi r^2 (2t+1)^2}$	M1,A1	(2)
(c)	$V = \int 1000(2t+1)^{-2} dt$ and integrate to $p(2t+1)^{-1}$, $= -500(2t+1)^{-1} (+c)$	M1, A1	
	Using V=0 when t=0 to find c , (c = 500 , or equivalent)	M1	
	$\therefore V = 500(1 - \frac{1}{2t+1})$ (any form)	A1	(4)
(d)	(i) Substitute t = 5 to give V,	M1,	
	then use $r = \sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give r, = *4.77*	M1, A1	(3)
	(ii) Substitutes t = 5 and r = 'their value' into 'their' part (b)	M1	
	$\frac{dr}{dt} = 0.0289$ $(\approx 2.90x10^{-2})$ (cm/s) * AG	A1	(2) **[12]**

Question 3: June 06 Q7

Question Number	Scheme		Marks
(a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \underline{\frac{8}{12x}}\,; = \frac{\frac{2}{3}}{x} \quad \Rightarrow \left(k = \frac{2}{3}\right)$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}$; $\underline{\frac{8}{12x}}$	M1; A1oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$; λx	M1; A1 $\sqrt{}$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\, dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\, dt$ on the other side. integral signs not necessary.	B1
	$\int V^{-\frac{1}{3}}\, dV = \int 2\, dt$		
	$\frac{3}{2}V^{\frac{2}{3}} = 2t \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and $2t$; Correct equation with/without + c.	M1; A1
	$\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6$	Use of V = 8 and t = 0 in a changed equation containing c ; c = 6	M1 $*$; A1
	Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$		
	$\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6 \quad \Rightarrow \quad 12 = 2t + 6$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 $*$
	giving $t = 3$.	t = 3	A1 cao
			[7]
			15 marks

Question 4: Jan 07 Q4

Question Number	Scheme		Marks
4. (a)	$\dfrac{2x-1}{(x-1)(2x-3)} \equiv \dfrac{A}{(x-1)} + \dfrac{B}{(2x-3)}$		
	$2x-1 \equiv A(2x-3)+B(x-1)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $x=\frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B = 4$		
	Let $x=1$, $1 = A(-1) \Rightarrow A = -1$	either one of $A=-1$ or $B=4$. both correct for their A, B.	A1 A1
	giving $\dfrac{-1}{(x-1)} + \dfrac{4}{(2x-3)}$		**[3]**
(b) & (c)	$\displaystyle\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown. Can be implied	B1
	$\displaystyle = \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{\ }$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{\ }$ A1 **[5]**
	$y=10,\ x=2$ gives $c=\ln 10$	$c = \ln 10$	B1
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$		
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\dfrac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or $\ln y = \ln\left(\dfrac{10(2x-3)^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\underline{y = \dfrac{10(2x-3)^2}{(x-1)}}$	$\underline{y = \dfrac{10(2x-3)^2}{(x-1)}}$ or aef. isw	A1 aef **[4]**
			12 marks

Question 5: June 07 Q8

Question Number	Scheme		Marks
(a)	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt; (+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
(b)	$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399 \text{ min}}$ or $t = \underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6 \text{ hr } 39 \text{ mins}}$	A1
			[3]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

(c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin \lambda t; (+c)$	Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1 **[4]**
(d)	$P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ ...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}$	Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5} \sin^{-1}(\ln 2)}$	Then rearranges to make t the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477...$		
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441 \text{min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7 \text{ hr } 21 \text{ mins}}$	A1 **[3]**
			14 marks

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

***Aliter* (a) Way 2**	$\dfrac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)	
	$\displaystyle\int \frac{dP}{kP} = \int 1\,dt$ — Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln P = t;(+c)$ — Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$ — Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$	
	Hence, $\underline{P = P_0\,e^{kt}}$ — $\underline{P = P_0\,e^{kt}}$	A1
		[4]
***Aliter* (a) Way 3**	$\displaystyle\int \frac{dP}{kP} = \int 1\,dt$ — Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln(kP) = t;(+c)$ — Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$ $\left(\text{or } kP = Ae^{kt} \Rightarrow kP_0 = A\right)$ — Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$ $\Rightarrow e^{\ln(kP)} = e^{kt + \ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$ $\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0\,e^{kt}$ $\left(\text{or } kP = kP_0e^{kt}\right)$	
	Hence, $\underline{P = P_0\,e^{kt}}$ — $\underline{P = P_0\,e^{kt}}$	A1
		[4]

	$\frac{dP}{dt} = \lambda P\cos\lambda t$ and $t=0,\ P=P_0$ (1)		
Aliter **(c)** **Way 2**	$\int \frac{dP}{\lambda P} = \int \cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos\lambda t\, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t; (+c)$	Must see $\frac{1}{\lambda}\ln P$ and $\frac{1}{\lambda}\sin\lambda t$; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \frac{1}{\lambda}\ln P_0 = c$ $\left(\text{or } P = Ae^{\sin\lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t + \frac{1}{\lambda}\ln P_0 \Rightarrow \ln P = \sin\lambda t + \ln P_0$		
	$\Rightarrow e^{\ln P} = e^{\sin\lambda t + \ln P_0} = e^{\sin\lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin\lambda t}}$	$\underline{P = P_0 e^{\sin\lambda t}}$	A1 **[4]**

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

$\underline{P = P_0 e^{\sin\lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

	$\frac{dP}{dt} = \lambda P\cos\lambda t$ and $t=0,\ P=P_0$ (1)		
***Aliter* (c) Way 3**	$\int \frac{dP}{\lambda P} = \int \cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos\lambda t\, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin\lambda t; (+c)$	Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin\lambda t$; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$ $\left(\text{or } \lambda P = Ae^{\sin\lambda t} \Rightarrow \lambda P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin\lambda t + \frac{1}{\lambda}\ln(\lambda P_0)$		
	$\Rightarrow \ln(\lambda P) = \sin\lambda t + \ln(\lambda P_0)$		
	$\Rightarrow e^{\ln(\lambda P)} = e^{\sin\lambda t + \ln(\lambda P_0)} = e^{\sin\lambda t}.e^{\ln(\lambda P_0)}$		
	$\Rightarrow \lambda P = e^{\sin\lambda t}.(\lambda P_0)$ $\left(\text{or } \lambda P = \lambda P_0 e^{\sin\lambda t}\right)$		
	Hence, $\underline{P = P_0 e^{\sin\lambda t}}$	$\underline{P = P_0 e^{\sin\lambda t}}$	A1 **[4]**

Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.
ft denotes "follow through"
cao denotes "correct answer only"
aef denotes "any equivalent form"

Question 6: Jan 08 Q8

Question Number	Scheme		Marks
(a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$(V = 4000h \Rightarrow)\ \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Correct proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
(d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$		
	$\frac{\mathrm{d}h}{\mathrm{d}x}=2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x}=-2(20-x)$	Correct $\frac{\mathrm{d}h}{\mathrm{d}x}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$	$\pm\lambda\int \frac{20-x}{x}\,\mathrm{d}x$ or $\pm\lambda\int \frac{20-x}{20-(20-x)}\,\mathrm{d}x$ where λ is a constant	M1
	$=100\int \frac{x-20}{x}\,\mathrm{d}x$		
	$=100\int \left(1-\frac{20}{x}\right)\mathrm{d}x$		
	$=100(x-20\ln x)\ (+c)$	$\pm\alpha x \pm \beta\ln x$; $\alpha,\beta \neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100x-2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$=(1000-2000\ln 10)-(2000-2000\ln 20)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$=2000\ln 20-2000\ln 10-1000$	Combining logs to give...	
	$=2000\ln 2-1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 aef
			[6]
(e)	Time required $=2000\ln 2-1000 = 386.2943611...$ sec		
	$=386$ seconds (nearest second)		
	$=6$ minutes and 26 seconds (nearest second)	6 minutes, 26 seconds	B1 [1]
			13 marks

Question 7: Jan 10 Q5

Question Number	Scheme		Marks	
	(a) $\int \frac{9x+6}{x}\,dx = \int\left(9+\frac{6}{x}\right)dx$		M1	
	$= 9x + 6\ln x \ (+C)$		A1	(2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,dy = \int \frac{9x+6}{x}\,dx$	Integral signs not necessary	B1	
	$\int y^{-\frac{1}{3}}\,dy = \int \frac{9x+6}{x}\,dx$			
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \ (+C)$	$\pm ky^{\frac{2}{3}} =$ their (a)	M1	
	$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \ (+C)$	ft their (a)	A1ft	
	$y = 8,\ x = 1$			
	$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

Question 8: **June 10 Q8**

Question Number	Scheme		Marks
	(a) $\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$		B1
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1
	Leading to $75\dfrac{dh}{dt} = 4 - 5h$ ✱	cso	A1 **(5)**
	(b) $\displaystyle\int \frac{75}{4-5h} dh = \int 1 dt$	separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$		M1 A1
	$-15\ln(4-5h) = t + C$		
	When $t = 0$, $h = 0.2$		
	$-15\ln 3 = C$		M1
	$t = 15\ln 3 - 15\ln(4-5h)$		
	When $h = 0.5$		
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1
	Alternative for last 3 marks		
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$		
	$= -15\ln 1.5 + 15\ln 3$		M1 M1
	$= 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1 **(6)**

Question 9: **Jan11 Q3**

Question Number	Scheme	Marks
(a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$ $5 = A(3x+2) + B(x-1)$ $x \to 1 \qquad 5 = 5A \Rightarrow A = 1$ $x \to -\dfrac{2}{3} \qquad 5 = -\dfrac{5}{3}B \Rightarrow B = -3$	 M1 A1 A1 (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{x-1} - \frac{3}{3x+2}\right) dx$ $= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	 M1 A1ft A1ft (3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{y}\right) dy$ $\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$ $y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c) Using $(2, 8)$ $\qquad 8 = \dfrac{K}{8}$ depends on first two Ms in (c) $y = \dfrac{64(x-1)}{3x+2}$	M1 M1 A1 M1 dep M1 dep A1 (6) [12]

Question 10: **June 11 Q8**

Question Number	Scheme	Marks
	(a) $\int (4y+3)^{-\frac{1}{2}}\,dx = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,dy = \int \frac{1}{x^2}\,dx$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,dy = \int x^{-2}\,dx$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ $\quad$ or equivalent	A1 **(6)**
		[8]