Integration : Solving Differential Equations - Edexcel Past Exam Questions

1. Liquid is pouring into a container at a constant rate of 20 $\text{cm}^3\ \text{s}^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(a) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant. **(2)**

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + B\mathrm{e}^{-kt},$$

giving the values of A and B in terms of k. **(6)**

Given also that $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start. **(5)**

June 05 Q8

2. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$. **(1)**

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{\mathrm{d}r}{\mathrm{d}t}$. **(2)**

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t. **(4)**

(d) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures, **(3)**

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}. **(2)**

Jan 06 Q7

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3. At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k}{x}$, where k is a constant to be found, **(4)**

(b) $\frac{\mathrm{d}V}{\mathrm{d}t} = 2V^{\frac{1}{3}}$. **(4)**

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$. **(7)**

June 06 Q7

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4. (*a*) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions. **(3)**

(*b*) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y. \qquad \textbf{(5)}$$

(*c*) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$. **(4)**

Jan 07 Q4

5. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(*a*) solve the differential equation, giving P in terms of P_0, k and t. **(4)**

Given also that $k = 2.5$,

(*b*) find the time taken, to the nearest minute, for the population to reach $2P_0$. **(3)**

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(*c*) solve the second differential equation, giving P in terms of P_0, λ and t. **(4)**

Given also that $\lambda = 2.5$,

(*d*) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model. **(3)**

June 07 Q8

6. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 cm^3s^{-1} and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(*a*) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - k\sqrt{h},$$

where k is a positive constant. **(3)**

When $h = 25$, water is leaking out of the hole at 400 cm^3s^{-1}.

(*b*) Show that $k = 0.02$. **(1)**

(*c*) Separate the variables of the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - 0.02\sqrt{h}$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(*d*) find the exact value of $\displaystyle\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h$. **(6)**

(*e*) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. **(1)**

Jan 08 Q8

7. (*a*) Find $\int \frac{9x+6}{x}\,dx$, $x > 0$. **(2)**

(*b*) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$. **(6)**

Jan 10 Q5

8.

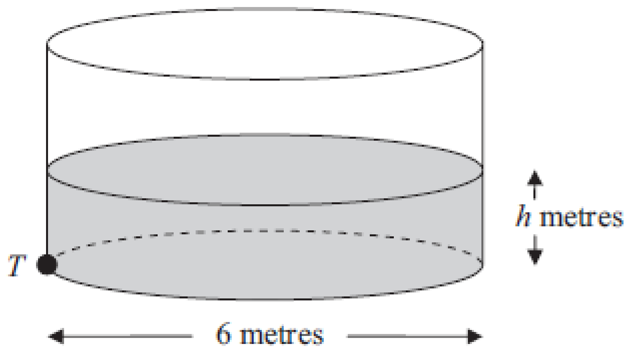

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(*a*) Show that, t minutes after the tap has been opened,

$$75\frac{dh}{dt} = (4 - 5h).$$ **(5)**

When $t = 0$, $h = 0.2$

(*b*) Find the value of t when $h = 0.5$ **(6)**

June 10 Q8

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9. (*a*) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions. **(3)**

(*b*) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx$, where $x > 1$. **(3)**

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$. **(6)**

Jan 11 Q3

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10. (*a*) Find $\displaystyle\int (4y+3)^{-\frac{1}{2}}\,dy$. **(2)**

(*b*) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2},$$

giving your answer in the form $y = f(x)$. **(6)**

June 11 Q8
