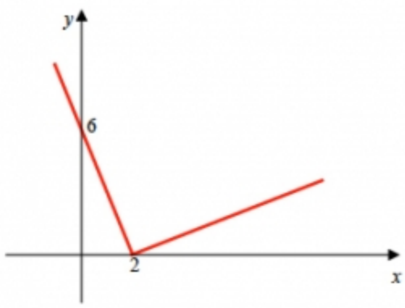
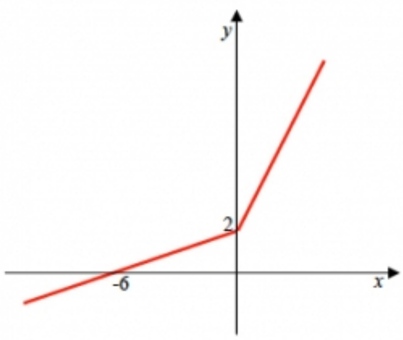
Pure Mathematics 2 Practice Paper J11 MARK SCHEME

Question 1

Question Number	Scheme		Marks
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \dfrac{3+5x}{x+2}$	$\dfrac{3+5x}{x+2}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	Correct Range	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	$\mathrm{g}\,\mathrm{g}(2) = \mathrm{g}(0) = -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = 5$	5	A1
			(2)
(e)(i)		Correct shape 	B1
		$(2, \{0\}), (\{0\}, 6)$	B1
Question Number	**Scheme**		**Marks**
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1
			(4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) [13]

Question 2

Question Number	Scheme		Marks
(a)	$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$		
	$= \frac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \frac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \frac{4x+1}{2x-1}$		A1
			(4)
(b)	$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x>1$		
	$f(x) = \frac{(4x+1)}{(2x-1)} - 2$		
	$= \frac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \frac{4x+1-4x+2}{(2x-1)}$		
	$= \frac{3}{(2x-1)}$	Correct result	A1 *
			(2)
(c)	$f(x) = \frac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$		
		$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$f'(2) = \frac{-6}{9} = -\frac{2}{3}$	Either $\frac{-6}{9}$ or $-\frac{2}{3}$	A1
			(3)
			[9]

Question 3

Question Number	Scheme	Marks
(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \ldots$	M1 A1
	$= 1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \ldots$	
	$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots$	M1 A1 (5)
(b)	$\mathrm{f}(x) = (a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots\right)$	
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0 \quad (3a+b=0)$	M1
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16} \quad (9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a=-1,\ b=3$	M1 A1 (5)
(c)	Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$	M1 A1ft
	$=\frac{27}{16}$ cao	A1 (3)
		[13]

Question 4

Question Number	Scheme		Marks
(a)	$7\cos x - 24\sin x = R\cos(x+\alpha)$		
	$7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$		
	$R = \sqrt{7^2+24^2}\,;= 25$	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$	$\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x = 25\cos(x+1.287)$		(3)
(b)	Minimum value $= \underline{-25}$	-25 or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$		
	$25\cos(x+1.287) = 10$		
	$\cos(x+1.287) = \frac{10}{25}$	$\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$	M1
	$\text{PV} = 1.159279481...^c$ or $66.42182152...^\circ$	For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$	either 2π + or − their PV^c or 360° + or − their PV°	M1
	gives, $x = \{3.836906..., 6.155465...\}$	awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	A1 A1 (5) **[9]**

Question 5

Question Number	Scheme		Marks
(a)	$y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin \pi}{2 + \cos \pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $\mathrm{m(T)} = \dfrac{6\sin \pi + 4\cos \pi + 2}{(2 + \cos \pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\mathrm{m(T)} = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) [8]

Question 6

Question Number	Scheme	Marks
	$\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$	M1 A1 A1
	$= \quad \ldots \quad + \frac{\sin 2x}{4}$	M1
	$[\ \ldots\ ]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]

Question 7

Question Number	Scheme	Marks
(a)	$x = 3 \Rightarrow y = 0.1847$ awrt	B1
	$x = 5 \Rightarrow y = 0.1667$ awrt or $\frac{1}{6}$	B1
		(2)
(b)	$I \approx \underline{\underline{\frac{1}{2}}}\left[0.2 + 0.1667 + 2(0.1847 + 0.1745)\right]$	$\underline{\underline{\text{B1}}}$ M1 A1ft
	≈ 0.543 0.542 or 0.543	A1 **(4)**
(c)	$\frac{dx}{du} = 2(u-4)$	B1
	$\int \frac{1}{4+\sqrt{(x-1)}}\,dx = \int \frac{1}{u} \times 2(u-4)\,du$	M1
	$= \int \left(2 - \frac{8}{u}\right) du$	A1
	$= 2u - 8\ln u$	M1 A1
	$x = 2 \Rightarrow u = 5,\ \ x = 5 \Rightarrow u = 6$	B1
	$\left[2u - 8\ln u\right]_5^6 = (12 - 8\ln 6) - (10 - 8\ln 5)$	M1
	$= 2 + 8\ln\left(\frac{5}{6}\right)$	A1
		(8)
		[14]

Question 8

Question Number	Scheme		Marks
	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1 A1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^{\circ}$ or $\alpha_2 = -18^{\circ}$		
	$\theta = \{54, 126, 198, 342\}$	Any one correct answer 180-their pv All four solutions correct.	A1 dM1(*) A1
			(7)

Question 9

Question Number	Scheme		Marks
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 AG
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

Question 10

Question Number	Scheme		Marks
(a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$		
	$5 = A(3x+2) + B(x-1)$		
	$x \to 1$ $5 = 5A \Rightarrow A = 1$		M1 A1
	$x \to -\dfrac{2}{3}$ $5 = -\dfrac{5}{3}B \Rightarrow B = -3$		A1 (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int\left(\frac{1}{x-1} - \frac{3}{3x+2}\right)\mathrm{d}x$		
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$	ft constants	M1 A1ft A1ft
			(3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int\left(\frac{1}{y}\right)\mathrm{d}y$		M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$		M1 A1
	$y = \dfrac{K(x-1)}{3x+2}$	depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $8 = \dfrac{K}{8}$	depends on first two Ms in (c)	M1 dep
	$y = \dfrac{64(x-1)}{3x+2}$		A1 (6)
			[12]