Pure Mathematics 2

Advanced Level

Practice Paper J11

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 10 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The function f is defined by

$$f: x \rightarrow \frac{3 - 2x}{x - 5} \ , \ x \in R, \ x \neq 5$$
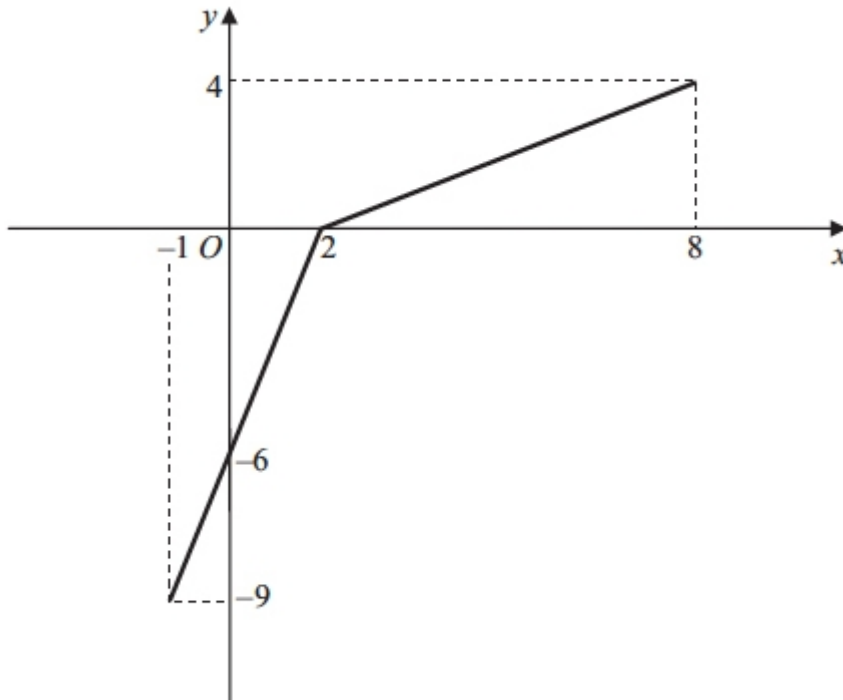
(a) Find $f^{-1}(x)$. (3)

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g. (1)

(c) Find gg(2). (2)

(d) Find fg(8). (2)

(e) On separate diagrams, sketch the graph with equation

   (i) $y = |g(x)|$,

   (ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes. (4)

(f) State the domain of the inverse function g^{-1} . (1)

(Total 13 marks)

Question 2

(a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form. **(4)**

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate f (x) and find f '(2) **(3)**

(Total 9 marks)

Question 3

(*a*) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \qquad |x| < \frac{2}{3},$$

in ascending powers of *x*, up to and including the term in x^3. Give each coefficient as a simplified fraction. **(5)**

$$f(x) = \frac{a + bx}{(2 - 3x)^2}, \qquad |x| < \frac{2}{3}, \qquad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of f(*x*), in ascending powers of *x*, the coefficient of *x* is 0 and the coefficient of x^2 is $\frac{9}{16}$.

Find

(*b*) the value of *a* and the value of *b*, **(5)**

(*c*) the coefficient of x^3 , giving your answer as a simplified fraction. **(3)**

(Total 13 marks)

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Question 4

(a) Express 7 cos x - 24sin x in the form $R \cos(x + a)$ where $R > 0$ and $< a < \dfrac{\pi}{2}$.

Give the value of a to 3 decimal places. **(3)**

(b) Hence write down the minimum value of 7 cos x - 24 sin x. **(1)**

(c) Solve, for $0 \leqslant x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10$$

giving your answers to 2 decimal places. **(5)**

(Total 9 marks)

Question 5

The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \dfrac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants. **(4)**

(Total 8 marks)

Question 6

Use integration to find the exact value of

$$\int_{0}^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

(Total 6 marks)

Question 7

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer

to 3 decimal places. (4)

(c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I. (8)

(Total 14 marks)

Question 8

Find all the solutions

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leqslant \theta < 360°$ (7)

(Total 6 marks)

Question 9

(a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\dfrac{d}{dx}(\sec x)\ \sec x\ \tan x$. (3)

Given that

$$x = \sec 2y$$

(b) find $\dfrac{dx}{dy}$ in terms of y. (2)

(c) Hence find $\dfrac{dy}{dx}$ in terms of x. (4)

(Total 9 marks)

Question 10

(a) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions. **(3)**

(b) Hence find $\displaystyle\int \dfrac{5}{(x-1)(3x+2)}\ dx$, where $x > 1$. **(3)**

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\dfrac{dy}{dx} = 5y, \qquad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$. **(6)**

(Total 12 marks)

TOTAL FOR PAPER IS 100 MARKS