Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper J8

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 10 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \ x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \ x > 0, \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1} . **(2)**

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve gf $(x) = 0$. **(2)**

(Total 13 marks)

Question 2

(a) Use the binomial theorem to expand

$$(8 - 3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction. **(5)**

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places. **(2)**

(Total 7 marks)

Question 3

A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2} .$$

(a) Show that the turning points on C occur where $\tan x = -1$. **(6)**

(b) Find an equation of the tangent to C at the point where $x = 0$. **(2)**

(Total 8 marks)

Question 4

A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$. (3)

(b) Find the gradient of the curve at each of these points. (6)

(Total 9 marks)

Question 5

A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \le x \le \pi.$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A. (5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures. (4)

Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places. (4)

(Total 13 marks)

Question 6

$f(x) = \ln(x + 2) - x + 1, \quad x > -2, x \in \mathbb{R}.$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$. (2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \ x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places. (3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places. (2)

(Total 7 marks)

Question 7

(i) Find $\int \ln(\frac{x}{2})\,dx$. (4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$ (5)

(Total 9 marks)

Question 8

(a) Use the double angle formulae and the identity

$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only. (4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2} .$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4 .$$

(3)

(Total 11 marks)

Question 9

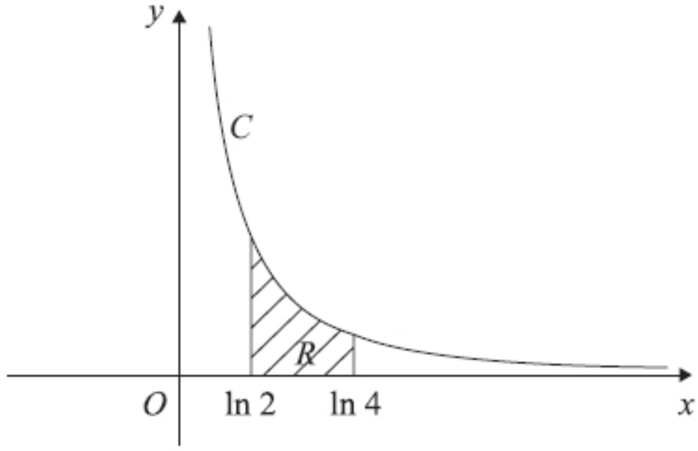

Figure 3

The curve *C* has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region *R* between the curve *C* and the *x*-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of *R* is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,dt.$$

(4)

(b) Hence find an exact value for this area. **(6)**

(c) Find a cartesian equation of the curve *C*, in the form $y = f(x)$. **(4)**

(d) State the domain of values for *x* for this curve. **(1)**

(Total 15 marks)

Question 10

Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 cm^3 s^{-1} and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$\frac{dh}{dt} = 0.4 - k\sqrt{h}$, where k is a positive constant. **(3)**

When $h = 25$, water is leaking out of the hole at 400 cm^3 s^{-1}.

(b) Show that $k = 0.02$ **(1)**

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$ **(6)**

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. **(1)**

(Total 13 marks)

TOTAL FOR PAPER IS 100 MARKS