Pure Mathematics 2 Practice Paper J9 MARK SCHEME

Question 1

Question Number	Scheme	Marks
(a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$f'(x)=\frac{3}{2}x^{-\frac{1}{2}}-9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f′ (1.1) = -6.37086…	B1 B1
	$x_1=1.1-\frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) [9]

Notes:
(a) awrt 0.3 and −0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt −6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question 2

Question Number	Scheme	Marks
(a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$ Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$ Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
	Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A$ $\Rightarrow A=0$ $x=0,\quad 16=2A+B+4C$ $\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$ Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
		(4)
(b)	$f(x)=\dfrac{4}{(3x+2)^2}+\dfrac{3}{(1-x)}$	
	$=4(3x+2)^{-2}+3(1-x)^{-1}$ Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$	
	$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$	
	$=1\left\{1+(-2)\left(\frac{3x}{2}\right);+\frac{(-2)(-3)}{2!}\left(\frac{3x}{2}\right)^2+\ldots\right\}$ Either $1\pm(-2)\left(\frac{3x}{2}\right)$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots\right\}$ Ignoring 1 and 3, any one correct $\{\ldots\ldots\ldots\}$ expansion.	A1
	Both $\{\ldots\ldots\ldots\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$	
	$=4+0x\ ;+\frac{39}{4}x^2$ $4+(0x)\ ;\ \frac{39}{4}x^2$	A1; A1
		(6)

Question Number	Scheme	Marks
(c)	$\text{Actual}=f(0.2)=\dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $=\dfrac{23.48}{5.408}=4.341715976\ldots=\dfrac{2935}{676}$ Attempt to find the actual value of $f(0.2)$ or seeing awrt 4.3 and believing it is candidate's actual $f(0.2)$. Or $\text{Actual}=f(0.2)=\dfrac{4}{(3(0.2)+2)^2}+\dfrac{3}{(1-0.2)}$ $=\dfrac{4}{6.76}+3.75=4.341715976\ldots=\dfrac{2935}{676}$ Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)}+\dfrac{B}{(3x+2)^2}+\dfrac{C}{(1-x)}$ with their A, B and C.	M1
	$\text{Estimate}=f(0.2)=4+\frac{39}{4}(0.2)^2$ $=4+0.39=4.39$ Attempt to find an estimate for $f(0.2)$ using their answer to (b)	M1$\sqrt{}$
	$\%\text{age error}=\dfrac{\lvert 4.39-4.341715976\ldots\rvert}{4.341715976\ldots}\times 100$ $\left\lvert\dfrac{\text{their estimate - actual}}{\text{actual}}\right\rvert\times 100$	M1
	$=1.112095408\ldots=1.1\%\,(2\text{sf})$ 1.1%	A1 cao
		(4)
		[14]

Question 3

Question Number	Scheme		Marks
(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$		
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ Correct $\frac{dy}{dx}$	M1 A1
	At A, $m(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T(x-(\text{their } 7))$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 cso (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^3-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]

Question 4

Question Number	Scheme	Marks
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\frac{\text{A1+A1}}{\text{A1}}$
		(4) [10]
	Alternative to (b)	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^2}-\frac{2\sin 2x}{x^3}\right)$	A1 (4)

Question 5

Question Number	Scheme		Marks	
	$x = \cos(2y + \pi)$			
	$\dfrac{dx}{dy} = -2\sin(2y + \pi)$		M1 A1	
	$\dfrac{dy}{dx} = -\dfrac{1}{2\sin(2y + \pi)}$	Follow through their $\dfrac{dx}{dy}$ before or after substitution	A1ft	
	At $y = \dfrac{\pi}{4}$, $\quad \dfrac{dy}{dx} = -\dfrac{1}{2\sin\frac{3\pi}{2}} = \dfrac{1}{2}$		B1	
	$y - \dfrac{\pi}{4} = \dfrac{1}{2}x$		M1	
	$y = \dfrac{1}{2}x + \dfrac{\pi}{4}$		A1	(6)
				[6]

Question 6

Question Number	Scheme		Marks
(a)	Similar triangles $\Rightarrow \underline{\dfrac{r}{h}=\dfrac{16}{24}} \Rightarrow \underline{r=\dfrac{2h}{3}}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			(2)
(b)	From the question, $\dfrac{dV}{dt}=8$	$\dfrac{dV}{dt}=8$	B1
	$\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}=\underline{\dfrac{4\pi h^2}{9}}$	$\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}$ or $\underline{\dfrac{4\pi h^2}{9}}$	B1
	$\dfrac{dh}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dh} = \underline{8\times\dfrac{9}{4\pi h^2}} = \underline{\dfrac{18}{\pi h^2}}$	Candidate's $\dfrac{dV}{dt}\div\dfrac{dV}{dh}$;	M1;
		$\underline{8\div\left(\dfrac{12\pi h^2}{27}\right)}$ or $\underline{8\times\dfrac{9}{4\pi h^2}}$ or $\underline{\dfrac{18}{\pi h^2}}$ oe	A1
	When $h=12$, $\dfrac{dh}{dt}=\underline{\dfrac{18}{144\pi}}=\underline{\dfrac{1}{8\pi}}$	$\underline{\dfrac{18}{144\pi}}$ or $\underline{\dfrac{1}{8\pi}}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\underline{\dfrac{18}{144\pi}}$.		(5)
			[7]

Question 7

Question Number	Scheme		Marks
(a)	$\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	*Integrating* $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1
		Correct integration. Ignore limits.	A1
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\ (\text{units})^2$	$\underline{3}$	$\underline{\text{A1}}$
			(4)
	(Answer of 3 with no working scores M0A0M0A0.)		

Question 8

Question Number	Scheme		Marks
(a)	$\int \tan^2 x \, dx$		
	$\left[NB: \underline{\sec^2 A = 1+\tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x - x} \, (+c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\begin{Bmatrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow & v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{Bmatrix}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}, n \in \square, n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} (+c)$	... "integrate" (process the result);	M1
		correct solution with/without + c	A1 oe
			(4)

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x} \, dx$		
	$\left\{ u = 1 + e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \ \underline{\frac{dx}{du} = \frac{1}{e^x}}, \ \underline{\frac{dx}{du} = \frac{1}{u-1}} \right\}$	Differentiating to find any one of the three underlined	B1
	$= \int \frac{e^{2x} \cdot e^x}{1+e^x} \, dx = \int \frac{(u-1)^2 \cdot e^x}{u} \cdot \frac{1}{e^x} \, du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$	
	or $= \int \frac{(u-1)^3}{u} \cdot \frac{1}{(u-1)} \, du$	or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u} \, du$	$\underline{\int \frac{(u-1)^2}{u} \, du}$	A1
	$= \int \frac{u^2 - 2u + 1}{u} \, du$	An attempt to multiply out their numerator to give at least three terms	
	$= \int u - 2 + \frac{1}{u} \, du$	and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 cso
			(7)
			[13]

Question 9

Question Number	Scheme		Marks	
(a)(i)	$\sin 3\theta = \sin(2\theta + \theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ *	cso	A1	(4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	(5)
(b)	$\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ *	cso	A1	(4)
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ *	cso	A1	(4)
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ *	cso	A1	(4)

Question 10

Question Number	Scheme		Marks	
(a)	$R^2 = 3^2 + 4^2$		M1	
	$R = 5$		A1	
	$\tan\alpha = \frac{4}{3}$		M1	
	$\alpha = 53\ ...\ ^\circ$	awrt 53°	A1	(4)
(b)	Maximum value is 5	ft their R	B1 ft	
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
	$\theta = \alpha = 53\ ...\ ^\circ$	ft their α	A1 ft	(3)
(c)	$\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$			
	Minimum occurs when $\cos(15t - \alpha)^\circ = -1$		M1	
	The minimum temperature is $(10 - 5)^\circ = 5^\circ$		A1 ft	(2)
(d)	$15t - \alpha = 180$		M1	
	$t = 15.5$	awrt 15.5	M1 A1	(3)
				[12]