Pure Mathematics 2 Practice Paper M10 MARK SCHEME

Question 1

Question Number	Scheme	Marks
	(a) $25\,000 \times 1.03 = 25750$ $\left\{25000 + 750 = 25750, \text{ or } 25000\dfrac{(1-0.03^2)}{1-0.03} = 25750\right\}$ (*)	B1 (1)
	(b) $r = 1.03$ Allow $\dfrac{103}{100}$ or $1\dfrac{3}{100}$ but no other alternatives	B1 (1)
	(c) $25000r^{N-1} > 40000$ (Either letter r or their r value) Allow '=' or '<' $r^M > 1.6 \Rightarrow \log r^M > \log 1.6$ Allow '=' or '<' (See below) OR (by change of base), $\log_{1.03} 1.6 < M \Rightarrow \dfrac{\log 1.6}{\log 1.03} < M$ $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) Accept work for part (c) seen in part (d)	M1 M1 A1 cso (3)
	(d) Attempt to evaluate $\dfrac{\log 1.6}{\log 1.03} + 1$ {or $25000(1.03)^{15}$ and $25000(1.03)^{16}$} $N = 17$ (not 16.9 and not e.g. $N \geq 17$) Allow '17th year' Accept work for part (d) seen in part (c)	M1 A1 (2)
	(e) Using formula $\dfrac{a(1-r^n)}{1-r}$ with values of a and r, and $n = 9$, 10 or 11 $\dfrac{25\,000(1-1.03^{10})}{1-1.03}$ 287 000 (must be rounded to the nearest 1 000) Allow 287000.00	M1 A1 A1 (3) 10

(c) 2nd M: Requires $\dfrac{40000}{25000}$ to be dealt with, and 'two' logs introduced.

With, say, N instead of $N-1$, this mark is still available.

> Jumping straight from $1.03^{N-1} > 1.6$ to $(N-1)\log 1.03 > \log 1.6$ can score only M1 M0 A0.
> (The intermediate step $\log 1.03^{N-1} > \log 1.6$ must be seen).

Longer methods require correct log work throughout for 2nd M, e.g.:

$\log(25000r^{N-1}) > \log 40000 \Rightarrow \log 25000 + \log r^{N-1} > \log 40000 \Rightarrow$

$\log r^{N-1} > \log 40000 - \log 25000 \Rightarrow \log r^{N-1} > \log 1.6$

(d) Correct answer with no working scores both marks.

Evaluating $\log\left(\dfrac{1.6}{1.03}\right) + 1$ does not score the M mark.

(e) M1 can also be scored by a "year by year" method, with terms added.

(Allow the M mark if there is evidence of adding 9, 10 or 11 terms).

1st A1 is scored if the 10 correct terms have been added (allow terms to be to the nearest 100).

To the nearest 100, these terms are:

25000, 25800, 26500, 27300, 28100, 29000, 29900, 30700, 31700, 32600

No working shown: Special case: 287 000 scores 1 mark, scored on ePEN as 1, 0, 0.

(Other answers with no working score no marks).

Question 2

Question Number	Scheme	Marks
	(a) $A = 2$	
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1 \qquad -3 = 3B \;\Rightarrow\; B = -1$	M1 A1
	$x \to -2 \qquad -12 = -3C \;\Rightarrow\; C = 4$	A1 (3)
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1} = 1 + x + x^2 + \ldots$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$	M1
	$= 5 + \ldots$ ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $0x$ stated or implied	A1 A1 (6)
		(9)

Question 3

Question Number	Scheme	Marks
	At P, $y = \underline{3}$	B1
	$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1A1
	$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(N) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
	N: $y - 3 = \frac{1}{18}(x-2)$	M1
	N: $\underline{x - 18y + 52 = 0}$	A1 (6)
		[7]
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $m(N) = -\frac{1}{\text{their } m(T)}$. 4th M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Question 4

Question Number	Scheme	Marks
	$\frac{d}{dx}(2^x) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = 4\ln 2 - 2$ Accept exact equivalents	A1
		(6)

Question 5

Question Number	Scheme	Marks
(a)	Either $y = 2$ or $(0, 2)$	B1
		(1)
(b)	When $x = 2,\ y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1
		(3)
(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1
		(3)
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x-7)(x-1) = 0$	M1
	$x = \frac{7}{2}, 1$	A1
	When $x = \frac{7}{2},\ y = 9e^{-\frac{7}{2}}$, when $x = 1,\ y = -e^{-1}$	ddM1A1
		(5) [12]
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\},\ y = -e^{-1}$ and $\{x = \frac{7}{2}\},\ y = 9e^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

Question 6

Question Number	Scheme		Marks	
	(a) $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ *	cso	A1	(3)
	(b) $\int \theta \cos 2\theta \, d\theta = \frac{1}{2}\theta \sin 2\theta - \frac{1}{2}\int \sin 2\theta \, d\theta$		M1 A1	
	$= \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta f(\theta) \, d\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta \sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\; \dots \; \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16} - \frac{7}{4}$		A1	(7)
				[10]

Question 7

Question Number	Scheme	Marks
(a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 cso
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	$\theta_1 =$ awrt 26.6°	A1
	$\theta_2 =$ awrt -153.4°	A1$\sqrt{\ }$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1$\sqrt{\ }$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	

Question 8

Question Number	Scheme	Marks
	$\dfrac{du}{dx} = -\sin x$	B1
	$\int \sin x\, e^{\cos x+1}dx = -\int e^u du$	M1 A1
	$= -e^u$ ft sign error	A1ft
	$= -e^{\cos x+1}$	
	$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$ or equivalent with u	M1
	$= e(e-1)$ ✱ cso	A1 (6)
		[6]

Question 9

Question Number	Scheme		Marks	
	(a) $\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$		B1 B1	
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}\left(= \frac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1	(4)
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$		B1	
	$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$		M1 A1	
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$		M1	
	$y = 0 \ \Rightarrow\ x = \frac{3}{8}$		M1 A1	(6)
				[10]

Question 10

Question Number	Scheme		Marks	
	(a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1	
	$V = 9\pi h \quad \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$		B1	
	$9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1	
	Leading to $75\frac{dh}{dt} = 4 - 5h$ *	cso	A1	(5)
	(b) $\int \frac{75}{4-5h} dh = \int 1 dt$	separating variables	M1	
	$-15\ln(4-5h) = t \quad (+C)$		M1 A1	
	$-15\ln(4-5h) = t + C$			
	When $t = 0$, $h = 0.2$			
	$-15\ln 3 = C$		M1	
	$t = 15\ln 3 - 15\ln(4-5h)$			
	When $h = 0.5$			
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1	
	Alternative for last 3 marks			
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$			
	$= -15\ln 1.5 + 15\ln 3$		M1 M1	
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1	(6)

Question 11

Question Number	Scheme	Marks
(a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = $ awrt 0.6435	M1A1
		(3)
(b) (i)	Max Value $= 2.5$	B1$\sqrt{}$
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = $ awrt 2.21	$\underline{\text{M1}}$;A1 $\sqrt{}$
		(3)
(c)	$H_{\text{Max}} = 8.5$ (m)	B1$\sqrt{}$
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}$; $\Rightarrow t = $ awrt 4.41	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = $ awrt 2.1 or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\right\}$	ddM1
	Times $= \{14:06,\ 18:43\}$	A1 (6)
		[15]
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0. M1: $\tan\alpha = \pm\frac{1.5}{2}$ or $\tan\alpha = \pm\frac{2}{1.5}$ A1: $\alpha = $ awrt 0.6435 (b) B1$\sqrt{}$: 2.5 or follow through the value of R in part (a). M1: For $\sin(\theta - \text{their } \alpha) = 1$ A1$\sqrt{}$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf. (c) B1$\sqrt{}$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6. M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = $ their (b) answer A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40. (d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$ A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different α's. Note this mark can be implied by seeing 1.055. A1: Either $t = $ awrt 2.1 or $t = $ awrt 6.7 ddM1: either π – their PV^{c}. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730… or 3.373… A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.	