Pure Mathematics 2 Practice Paper M11 MARK SCHEME

Question 1

Question Number	Scheme	Marks
	Assume the opposite is true that $\sqrt{3}$ can be expressed in the form $\frac{a}{b}$, where a and b are integers with no common factor except 1	B1
	$\sqrt{3} = \frac{a}{b}$ $3 = \frac{a^2}{b^2}$	M1
	$a^2 = 3b^2$ therefore a^2 must be a multiple of 3. Therefore a must be a multiple of 3 a can be expressed as $3n$ since it is a multiple of 3 $3b^2 = (3n)^2$ $3b^2 = 9n^2$ $b^2 = 3n^2$	M1
	Therefore b^2 must be a multiple of 3. Therefore b must be a multiple of 3	
	This contradicts our assumption since a and b both have common factor 3. Therefore $\sqrt{3}$ is irrational	A1

Question 2

Question Number	Scheme		Marks
(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 (cm)^2	Using $\frac{1}{2}r^2\theta$ (See notes) 6π or 18.85 or awrt 18.8	M1 A1 [2]
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\frac{1}{2} = \frac{r}{6-r}$ $6 - r = 2r \Rightarrow r = 2$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$ Replaces sin by numeric value $r = 2$	M1 dM1 A1 cso [3]
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 (cm)^2	their area of sector $- \pi r^2$ 2π or awrt 6.3	M1 A1 cao [2] 7
(a)	M1: Needs θ in radians for this formula. Candidate could convert to degrees and use the degrees formula. A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8 Correct answer with no working is M1A1. This M1A1 can only be awarded in part (a).		
(b)	M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$. 1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark. dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1. A1: For $r = 2$ from correct solution only. **Alternative:** 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$. **Note** seeing $OC = 2r$ is M1M1. **Special Case:** If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).		
(c)	M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b). Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative. Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates. **Note:** Candidates can get M1 by writing "their part (a) answer $- \pi r^2$", where the radius of the circle is not substituted. A1: cao – accept exact answer or awrt 6.3 Correct answer only with no working in (c) gets M1A1 **Beware: The answer in (c) is the same as the arc length of the pendant**		

Question 3

Question Number	Scheme		Marks
(a)	$\{ar = 192 \text{ and } ar^2 = 144\}$		
	$r = \dfrac{144}{192}$	Attempt to eliminate a. (See notes.)	M1
	$r = \frac{3}{4}$ or 0.75	$\frac{3}{4}$ or 0.75	A1
			[2]
(b)	$a(0.75) = 192$		M1
	$a\left\{= \dfrac{192}{0.75}\right\} = 256$	256	A1
			[2]
(c)	$S_\infty = \dfrac{256}{1-0.75}$	Applies $\dfrac{a}{1-r}$ correctly using both their a **and** their $\lvert r\rvert < 1$.	M1
	So, $\{S_\infty =\}$ 1024	1024	A1 cao
			[2]
(d)	$\dfrac{256(1-(0.75)^n)}{1-0.75} > 1000$	Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \dfrac{1000(0.25)}{256}\ \left\{= \dfrac{6}{256}\right\}$	Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >).	M1
	$n\log(0.75) < \log\left(\dfrac{6}{256}\right)$	Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \dfrac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	See notes **and** $n = 14$	A1 cso
			[4]
			10

(a)	M1: **for eliminating a** by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or dividing $ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\frac{1}{r}$. Note that $r^2 - r = \frac{144}{192}$ is M0.
	Note also that any of $r = \frac{144}{192}$ or $r = \frac{192}{144}\left\{= \frac{4}{3}\right\}$ or $\frac{1}{r} = \frac{192}{144}$ or $\frac{1}{r} = \frac{144}{192}$ are fine for the award of M1. **Note:** A candidate just writing $r = \frac{144}{192}$ with no reference to a can also get the method mark.
	Note: $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b).
(b)	M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a =\}\frac{192}{r}$ or $ar^2 = 144$ or $\{a =\}\frac{144}{r^2}$. No slips allowed here for M1.
	M1: can also be awarded for writing down $144 = a\left(\frac{192}{a}\right)^2$
	A1 for $a = 256$ only. Note 256 from any working scores M1A1.
	Note: Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.

Question Number	Scheme	Marks
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their a **and** their r, where $\lvert r\rvert < 1$.	
	A1: for 1024, cao.	
	In parts (a) or (b) or (c), the correct answer with no working scores full marks.	
(d)	1st M1: For applying S_n with their a and either "the letter r" or their r and "uses" 1000.	
	2nd M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality.	
	$+(r)^n$ must be derived from the S_n formula.	
	3rd M1: For applying the power law to $\lambda^k = \mu$ to give $k\log\lambda = \log\mu$ oe. where $\lambda, \mu > 0$.	
	or 3rd M1: For solving $\lambda^k = \mu$ to give $k = \log_\lambda \mu$, where $\lambda, \mu > 0$.	
	A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an <u>incorrect</u> inequality statement at any stage in a candidate's working for this part loses this mark.	
	Note: Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution.	
	Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to state in the final line of their working that $n = 13.04$ (truncated) or $n =$ awrt $13.05 \Rightarrow n = 14$ for A1.	
	$n = 14$ from no working gets SC: M0M0M1A1.	
	A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of the power law of logarithms.	
	<u>Trial & Improvement Method:</u>	
	For $a = 256$ and $r = 0.75$, apply the following scheme:	
	$S_{13} = \frac{256(1-(0.75)^{13})}{1-0.75} = 999.6725616...$ — Attempt to find either S_{13} or S_{14}.	M1
	EITHER (1) $S_{13} =$ awrt 999.7 or truncated 999 OR (2) $S_{14} =$ awrt 1005.8 or truncated 1005.	M1
	$S_{14} = \frac{256(1-(0.75)^{14})}{1-0.75} = 1005.754421...$ — Attempt to find both S_{13} and S_{14}.	M1
	So, $n = 14$. — BOTH (1) $S_{13} =$ awrt 999.7 or truncated 999 AND (2) $S_{14} =$ awrt 1005.8 or truncated 1005 AND $n = 14$.	A1

Question 4

Question Number	Scheme	Marks
(a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\dfrac{4x-5}{(2x+1)(x-3)} - \dfrac{2x}{(x+3)(x-3)}$	
	$= \dfrac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \dfrac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$= \dfrac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$= \dfrac{5\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}(x+3)} = \dfrac{5}{(2x+1)(x+3)}$	A1*
		(5)

(b)	$$f(x) = \frac{5}{2x^2 + 7x + 3}$$	
	$$f'(x) = \frac{-5(\mathbf{4x + 7})}{(2x^2 + 7x + 3)^2}$$	M1**M1**A1
	$$f'(-1) = -\frac{15}{4}$$	M1A1
	Uses m_1m_2=-1 to give gradient of normal=$\frac{4}{15}$	M1
	$$\frac{y - (-\frac{5}{2})}{(x - -1)} = their\ \frac{4}{15}$$	M1
	$y + \frac{5}{2} = \frac{4}{15}(x + 1)$ or any equivalent form	A1
		(8)
		13 Marks

Question 5

Question Number	Scheme		Marks	
		$\frac{1}{y}\frac{dy}{dx} = \ldots$	B1	
		$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
	At $x = 2$,	$\ln y = 2(2)\ln 2$	M1	
	leading to	$y = 16$ Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$	$\frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$	M1	
		$\frac{dy}{dx} = 16(2 + 2\ln 2)$	A1	(7)
				[7]
	Alternative			
		$y = e^{2x\ln x}$	B1	
		$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
		$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1	
	At $x = 2$,	$\frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1	
		$= 16(2 + 2\ln 2)$	A1	(7)

Question 6

Question Number	Scheme		Marks	
	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

Question 7

Question Number	Scheme		Marks	
	(a) $u = x^2 + 2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$		B1	
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,\mathrm{d}x$		B1	
	$\int x^3 \ln(x^2+2)\,\mathrm{d}x = \int x^2 \ln(x^2+2)\,x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\,\mathrm{d}u$		M1	
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱		A1	(4)
	cso			
	(b) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u^2}{2} - 2u\right)\frac{1}{u}\,\mathrm{d}u$		M1 A1	
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u}{2} - 2\right)\mathrm{d}u$			
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right) \quad (+C)$		M1 A1	
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right)\right]_2^4$			
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - ((2-4)\ln 2 - 1 + 4)\right]$		M1	
	$= \tfrac{1}{2}(2\ln 2 + 1)$	$\ln 2 + \tfrac{1}{2}$	A1	(6)
				(10)

Question 8

Question Number	Scheme		Marks	
(a)	$\dfrac{1}{\sin 2\theta}-\dfrac{\cos 2\theta}{\sin 2\theta}=\dfrac{1-\cos 2\theta}{\sin 2\theta}$		M1	
	$=\dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$		M1A1	
	$=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	cso	A1*	
				(4)
(b)(i)	$\tan 15°=\dfrac{1}{\sin 30°}-\dfrac{\cos 30°}{\sin 30°}$		M1	
	$\tan 15°=\dfrac{1}{\frac{1}{2}}-\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=2-\sqrt{3}$	cso	dM1 A1*	
				(3)

(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
		12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°) \;\; or \;\; \tan(45° - 30°)$	
	$\tan 15° = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} \; or \; \dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15° = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} \; or \; \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2 - \sqrt{3}$	A1*

Question 9

Question Number	Scheme	Marks
(a)	$R^2 = 2^2 + 3^2$	M1
	$R = \sqrt{13}$ or 3.61 …..	A1
	$\tan\alpha = \frac{3}{2}$	M1
	$\alpha = 0.983 \ldots$	A1
		(4)
(b)	$f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1A1A1
	$= e^{2x}(2\cos 3x - 3\sin 3x)$	M1
	$= e^{2x}(R\cos(3x + \alpha)$	
	$= Re^{2x}\cos(3x + \alpha)$	A1* cso
		(5)
(c)	$f'(x) = 0 \Rightarrow \cos(3x + \alpha) = 0$	M1
	$3x + \alpha = \frac{\pi}{2}$	M1
	x=0.196… awrt 0.20	A1
		(3)
		12 Marks

	Alternative to part (c) $\Rightarrow$		
	$f'(x) = 0 \Rightarrow 2\cos 3x - 3\sin 3x = 0$	M1	
	$\tan 3x = \frac{2}{3}$	M1	
	$x = 0.196\ldots$ awrt 0.20	A1	
			(3)

Question 10

Question Number	Scheme		Marks	
	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$		M1	
	$\theta = \frac{\pi}{3}$	awrt 1.05	A1	(2)
	(b) $\frac{dx}{d\theta} = \sec^2\theta$, $\frac{dy}{d\theta} = \cos\theta$			
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}$ $\left(= \cos^3\theta\right)$		M1 A1	
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$	Can be implied	A1	
	Using $mm' = -1$, $m' = -8$		M1	
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		M1	
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$			
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$	1.0625	A1	(6)

Question 11

Question Number	Scheme		Marks	
(a)	$p=7.5$		B1	
				(1)
(b)	$2.5 = 7.5e^{-4k}$		M1	
	$e^{-4k} = \frac{1}{3}$		M1	
	$-4k = \ln(\frac{1}{3})$		dM1	
	$-4k = -\ln(3)$			
	$k = \frac{1}{4}\ln(3)$		A1*	
	See notes for additional correct solutions and the last A1			
				(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$	ft on their p and k	M1A1ft	
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$			
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$		M1A1	
	$-\frac{1}{4}(ln3)t = \ln(0.32)$		dM1	
	t=4.1486….	4.15 or awrt 4.1	A1	
				(6)
				11Marks