Pure Mathematics 2 Practice Paper M7 MARK SCHEME

Question 1

Question Number	Scheme	Marks
1.	'Assume that there exists a product of two odd numbers that is even	B1
	$(2m+1)(2n+1) = 4mn + 2m + 2n + 1$	M1
	$= 2(2mn + m + n) + 1$	
	$2(2mn + m + n)$ is even so $2(2mn + m + n) + 1$ must be odd	
	This contradicts the assumption that the product of two odd numbers is even, therefore the product of two odd numbers is odd	B1

Question 2

Question Number	Scheme		Marks
(a) **Way 1**	A method of long division gives,		
	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \dfrac{4}{(2x+1)(2x-1)}$	$A = 2$	B1
	$\dfrac{4}{(2x+1)(2x-1)} \equiv \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$	Forming any one of these two identities. Can be implied.	M1
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	**See note below** either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 **[4]**
Aliter (a) **Way 2**	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	See below for the award of B1	***decide to award B1 here!! …*** … *for* $A = 2$	B1
	$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$	Forming this identity. Can be implied.	M1
	Equate x^2, $8 = 4A \Rightarrow A = 2$		
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	**See note below** either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 **[4]**
	If a candidate states one of either B or C correctly then the method mark M1 can be implied.		

(b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$	M1 *
		$A \to Ax$	B1 $\sqrt{}$
		$-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1) + \ln(2x-1)$ **See note below.**	A1 cso & aef
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	depM1 *
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for ***their numerical*** expression.	M1
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$ Or $2 - \ln\left(\frac{5}{9}\right)$ **and** k stated as $\frac{9}{5}$.	A1 [6]
			10 marks

Some candidates may find rational values for B and C. They may combine the denominator of their B or C with (2x +1) or (2x – 1). Hence: Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$ is okay for M1.

Candidates are not allowed to fluke $-\ln(2x+1) + \ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error ***in applying the laws of logarithms*** would then earn M0.

Note: This is not a dependent method mark.

Question 3

Question Number	Scheme		Marks
(a)	$x = \tan^2 t$, $\quad y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t$, $\frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t}$ $\left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \frac{\frac{1}{\sqrt{2}}}{\underline{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x-1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = \underline{(\text{their gradient})}x + "\underline{c}"$.	M1 $\sqrt{}$ aef
	T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of $\underline{\text{tangent}}$	$\underline{\text{A1}}$ aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence **T**: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

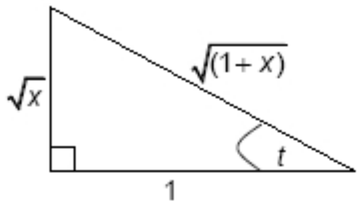
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

(c) **Way 1**	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1 - \sin^2 t}$	Uses $\cos^2 t = 1 - \sin^2 t$	M1
	$x = \dfrac{y^2}{1 - y^2}$	Eliminates 't' to write an equation involving x and y.	M1
	$x(1 - y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1 + x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1 + x}$	$\dfrac{x}{1 + x}$	A1
			[4]
Aliter (c) **Way 2**	$1 + \cot^2 t = \operatorname{cosec}^2 t$	Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\operatorname{cosec}^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $\quad 1 + \dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	$1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	A1
			[4]

$\dfrac{1}{1 + \frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

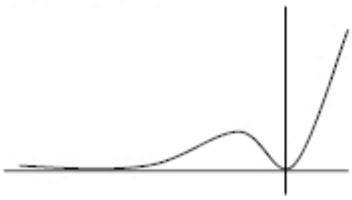
$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 5**	$x = \tan^2 t \qquad y = \sin t$		
	$x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$		
		Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates 't' *to* write an equation involving x and y.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

Question 4

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(b)	If $\frac{dy}{dx} = 0$, $e^x(x^2+2x) = 0$ setting $(a) = 0$ $[e^x \neq 0]$ $x(x+2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4e^{-2}$ $(= 0.54\ldots)$	M1 A1 A1 √ (3)
(c)	$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $\left[= (x^2+4x+2)e^x\right]$	M1, A1 (2)
(d)	$x = 0$, $\frac{d^2y}{dx^2} > 0$ (=2) $x = -2$, $\frac{d^2y}{dx^2} < 0$ $[= -2e^{-2}$ $(= -0.270\ldots)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$ minimum $\therefore$ maximum	M1 A1 (cso) (2)
Alt.(d)	For M1: Evaluate, or state sign of, $\frac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	 **(10 marks)**

Notes: (a) M for attempt at $f(x)g'(x) + f'(x)g(x)$
1st A1 for one correct, 2nd A1 for the other correct.
Note that x^2e^x on its own scores no marks
(b) 1st A1 $(x = 0)$ may be omitted, but for
2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

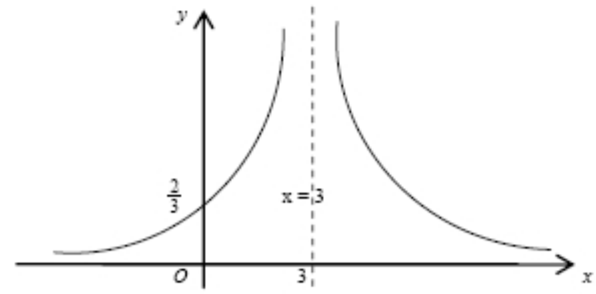
(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen,
or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.
Need only consider the quadratic, as may assume $e^x > 0$.
If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question 5

Question number	Scheme	Marks
	(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))	B1 (1)
	(b) $50\,000r^{n-1} > 200000$ (Using answer to (a), which must include r and n, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)	M1
	$r^{n-1} > 4 \quad \Rightarrow \quad (n-1)\log r > \log 4$ (Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)	M1
	$n > \dfrac{\log 4}{\log r} + 1$ (*)	A1cso (3)
	(c) $r = 1.09$: $n > \dfrac{\log 4}{\log 1.09} + 1$ or $n - 1 > \dfrac{\log 4}{\log 1.09}$ ($n > 17.086...$) (Allow equality)	M1
	Year 18 or 2023 (If one of these is correct, ignore the other)	A1 (2)
	(d) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{50000(1-1.09^{10})}{1-1.09}$	M1 A1
	£760 000 (Must be this answer… nearest £10000)	A1 (3) **9**
	(b) <u>Incorrect</u> inequality sign at any stage loses the A mark. Condone missing brackets if otherwise correct, e.g $n-1\log r > \log 4$. <u>A common mistake:</u> $50\,000r^{n-1} > 200000$ M1 $(n-1)\log 50\,000r > \log 200000$ M0 ('Recovery' from here is not possible). (c) Correct answer with no working scores full marks. Year 17 (or 2022) with no working scores M1 A0. Treat other methods (e.g. "year by year" calculation) as if there is no working. (d) M1: Use of the correct formula with $a = 50000$, 5000 or 500000, and $n = 9$, 10, 11 or 15. M1 can also be scored by a "year by year" method, <u>with terms added</u>. (Allow the M mark if there is evidence of adding 9, 10, 11 or 15 terms). 1st A1 is scored if 10 correct terms have been added (allow "nearest £100"). (50000, 54500, 59405, 64751, 70579, 76931, 83855, 91402, 99628, 108595) <u>No</u> working shown: Special case: 760 000 scores 1 mark, scored as 1, 0, 0. (Other answers with no working score no marks).	

Question 6

Question Number	Scheme	Marks
(*a*)	Finding g(4) = k and f(k) = …. or $\text{fg}(x) = \ln\left(\frac{4}{x-3} - 1\right)$	M1
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] $= \ln 3$	A1 (2)
(*b*)	$y = \ln(2x-1) \;\Rightarrow\; e^y = 2x - 1$ or $e^x = 2y - 1$	M1, A1
	$f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$	A1
	Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$] independent	B1 (4)
(*c*)	 Shape, and *x*-axis should appear to be asymptote	B1
	Equation $x = 3$ needed, may see in diagram (ignore others)	B1 ind.
	Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(*d*)	$\frac{2}{x-3} = 3 \;\Rightarrow\; x = 3\frac{2}{3}$ or exact equiv.	B1
	$\frac{2}{x-3} = -3$, $\Rightarrow x = 2\frac{1}{3}$ or exact equiv.	M1, A1 (3)
	Note: $2 = 3(x + 3)$ or $2 = 3(-x - 3)$ o.e. is M0A0	
Alt:	Squaring to quadratic $(9x^2 - 54x + 77 = 0)$ **and** solving M1; B1A1	**(12 marks)**

Question 7

Question Number	Scheme	Marks
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$, with substitution $u = 2^x$	
	$\frac{du}{dx} = 2^x.\ln 2 \;\Rightarrow\; \frac{dx}{du} = \frac{1}{2^x.\ln 2}$ $\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$	B1
	$\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$ $k\int \frac{1}{(u+1)^2}\,du$ where *k* is constant	M1*

$= \left(\frac{1}{\ln 2}\right)\left(\frac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \to a(u+1)^{-1}$ $(u+1)^{-2} \to -1.(u+1)^{-1}$	M1 A1
change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1 *
$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef [6]
Alternatively candidate can revert back to x …		
$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1 *
$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef
		6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

Question 8

Question Number	Scheme		Marks
(a)	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow & v = \frac{1}{2}\sin 2x \end{array}\right\}$		
	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	***(see note below)*** Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1$, $k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1 **[4]**
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \; (+c)$	Completely correct expression with/without $+c$	A1 **[3]**
			7 marks
Notes:			
(b)	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1 \, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow & v = \lambda\sin 2x \end{array}\right\}$		
	$\text{Int} = \int x\cos 2x \, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1 \, dx$	This is also acceptable for M1	M1

Aliter (b) **Way 2**	$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x+1}{2}\right)dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral …	M1
	$\begin{Bmatrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow & v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{Bmatrix}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
Aliter (b) **Way 3**	$\int x\cos 2x\,dx = \int x\left(2\cos^2 x - 1\right)dx$	Substitutes **correctly** for $\cos 2x$ in $\int x\cos 2x\,dx$	M1
	$\Rightarrow 2\int x\cos^2 x\,dx - \int x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\,dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Question 9

Question Number	Scheme	Marks
(a)	Complete method for R: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2+2^2)}$	M1
	$R = \sqrt{13}$ or 3.61 (or more accurate)	A1
	Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]	M1
	$\alpha = 0.588$ (Allow 33.7°)	A1 (4)
(b)	Greatest value $= \left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
(c)	$\sin(x+0.588) = \frac{1}{\sqrt{13}}$ $(= 0.27735\ldots)$ $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
	$(x+0.588) = 0.281(03\ldots)$ or 16.1°	A1
	$(x+0.588) = \pi - 0.28103\ldots$ Must be $\pi -$ their 0.281 or 180° − their 16.1°	M1
	or $(x+0.588) = 2\pi + 0.28103\ldots$ Must be $2\pi +$ their 0.281 or 360° + their 16.1°	M1
	$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only) If 0.281 or 16.1° not seen, correct answers imply this A mark	A1 (5) **(11 marks)**

Notes: (a) 1st M1 for correct method for R
2nd M1 for correct method for $\tan\alpha$
No working at all: M1A1 for √13, M1A1 for 0.588 or 33.7°.
N.B. $R\cos\alpha = 2$, $R\sin\alpha = 3$ used, can still score M1A1 for R, but loses the A mark for α.
$\cos\alpha = 3$, $\sin\alpha = 2$: apply the same marking.

(b) M1 for realising $\sin(x+\alpha) = \pm 1$, so finding R^4.

(c) Working in mixed degrees/rads : first two marks available
Working consistently in degrees: Possible to score first 4 marks
[Degree answers, just for reference only, are 130.2° and 342.4°]
Third M1 can be gained for candidate's 0.281 − candidate's 0.588 + 2π or equiv. in degrees
One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ — M1
$[13\cos^2 x - 4\cos x - 8 = 0,\quad 13\sin^2 x - 6\sin x - 3 = 0]$
Correct values for $\cos x =$ 0.953…, −0.646; or $\sin x = 0.767$, 2.27 awrt — A1
For any one value of $\cos x$ or $\sin x$, correct method for two values of x — M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1
Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding — M1

(ii) Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$
$9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ — M1
$(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0\ldots$ (α) — A1
$(2x - 1.176) = 2\pi - \alpha,\ 2\pi + \alpha, \ldots\ldots$ — M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1
Checking other values and discarding — M1

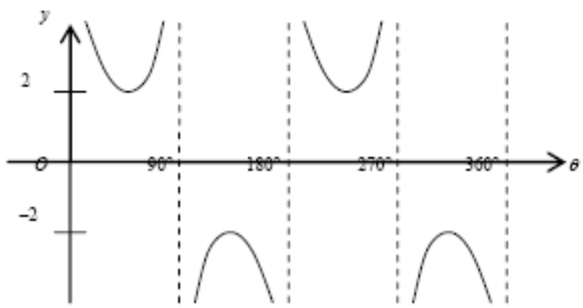
Question 10

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>(a)

Alt.(a)</td><td colspan="2">$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$
M1 Use of common denominator to obtain single fraction
$=\frac{1}{\cos\theta\sin\theta}$
M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)
$=\frac{1}{\frac{1}{2}\sin 2\theta}$ Use of $\sin 2\theta=2\sin\theta\cos\theta$
$=2\text{cosec}2\theta$ (✱)

$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}$ M1
$=\frac{\sec^2\theta}{\tan\theta}$ M1
$=\frac{1}{\cos\theta\sin\theta}=\frac{1}{\frac{1}{2}\sin 2\theta}$ M1
$=2\text{cosec}2\theta$ (✱) (cso) A1
If show two expressions are equal, need conclusion such as QED, tick, true.</td><td>M1

M1

M1
A1 cso (4)</td></tr>
<tr><td>(b)</td><td></td><td>Shape
(May be translated but need to see 4"sections")

T.P.s at $y=\pm 2$, asymptotic at correct x-values (dotted lines not required)</td><td>B1

B1 dep. (2)</td></tr>
</table>

Question 11

Question Number	Scheme		Marks
(a)	$\dfrac{dP}{dt} = kP$ and $t=0,\ P = P_0$ (1)		
	$\int \dfrac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt; (+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t=0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \quad \Rightarrow e^{\ln P} = e^{kt+\ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
(b)	$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow \quad t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399\text{min}}$ or $t = \underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6 \text{ hr } 39 \text{ mins}}$	A1
			[3]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

(c)	$\frac{dP}{dt} = \lambda P \cos\lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda\cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda\cos\lambda t\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin\lambda t;(+c)$	Must see $\ln P$ and $\sin\lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin\lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin\lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin\lambda t + \ln P_0} = e^{\sin\lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin\lambda t}}$	$\underline{P = P_0 e^{\sin\lambda t}}$	A1
			[4]
(d)	$P = 2P_0\ \&\ \lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ ...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin\lambda t = \ln 2}$	Eliminates P_0 and makes $\sin\lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5}\sin^{-1}(\ln 2)}$	Then rearranges to make t the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477...$		
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441}$min or $t = \underline{7\text{ hr } 21\text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7\text{ hr } 21\text{ mins}}$	A1
			[3]
			14 marks

$\underline{P = P_0 e^{\sin\lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

	$\frac{dP}{dt} = kP$ and $t=0,\ P=P_0$ (1)		
***Aliter* (a) Way 2**	$\int \frac{dP}{kP} = \int 1\,dt$	Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln P = t;(+c)$	Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow e^{\ln P} = e^{kt+\ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 **[4]**
***Aliter* (a) Way 3**	$\int \frac{dP}{kP} = \int 1\,dt$	Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln(kP) = t;(+c)$	Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$ $\left(\text{or } kP = Ae^{kt} \Rightarrow kP_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$ $\Rightarrow e^{\ln(kP)} = e^{kt+\ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$ $\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0 e^{kt}$ $\left(\text{or } kP = kP_0e^{kt}\right)$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 **[4]**

***Aliter* (c) Way 2**	$\dfrac{dP}{dt} = \lambda P\cos\lambda t$ and $t=0,\ P = P_0$ (1)		
	$\displaystyle\int \frac{dP}{\lambda P} = \int \cos\lambda t\ dt$	Separates the variables with $\displaystyle\int \frac{dP}{\lambda P}$ and $\displaystyle\int \cos\lambda t\,dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t;(+c)$	Must see $\frac{1}{\lambda}\ln P$ and $\frac{1}{\lambda}\sin\lambda t$; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \frac{1}{\lambda}\ln P_0 = c$ $\left(\text{or } P = Ae^{\sin\lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t + \frac{1}{\lambda}\ln P_0 \Rightarrow \ln P = \sin\lambda t + \ln P_0$		
	$\Rightarrow e^{\ln P} = e^{\sin\lambda t + \ln P_0} = e^{\sin\lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0\,e^{\sin\lambda t}}$	$\underline{P = P_0\,e^{\sin\lambda t}}$	A1 **[4]**

$\underline{P = P_0\,e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

$\underline{P = P_0\,e^{\sin\lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
***Aliter* (c) Way 3**	$\int \frac{dP}{\lambda P} = \int \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t; (+c)$	Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \ \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$ $\left(\text{or } \lambda P = Ae^{\sin \lambda t} \ \Rightarrow \lambda P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t + \frac{1}{\lambda}\ln(\lambda P_0)$ $\Rightarrow \ \ln(\lambda P) = \sin \lambda t + \ln(\lambda P_0)$ $\Rightarrow e^{\ln(\lambda P)} = e^{\sin \lambda t + \ln(\lambda P_0)} = e^{\sin \lambda t}.e^{\ln(\lambda P_0)}$ $\Rightarrow \lambda P = e^{\sin \lambda t}.(\lambda P_0)$ $\left(\text{or } \lambda P = \lambda P_0 e^{\sin \lambda t}\right)$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1 **[4]**

Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.
ft denotes "follow through"
cao denotes "correct answer only"
aef denotes "any equivalent form"