Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M7

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Use proof by contradiction to prove the statement: 'The product of two odd numbers is odd.' **(3)**

(Total 3 marks)

Question 2

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(a) Find the values of the constants *A*, *B* and *C*. **(4)**

(b) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,\mathrm{d}x$ is 2 + ln *k*, giving the value of the constant *k*. **(6)**

(Total 10 marks)

Question 3

A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *t*. You need not simplify your answer. **(3)**

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.
Give your answer in the form $y = ax + b$, where *a* and *b* are constants to be determined. **(5)**

(c) Find a cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$. **(4)**

(Total 12 marks)

Question 4

A curve C has equation

$$y = x^2e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation. **(3)**

(b) Hence find the coordinates of the turning points of C. **(3)**

(c) Find $\frac{d^2y}{dx^2}$. **(2)**

(d) Determine the nature of each turning point of the curve C. **(2)**

(Total 10 marks)

Question 5

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n. **(1)**

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$. **(3)**

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000, **(2)**

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000. **(3)**

(Total 9 marks)

Question 6

The functions f and g are defined by

$$\mathrm{f}: x \mapsto \ln(2x-1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$\mathrm{g}: x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(a) Find the exact value of fg(4). **(2)**

(b) Find the inverse function $\mathrm{f}^{-1}(x)$, stating its domain. **(4)**

(c) Sketch the graph of $y = |\,\mathrm{g}(x)\,|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis. **(3)**

(d) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3.$ **(3)**

(Total 12 marks)

Question 7

Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x .$$

(6)

(Total 6 marks)

Question 8

(a) Find $\int x\cos 2x\,\mathrm{d}x$. **(4)**

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x\cos^2 x\,\mathrm{d}x$. **(3)**

(Total 7 marks)

Question 9

(a) Express 3 sin x + 2 cos x in the form R sin(x + α) where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. **(2)**

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places. **(5)**

(Total 11 marks)

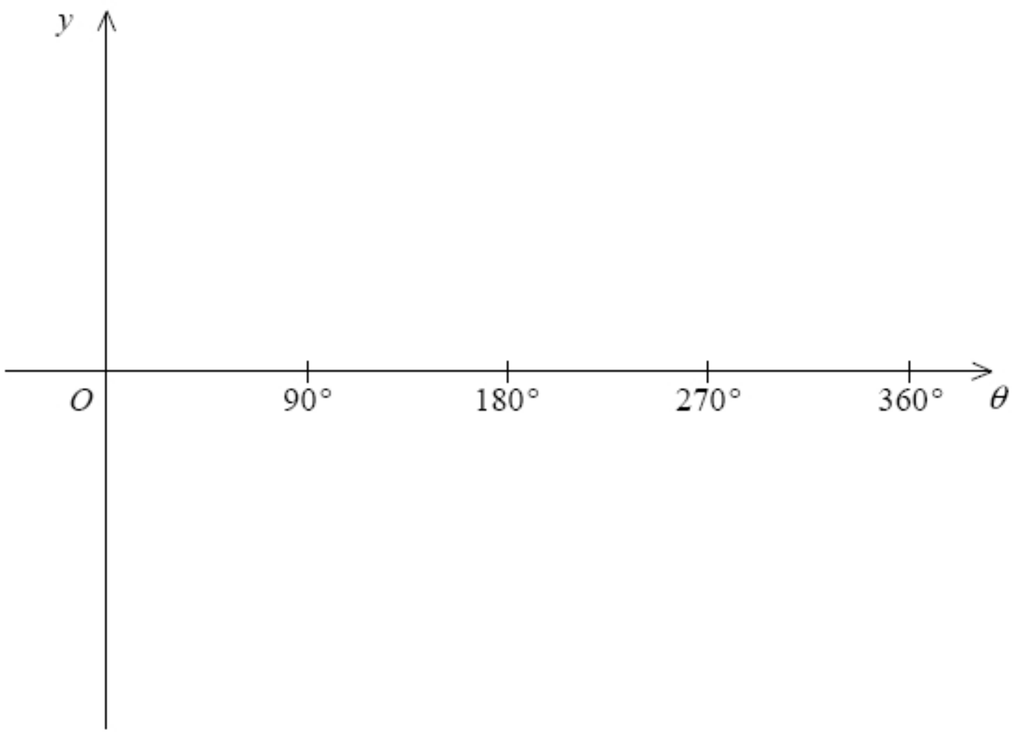
Question 10

(a) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta, \qquad \theta \neq 90n°.$$

(4)

(b) On the axes below, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$. **(2)**



(Total 12 marks)

Question 11

A population growth is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\ ,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t. **(4)**

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$. **(3)**

In an improved model the differential equation is given as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t\ ,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t. **(4)**

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model. **(3)**

(Total 14 marks)

TOTAL FOR PAPER IS 100 MARKS