Pure Mathematics 2 Practice Paper M8 MARK SCHEME

Question 1

Question Number	Scheme	Marks
(*a*)	5, 7, 9, 11 or $5+2+2+2=11$ or $5+6=11$	
	use $a=5$, $d=2$, $n=4$ and $t_4=5+3\times 2=11$	B1 (1)
(*b*)	$t_n=a+(n-1)d$ with one of $a=5$ or $d=2$ correct	M1
	$=5+2(n-1)$ or $2n+3$ or $1+2(n+1)$	A1 (2)
(*c*)	$S_n=\frac{n}{2}[2\times 5+2(n-1)]$ or use of $\frac{n}{2}(5+\text{"their } 2n+3\text{"})$	M1 A1
	$=\{n(5+n-1)\}=n(n+4)$ (*)	A1 cso (3)
(*d*)	$43=2n+3$	M1
	$[n]=20$	A1 (2)
(*e*)	$S_{20}=20\times 24$, $=\underline{480}$ (km)	M1 A1 (2)
		(10 marks)

Question 2

Question Number	Scheme	Marks
(a)	$x^2-2x-3=(x-3)(x+1)$	B1
	$\mathrm{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}\quad\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ *	A1 cso (4)
(b)	$\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\mathrm{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
(c)	Let $y=\mathrm{f}(x)$ $\quad y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\mathrm{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$	B1 ft (3)
(d)	$\mathrm{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		(12 marks)

Question 3

Question Number	Scheme	Marks
(a)	$\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ so $2 \equiv A(2+y) + B(2-y)$	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$, Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	M1
	giving $\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$	A1 cao (3)
(b)	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$	
	$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) \; + (c)$	B1 M1 A1 ft
	$y=0,\ x=\frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	M1
	$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$	
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$	
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$	
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	M1
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$	
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 (8)
		(11 marks)

Question 4

Question Number	Scheme	Marks
(a)	$f(1.6) = \ldots \qquad f(1.7) = \ldots$ (Evaluate both)	M1
	$0.08\ldots$ (or 0.09), $\quad -0.3\ldots$ One +ve, one −ve or sign change, $\therefore$ root	A1 (2)
(b)	$f'(x) = -4\sin x - e^{-x}$	B1
	$1.6 - \dfrac{f(1.6)}{f'(1.6)}$	M1
	$= 1.6 - \dfrac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})} \qquad \left(= 1.6 - \dfrac{0.085\ldots}{-4.2\ldots}\right)$	A1
		A1 (4)
		(6 marks)

Question 5

Question Number	Scheme	Marks
(a)(i)	$\dfrac{d}{dx}\left(e^{3x}(\sin x + 2\cos x)\right) = 3e^{3x}(\sin x + 2\cos x) + e^{3x}(\cos x - 2\sin x)$	M1 A1 A1 (3)
	$\left(= e^{3x}(\sin x + 7\cos x)\right)$	
(ii)	$\dfrac{d}{dx}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \dfrac{5x^3}{5x+2}$	M1 A1 A1 (3)
(b)	$\dfrac{dy}{dx} = \dfrac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\dfrac{\text{A1}}{\text{A1}}$
	$= \dfrac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$= \dfrac{20}{(x+1)^3}$ *	A1 cso (5)
(c)	$\dfrac{d^2y}{dx^2} = -\dfrac{60}{(x+1)^4} = -\dfrac{15}{4}$	M1
	$(x+1)^4 = 16$	M1
	$x = 1, -3$ both	A1 (3)
		(14 marks)

Question 6

Question Number	Scheme	Marks
(a)	From question, $\frac{dA}{dt} = 0.032$	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}$; $\left\{= \frac{0.016}{\pi x}\right\}$	M1
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$	
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	A1 cso (4)
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	B1 ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right)$; $\{= 0.24x\}$	M1
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\,\text{s}^{-1})$	A1 (4)
		(8 marks)

Question 7

Question Number	Scheme	Marks
(*a*)	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$	
	$\left\{\frac{dx}{dx} \times\right\} \quad \underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	M1 B1 A1
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x-y}{x-2y} = \frac{8}{3}$	M1
	giving $-18x - 3y = 8x - 16y$	
	giving $13y = 26x$	M1
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	A1 cso (6)
(*b*)	At P & Q, $y = 2x$. Substituting into eqn *	
	gives $3x^2 - (2x)^2 + x(2x) = 4$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	A1
	$y = 2x \Rightarrow y = \pm 4$, hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	A1 (3)
		(9 marks)

Question 8

Question Number	Scheme	Marks
(a)	$\left\{\begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$	
	$\int xe^x\, dx = xe^x - \int e^x.1\, dx$	M1 A1
	$= xe^x - \int e^x\, dx$	
	$= xe^x - e^x\ (+c)$	A1 (3)
(*b*)	$\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$	
	$\int x^2e^x\, dx = x^2e^x - \int e^x.2x\, dx$	M1 A1
	$= x^2e^x - 2\int xe^x\, dx$	
	$= x^2e^x - 2\left(xe^x - e^x\right) + c$	A1 (3)
		(6 marks)

Question 9

Question Number	Scheme	Marks
(a)	$\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta \qquad \frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\text{cosec}^2\theta$ ✱	A1 cso (2)
(b)	$2\left(\text{cosec}^2\theta-1\right)-9\,\text{cosec}\,\theta=3$	M1
	$2\,\text{cosec}^2\theta-9\,\text{cosec}\,\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$(2\,\text{cosec}\,\theta+1)(\text{cosec}\,\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$	M1
	$\text{cosec}\,\theta=5$ *or* $\sin\theta=\frac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6)
		(8 marks)

Question 10

Question Number	Scheme	Marks
(a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
(b)	$x = 8\cos t, \quad y = 4\sin 2t$	
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	
	Hence m(N) $= -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	M1
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
(c)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$	
	$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$ (*)	A1 (4)
(d)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
		(16 marks)