Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M8

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 10 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km. **(1)**

(b) Find an expression, in terms of n, for the length of her training run on the nth Saturday. **(2)**

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km. **(3)**

On the nth Saturday Sue runs 43 km.

(d) Find the value of n. **(2)**

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training. **(2)**

(Total 10 marks)

Question 2

The function f is defined by

$$f: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}$, $x > 3$. **(4)**

(b) Find the range of f. **(2)**

(c) Find $f^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$g: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$. **(3)**

(Total 12 marks)

Question 3

(a) Express $\frac{2}{4-y^2}$ in partial fractions. **(3)**

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4 - y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$. **(8)**

(Total 11 marks)

Question 4

$$f(x) = 4\cos x + e^{-x}.$$

(a) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7 **(2)**

(b) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures. **(4)**

(Total 6 marks)

Question 5

(a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$, **(3)**

(ii) $x^3 \ln(5x + 2)$. **(3)**

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq 1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. **(5)**

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. **(3)**

(Total 14 marks)

Question 6

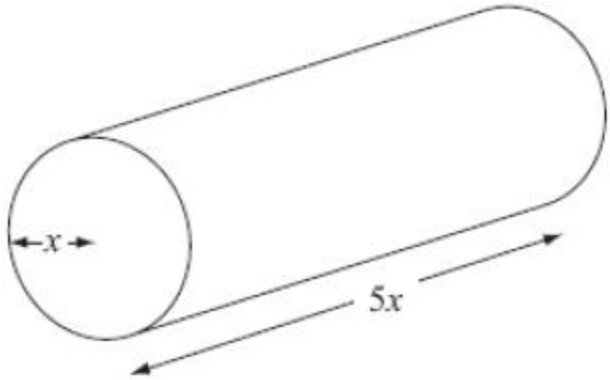

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of 0.032 cm^2 s^{-1}.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures. **(4)**

(b) Find the rate of increase of the volume of the rod when $x = 2$. **(4)**

(Total 8 marks)

Question 7

A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q. **(6)**

(b) Find the coordinates of P and Q. **(3)**

(Total 9 marks)

Question 8

(a) Use integration by parts to find $\int xe^x \ dx$. **(3)**

(b) Hence find $\int x^2e^x \ dx$. **(3)**

(Total 6 marks)

Question 9

(a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$ **(2)**

(b) Solve, for $0 \le \theta < 180°$, the equation

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place. **(6)**

(Total 8 marks)

Question 10

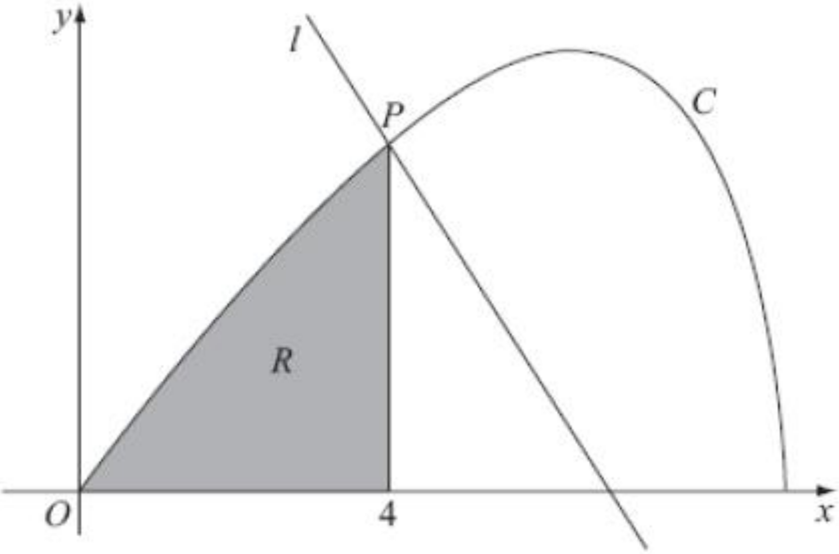

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P. **(2)**

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$. **(6)**

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$. **(4)**

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined. **(4)**

(Total 16 marks)

TOTAL FOR PAPER IS 100 MARKS