Pure Mathematics 2 Practice Paper M9 MARK SCHEME

Question Number	Scheme	٨	Marks
Q (a)	$(a_2 =)2k - 7$	B1	(1)
(b)	$(a_3 =)2(2k-7)-7 \text{ or } 4k-14-7, = 4k-21$ (*)	M1,	A1cso
		-	(2)
(c)	$(a_4 =) 2(4k-21)-7 = (8k-49)$	M1	
	$(a_4 =) 2(4k-21)-7 (=8k-49)$ $\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$ $k + (2k-7) + (4k-21) + (8k-49) = 15k-77 = 43 \qquad k = 8$	М1	
	k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43 $k = 8$	M1 A	A1 (4) [7]
(b)	M1 must see 2(their a_2) - 7 or $2(2k-7)-7$ or $4k-14-7$. Their a_2 must be a		
	function of k.		
	A1cso must see the $2(2k-7)-7$ or $4k-14-7$ expression and the $4k-21$ with no		
(c)	incorrect working 1^{st} M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k$ - 49		
	Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks. 2 nd M1 for attempting the sum of the 1 st 4 terms. Must have "+" not just, or clear		
	attempt to sum.		
	Follow through their a_2 and a_4 provided they are linear functions of k .		
	Must lead to linear expression in k. Condone use of their linear $a_3 \neq 4k-21$		
	here too. 3^{rd} M1 for forming a linear equation in k using their sum and the 43 and attempt to		
	solve for k as far as $pk = q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0		
	Answer Only (e.g. trial improvement)		
	Accept $k = 8$ only if $8 + 9 + 11 + 15 = 43$ is seen as well		
	$\underline{\text{Sum}}_{a_2} + a_3 + a_4 + a_5 \text{ or } a_2 + a_3 + a_4$		
	Allow: M1 if 8k - 49 is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1		
	if they use their sum along with the 43 to form a linear equation and attempt to solve but A0		



(a)	$g(x) = \frac{e^x - 3}{e^x - 2} x \in \mathbb{R}, \ x \neq \ln 2.$			
	Apply quotient rule: $\begin{cases} u = e^x - 3 \\ \frac{du}{dx} = e^x \end{cases}$	$v = e^{x} - 2$ $\frac{dv}{dx} = e^{x}$		
	$g'(x) = \frac{e^{x}(e^{x}-2) - e^{x}(e^{x}-3)}{(e^{x}-2)^{2}}$	Applying $\frac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1	
	$= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$			
	$=\frac{e^x}{(e^x-2)^2}$	Correct result	A1 AG	(3)

Question Number	Scheme		Marks
(b)	$g'(x) = 1 \implies \frac{e^x}{(e^x - 2)^2} = 1$		
	9 - 19 - /	eir differentiated numerator equal to their denominator.	M1
	$e^{2x} - 5e^x + 4 = 0$	$e^{2x} - 5e^x + 4$	A1
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e ^x	M1
	$e^x = 4$ or $e^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0$, $\ln 4$	A1
			[1

Question Number	Scheme	Marks	
Q (a)	Curve retains shape when $x > \frac{1}{2} \ln k$	B1	
	Curve reflects through the x-axis when $x < \frac{1}{2} \ln k$	B1	
	O $(\frac{1}{2}\ln k, 0)$ x $(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1	(2)
(b)	Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) $(1-k,0)$	В1	(3)
	$(1-k,0) \text{ and } (0,\frac{1}{2}\ln k)$	B1	
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	B1	(2)
(d)	$y = e^{2x} - k \implies y + k = e^{2x}$ Attempt to make x (or swapped y) the subject	м1	(1)
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$ Makes e^{2x} the subject and takes ln of both sides	M1	
	Hence $f^{-1}(x) = \frac{1}{2}\ln(x+k)$ or $\frac{\ln\sqrt{(x+k)}}{\ln(x+k)}$	A1 cao	(3)
(e)	Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or RANGE answer	B1√	(1)
		[10]



Question Number	Scheme		rks
Q (a)	$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$		
	4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)	M1	
	A method for evaluating one constant	M1	
	$x \to -\frac{1}{2}$, $5 = A(\frac{1}{2})(\frac{5}{2}) \Rightarrow A = 4$ any one correct constant	A1	
	$x \to -1$, $6 = B(-1)(2) \Rightarrow B = -3$		
	$x \to -3$, $10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct	A1	(4)
(b)	(i) $\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3}\right) dx$		
	$= \frac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ A1 two ln terms correct	M1 A1	ft
	All three ln terms correct and "+C"; ft constants	A1ft	(3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$		
	$= (2 \ln 5 - 3 \ln 3 + \ln 5) - (2 \ln 1 - 3 \ln 1 + \ln 3)$	M1	
	$= 3 \ln 5 - 4 \ln 3$		
	$=\ln\left(\frac{5^3}{3^4}\right)$	М1	
	$= \ln\left(\frac{125}{81}\right)$	A1	(3)
			[10]



Question Number	Scheme	Marks
Q (a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -4\sin 2t \; , \frac{\mathrm{d}y}{\mathrm{d}t} = 6\cos t$	B1, B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{6\cos t}{4\sin 2t} \left(= -\frac{3}{4\sin t} \right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}, \sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)} \left(=3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2 \qquad \qquad k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) where the parameter is eliminated	
	① $y = (18 - 9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2} (18 - 9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos \frac{2\pi}{3} = -1$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	$y^2 = 18 - 9x$	
	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = -9$	B1
	At $t = \frac{\pi}{3}$, $y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{2 \times 3 \sqrt{3}} = -\frac{\sqrt{3}}{2}$	



Question Number	Scheme	Marks
Q (i)(a)	$y = x^{2} \cos 3x$ Apply product rule: $\begin{cases} u = x^{2} & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{cases}$	
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$ Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$ Any one term correct Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	M1 A1 A1
(b)	$y = \frac{\ln(x^2 + 1)}{x^2 + 1}$	(3
	$u = \ln(x^2 + 1) \implies \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2 + 1}$ $\ln(x^2 + 1) \implies \frac{\mathrm{something}}{x^2 + 1}$ $\ln(x^2 + 1) \implies \frac{2x}{x^2 + 1}$	
	Apply quotient rule: $\begin{cases} u = \ln(x^2 + 1) & v = x^2 + 1 \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} & \frac{dv}{dx} = 2x \end{cases}$	
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}$ Applying $\frac{vu' - uv'}{v^2}$ Correct differentiation with correct bracketing but allow recovery.	M1 A1
	$\left\{ \frac{dy}{dx} = \frac{2x - 2x \ln(x^2 + 1)}{\left(x^2 + 1\right)^2} \right\}$ {Ignore subsequent working.}	



Question Number	Sch	eme	Marks
(ii)	$y = \sqrt{4x+1}, \ x > -\frac{1}{4}$		
	At P , $y = \sqrt{4(2) + 1} = \sqrt{9} = 3$	At P , $y = \sqrt{9}$ or 3	
	$\frac{dy}{dx} = \frac{1}{2} (4x+1)^{-\frac{1}{2}} (4)$	$\pm k (4x+1)^{-\frac{1}{2}}$ $2 (4x+1)^{-\frac{1}{2}}$	M1*
	dx = 2	$2(4x+1)^{\frac{1}{2}}$	A1 aef
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation	M1
	$dx = (4(2)+1)^{\frac{1}{2}}$	involving $\frac{dv}{dx}$;	mi
	Hence $m(T) = \frac{2}{3}$		
		$y - y_1 = m(x - 2)$	
	Either T: $y-3=\frac{2}{3}(x-2)$;	or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and	
	or $y = \frac{2}{3}x + c$ and	their y_1 ; or uses $y = mx + c$ with	dM1*;
	$3 = \frac{2}{3}(2) + c \implies c = 3 - \frac{4}{3} = \frac{5}{3};$	'their TANGENT gradient', their x and their y_1 .	
	Either T: $3y-9=2(x-2)$;		
	T: $3y-9=2x-4$		
	T: $2x - 3y + 5 = 0$	2x - 3y + 5 = 0	A1
		Tangent must be stated in the form $ax + by + c = 0$, where a, b and c	
		are integers.	
	7 2 5		
	or T: $y = \frac{2}{3}x + \frac{5}{3}$		
	T: $3y = 2x + 5$		
	T: $2x - 3y + 5 = 0$		
			[1



Question Number	Scheme	Mark	s
Q (a)	$e^{-2x} \frac{dy}{dx} - 2y e^{-2x} = 2 + 2y \frac{dy}{dx}$ A1 correct RHS $\frac{d}{dx} (y e^{-2x}) = e^{-2x} \frac{dy}{dx} - 2y e^{-2x}$	M1 A1	
	$\frac{dx}{dx}(ye^{-y}) = e^{-x} \frac{dy}{dx} - 2ye^{-2x}$ $(e^{-2x} - 2y)\frac{dy}{dx} = 2 + 2ye^{-2x}$	B1 - M1	
	$\frac{dy}{dx} = \frac{2 + 2y e^{-2x}}{e^{-2x} - 2y}$	A1	(5
(b)	At P, $\frac{dy}{dx} = \frac{2 + 2e^0}{e^0 - 2} = -4$	м1	
	Using $mm' = -1$ $m' = \frac{1}{4}$	M1	
	$y-1=\frac{1}{4}(x-0)$	M1	
	x-4y+4=0 or any integer multiple	A1	(4
			[9
	Alternative for (a) differentiating implicitly with respect to y.		
	$e^{-2x} - 2ye^{-2x}\frac{dx}{dy} = 2\frac{dx}{dy} + 2y$ A1 correct RHS	M1 A1	
	$\frac{\mathrm{d}}{\mathrm{d}y}\left(y\mathrm{e}^{-2x}\right) = \mathrm{e}^{-2x} - 2y\mathrm{e}^{-2x}\frac{\mathrm{d}x}{\mathrm{d}y}$	B1	
	$(2+2ye^{-2x})\frac{dx}{dy} = e^{-2x} - 2y$	М1	
	$\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2y e^{-2x}}$ $\frac{dy}{dx} = \frac{2 + 2y e^{-2x}}{e^{-2x} - 2y}$		
	$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	A1	(5



Question Number	Scheme	Mark	s
Q (a)	$\int \sqrt{(5-x)} dx = \int (5-x)^{\frac{1}{2}} dx = \frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} (+C)$ $\left(= -\frac{2}{3} (5-x)^{\frac{3}{2}} + C \right)$	M1 A1	(2)
(b)	(i) $\int (x-1)\sqrt{(5-x)} dx = -\frac{2}{3}(x-1)(5-x)^{\frac{1}{2}} + \frac{2}{3}\int (5-x)^{\frac{1}{2}} dx$ $= \qquad \qquad +\frac{2}{3} \times \frac{(5-x)^{\frac{1}{2}}}{-\frac{5}{2}} (+C)$ $= -\frac{2}{3}(x-1)(5-x)^{\frac{1}{2}} - \frac{4}{15}(5-x)^{\frac{1}{2}} (+C)$	M1 A1ft - M1 - A1	(4)
	(ii) $\left[-\frac{2}{3} (x-1) (5-x)^{\frac{3}{2}} - \frac{4}{15} (5-x)^{\frac{5}{2}} \right]_{1}^{5} = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}} \right)$ $= \frac{128}{15} \left(= 8 \frac{8}{15} \approx 8.53 \right) \text{awrt } 8.53$	M1 A1	(2
	Alternatives for (b) and (c) (b) $u^2 = 5 - x \implies 2u \frac{du}{dx} = -1$ $\left(\Rightarrow \frac{dx}{du} = -2u\right)$ $\int (x-1)\sqrt{(5-x)} dx = \int (4-u^2)u \frac{dx}{du} du = \int (4-u^2)u(-2u) du$ $= \int (2u^4 - 8u^2) du = \frac{2}{5}u^5 - \frac{8}{3}u^3 (+C)$ $= \frac{2}{5}(5-x)^{\frac{3}{2}} - \frac{8}{3}(5-x)^{\frac{3}{2}} (+C)$	- M1 A1 - M1 - A1	
	(c) $x = 1 \Rightarrow u = 2, x = 5 \Rightarrow u = 0$ $\left[\frac{2}{5}u^5 - \frac{8}{3}u^3\right]_2^0 = (0 - 0) - \left(\frac{64}{5} - \frac{64}{3}\right)$ $= \frac{128}{15} \left(=8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1	(2)



Question Number		Scheme	Ma	arks	
Q (a)	$\sin 2x = \underline{2\sin x \cos x}$	$\frac{2\sin x \cos x}{2}$ B	1 a	ef	(1
(b)	$\frac{1}{\sin x} - 8c$ $\frac{1}{\sin x} = 8c$		1		
	$\sin x$ $1 = 8\sin x$ $1 = 4(2\sin x)$	cosx			
	$1 = 4\sin 2x$ $\sin 2x = \frac{1}{4}$	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	1		
	,	$\sin 2x = \frac{1}{4}$ A $(268, 2.88891)$	1		
		Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. Roth 0.13 and 1.44	1 1 ca	20	
		Both 0.13 and 1.44 Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	1 6		[6]



Question Number	Si	cheme	Mari	KS
Q (a)	$\cos^2\theta + \sin^2\theta = 1 \ \left(+ \cos^2\theta \right)$			
	$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$	Dividing $\cos^2 \theta + \sin^2 \theta = 1$ by $\cos^2 \theta$ to give <u>underlined</u> equation.	M1	
	$1 + \tan^2 \theta = \sec^2 \theta$ $\tan^2 \theta = \sec^2 \theta - 1$ (as required) AG	Complete proof. No errors seen.	A1 cso	(2
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, (\text{eqn *})$	0 ≤ θ < 360°		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2 \theta = \sec^2 \theta - 1$ into eqn * to get a quadratic in $\sec \theta$ only	M1	
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$			
	$3\sec^2\theta + 4\sec\theta - 4 = 0$	Forming a three term "one sided" quadratic expression in $\sec \theta$.	M1	
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1	
	$\sec \theta = -2$ or $\sec \theta = \frac{2}{3}$			
	$\frac{1}{\cos\theta} = -2 \text{or} \frac{1}{\cos\theta} = \frac{2}{3}$			
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$	$\cos\theta = -\frac{1}{2}$	A1;	
	$\alpha = 120^{\circ}$ or $\alpha = \text{no solutions}$			
	$\theta_1 = \underline{120}^{\circ}$	120°	<u>A1</u>	
	$\theta_2 = 240^\circ$	$\underline{240}^{\circ}$ or $\theta_2 = 360^{\circ} - \theta_1$ when solving using $\cos \theta =$	B1√	
	$\theta = \{120^{\circ}, 240^{\circ}\}$	Note the final A1 mark has been changed to a B1 mark.		(
				[8



Question Number	Scheme			Mark	s
Q (a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \frac{\cos A \cos A - \sin A \sin A}{2}$	Applies $A = B$ to $\cos(A + B)$ to give the <u>underlined</u> equation or $\cos 2A = \frac{\cos^2 A - \sin^2 A}{2}$	М1		
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives				
	$\frac{\cos 2A}{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \frac{1 - 2\sin^2 A}{\cos^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1	AG	(2
(b)	$C_1 = C_2 \implies 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1		
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k \sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	М1		
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$				
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$				
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1	AG	(:
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ $3\sin 2x + 4\cos 2x = R\cos 2x\cos \alpha + R\sin 2x\sin \alpha$				
	Equate $\sin 2x$: $3 = R \sin \alpha$ Equate $\cos 2x$: $4 = R \cos \alpha$				
	$R = \sqrt{3^2 + 4^2} \; ;= \sqrt{25} = 5$	<i>R</i> = 5	B1		
	$\tan \alpha = \frac{3}{4} \implies \alpha = 36.86989765^{\circ}$	$\tan \alpha = \pm \frac{3}{4}$ or $\tan \alpha = \pm \frac{4}{3}$ or $\sin \alpha = \pm \frac{3}{4 \text{ their } R}$ or $\cos \alpha = \pm \frac{4}{4 \text{ their } R}$ awrt 36.87	M1		
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$				



Question Number	Scher	ne	Ma	arks
(d)	$3\sin 2x + 4\cos 2x = 2$			
	$5\cos(2x - 36.87) = 2$			
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	М1	
	(2x-36.87) = 66.42182	awrt 66	A1	
	(2x-36.87) = 360 - 66.42182			
	Hence, $x = 51.64591$, 165.22409	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1	
		Both awrt 51.6 AND awrt 165.2	A1	,
		If there are any EXTRA solutions inside the range $0 \le x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^{\circ}$.		(-
				[12