Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M9

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k. **(1)**

(b) Show that $a_3 = 4k - 21$. **(2)**

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k. **(4)**

(Total 7 marks)

Question 2

The function g is defined by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2}, \qquad x \in \mathbb{R},\ x \neq \ln 2$$

(a) Differentiate g(x) to show that $\mathrm{g}'(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$ **(3)**

(b) Find the exact values of x for which $\mathrm{g}'(x) = 1$ **(4)**

(Total 7 marks)

Question 3

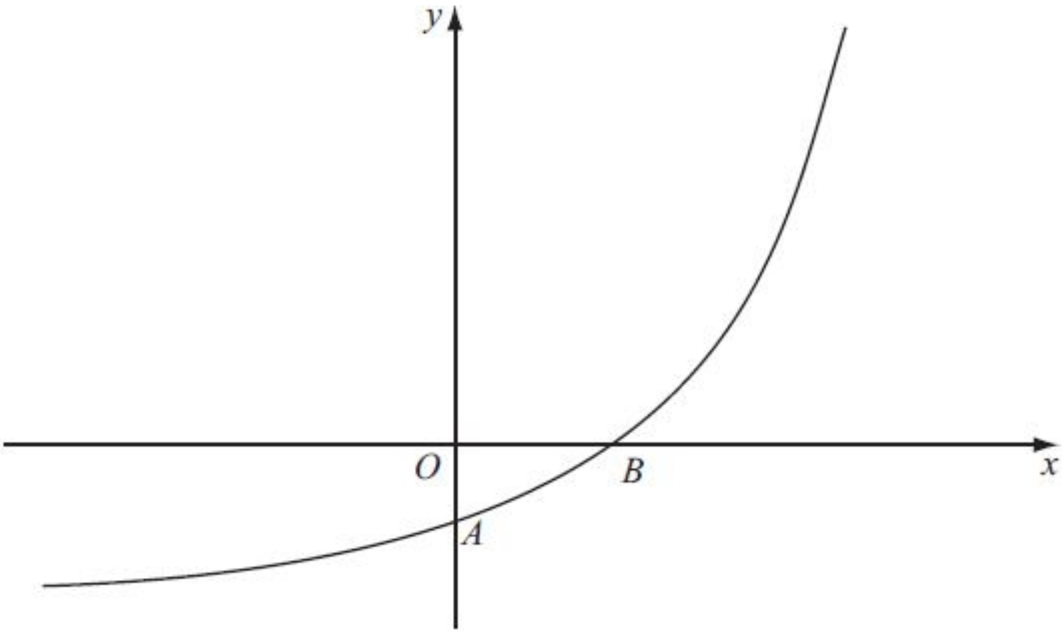

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x),\ x \in \mathbb{R}$. The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$, **(3)**

(b) $y = \mathrm{f}^{-1}(x)$. **(2)**

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f, **(1)**

(d) find $\mathrm{f}^{-1}(x)$, **(3)**

(e) write down the domain of f^{-1}. **(1)**

(Total 10 marks)

Question 4

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants A, B and C. (4)

(b) (i) Hence find $\int f(x)\,dx$. (3)

(ii) Find $\int_0^2 f(x)\,dx$ in the form $\ln k$, where k is a constant. (3)

(Total 10 marks)

Question 5

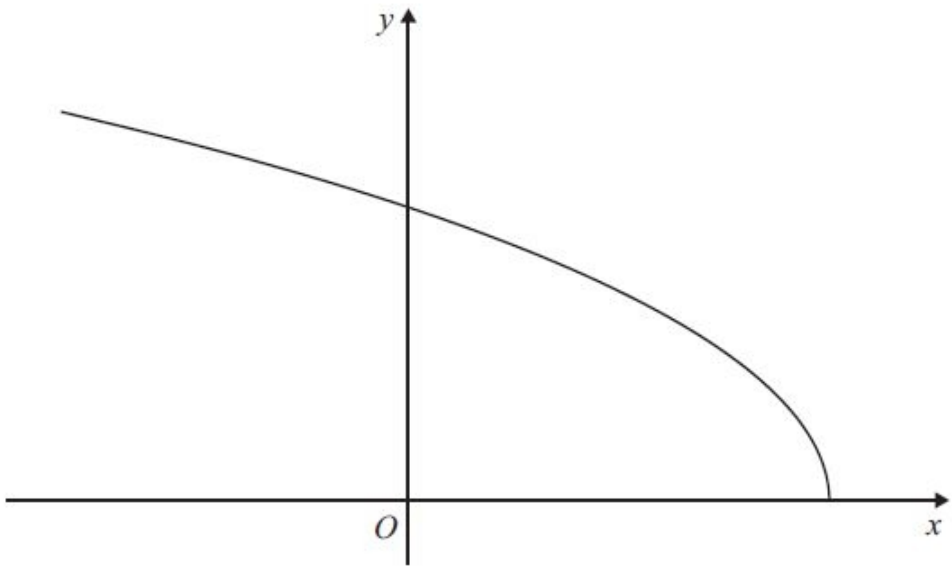

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$. (4)

(b) Find a cartesian equation of the curve in the form

$$y = f(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k. (4)

(c) Write down the range of $f(x)$. (2)

(Total 10 marks)

Question 6

(i) Differentiate with respect to x

(a) $x^2 \cos 3x$ (3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$ (4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(Total 13 marks)

Question 7

The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. (5)

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (4)

(Total 9 marks)

Question 8

(a) Find $\int \sqrt{(5-x)}\,dx$. **(2)**

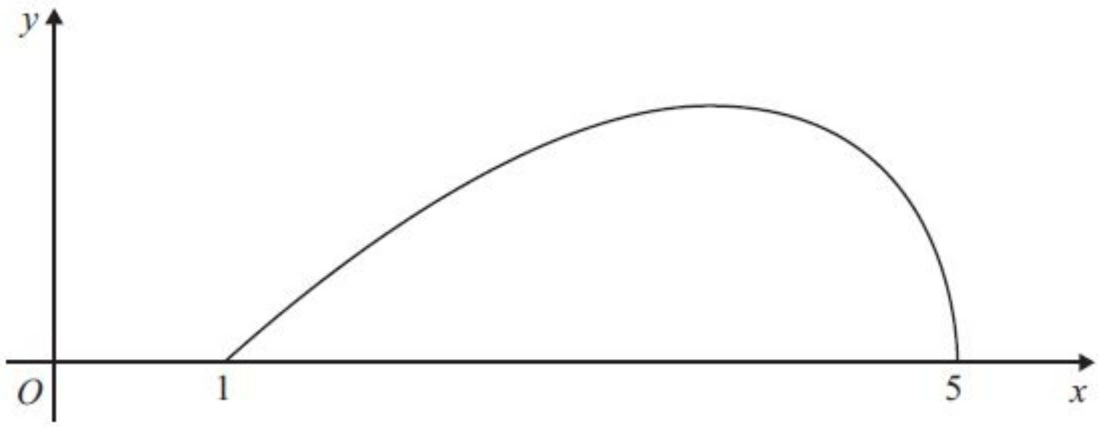

Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \qquad 1 \leqslant x \leqslant 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\,dx$$

(4)

(b) (ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,dx$. **(2)**

(Total 8 marks)

Question 9

(a) Write down sin 2*x* in terms of sin *x* and cos *x*. **(1)**

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places. **(5)**

(Total 6 marks)

Question 10

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$. **(2)**

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$ **(6)**

(Total 8 marks)

Question 11

(a) Use the identity $\cos(A + B) = \cos A\cos B - \sin A\sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$ **(2)**

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$ **(3)**

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places. **(3)**

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place. **(4)**

(Total 12 marks)

TOTAL FOR PAPER IS 100 MARKS