Name:

Total Marks:

Pure Mathematics 1

Advanced Subsidiary

Practice Paper J11

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

(a) On the axes below, sketch the graphs of

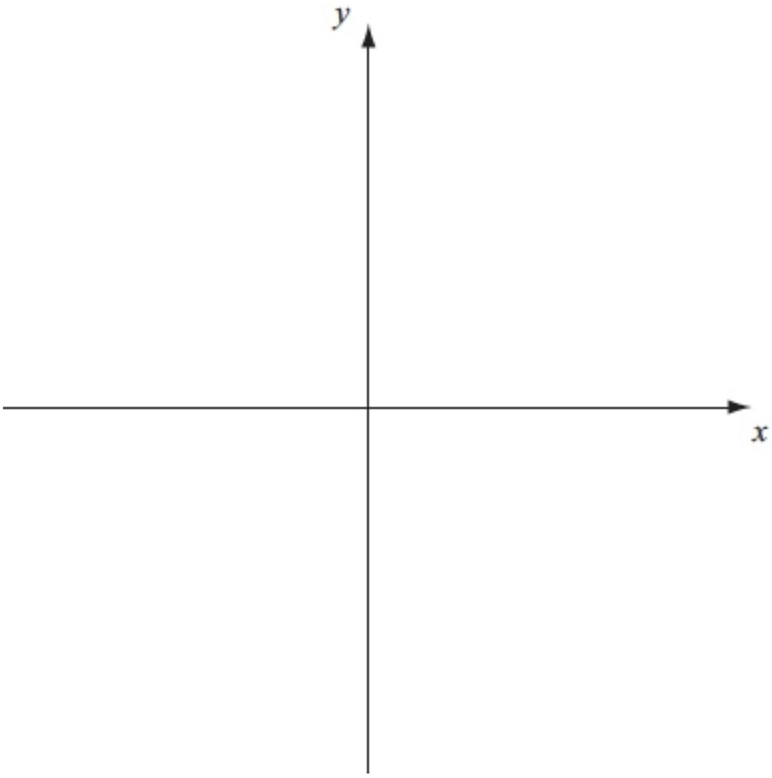
$y = x(x + 2)(3 - x)$

$y = -\dfrac{2}{x}$

showing clearly the coordinates of all the points where the curves cross the coordinate axes. **(5)**

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$x(x + 2)(3 - x) + \dfrac{2}{x} = 0$ **(2)**



(Total 8 marks)

Question 2

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A (1, 4) lies on L_1, find

(a) the value of k, **(1)**

(b) the gradient of L_1. **(2)**

The line L_2 passes through A and is perpendicular to L_1

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers **(4)**

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B. **(2)**

(e) Find the exact length of AB. **(2)**

(Total 11 marks)

Question 3

The points A and B have coordinates (−2, 11) and (8, 1) respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates (3, 6), **(1)**

(b) find an equation for C. **(4)**

(c) Verify that the point (10, 7) lies on C. **(1)**

(d) Find an equation of the tangent to C at the point (10, 7), giving your answer in the form $y = mx + c$, where m and c are constants. **(4)**

(Total 10 marks)

Question 4

The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$. **(4)**

(b) Show that the point $P(4,-8)$ lies on C. **(2)**

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(6)**

(Total 12 marks)

Question 5

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \qquad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$. **(4)**

(b) Hence find the maximum volume of the box. **(4)**

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. **(2)**

(Total 10 marks)

Question 6

(a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes. **(2)**

(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate. **(6)**

(Total 8 marks)

Question 7

Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A. **(2)**

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$. **(3)**

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places. **(3)**

(Total 8 marks)

Question 8

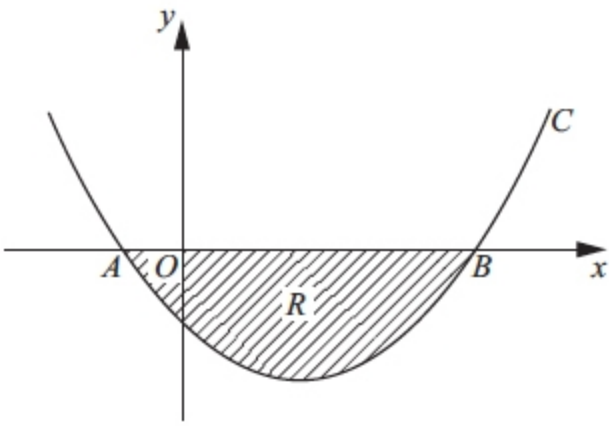


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B. **(1)**

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R. **(6)**

(Total 7 marks)

Question 9

(a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate. (5)

(Total 7 marks)

Question 10

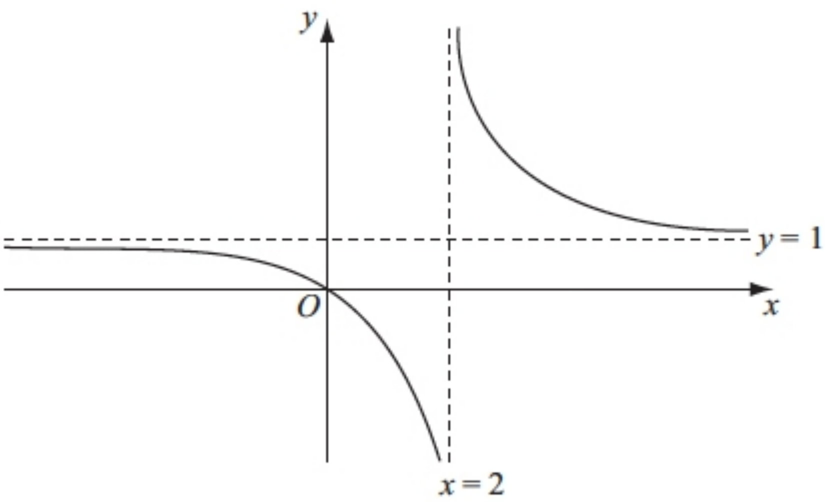


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x - 1)$ and state the equations of the asymptotes of this curve. (3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x - 1)$ crosses the coordinate axes. (4)

(Total 7 marks)

Question 11

The table shows the size of a population of house sparrows from 1980 to 2005.

Year	1980	1985	1990	1995	2000	2005
Population	25 000	22 000	18 750	16 250	13 500	12 000

The ‘red alert’ category for birds is used when a population has decreased by at least 50% in the previous 25 years

(i) Show that the information for this population is consistent with the house sparrow being on red alert in 2005. **(1)**

The size of the population may be modelled by a function of the form $P = a \times 10^{-kt}$, where P is the population, t is the number of years after 1980, and a and k are constants.

(ii) Write the equation $P = a \times 10^{-kt}$ in logarithmic form using base 10, giving your answer as simply as possible **(2)**

(iii) Complete the table and draw the graph of $\log_{10} P$ against t, drawing a line of best fit. **(3)**

(iv) Use your graph to find the values of a and k and hence the equation for P in terms of t **(4)**

(v) Find the size of the population in 2015 as predicted by this model

(vi) Would the house sparrow still be on red alert? Give a reason for your answer **(3)**

(Total 13 marks)

TOTAL FOR PAPER IS 100 MARKS