Pure Mathematics 1 Practice Paper M11 MARK SCHEME

Question1

Question Number	Scheme	Marks
(a)	Discriminant: $b^2-4ac=(k+3)^2-4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2-4k=k^2+2k+9=(k+1)^2+8$	M1 A1 (2)
(c)	For real roots, $b^2-4ac\geq 0$ or $b^2-4ac>0$ or $(k+1)^2+8>0$ $(k+1)^2\geq 0$ for all k, so $b^2-4ac>0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula b^2-4ac is seen at least 2 of a, b and c must be correct If formula b^2-4ac is **not** seen all 3 of a, b and c must be correct Use of b^2+4ac is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2+8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2\geq 0$ and conclusion. We will allow $(k+1)^2>0$ (or word positive) also allow $b^2-4ac\geq 0$ and conclusion.	

Question 2

Question Number	Scheme	Marks
(a)	$f(x) = 2x^3 - 7x^2 - 5x + 4$ Remainder $= f(1) = 2 - 7 - 5 + 4 = -6$ Attempts $f(1)$ or $f(-1)$. $= -6$ -6	M1 A1 [2]
(b)	$f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ Attempts $f(-1)$. and so $(x+1)$ is a factor. $f(-1) = 0$ with no sign or substitution errors **and for conclusion.**	M1 A1 [2]
(c)	$f(x) = \{(x+1)\}(2x^2 - 9x + 4)$ $= (x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).	M1 A1 dM1 A1 **[4]** **8**
(a) (b) (c)	M1 for *attempting* either $f(1)$ or $f(-1)$. Can be implied. Only one slip permitted. M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of x. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6. Award M1A1 for -6 without any working. M1: attempting only $f(-1)$. A1: must correctly show $f(-1) = 0$ and give a conclusion ***in part (b) only***. **Note**: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a preamble, eg: "If $f(-1) = 0$, $(x+1)$ is a factor…." **Note: Long division scores no marks in part (b). The factor theorem is required.** 1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a). 1st A1: For seeing $(2x^2 - 9x + 4)$. 2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly. 2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}, x = \frac{1}{2}, 4$, and not list all three factors. Award these responses M1A1M1A0. **Alternative:** 1st M1: For finding either $f(4) = 0$ or $f\left(\frac{1}{2}\right) = 0$. 1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark. 2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$. 2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$. **Alternative: (for the first two marks)** 1st M1: Expands $(x+1)(2x^2 + ax + b)$ {giving $2x^3 + (a+2)x^2 + (b+a)x + b$ } then compare coefficients to find values for a and b. 1st A1: $a = -9, b = 4$ **Not dealing with a factor of 2:** $(x+1)(x-\frac{1}{2})(x-4)$ or $(x+1)(x-\frac{1}{2})(2x-8)$ scores M1A1M1A0. **Answer only, with one sign error:** eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. **(c) Award M1A1M1A1 for Listing all three correct factors with no working.**	

Question 3

Question Number	Scheme	Marks
(a)	$x^2 + y^2 + 4x - 2y - 11 = 0$ $\{\underline{(x+2)^2 - 4} + \underline{(y-1)^2 - 1} - 11 = 0\}$ $(\pm 2, \pm 1)$, see notes. Centre is $(-2, 1)$. $(-2, 1)$.	M1 A1 cao [2]
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$ $r = \sqrt{11 \pm "1" \pm "4"}$ So $r = \sqrt{11+1+4} \Rightarrow r = 4$ 4 or $\sqrt{16}$ (Award A0 for ± 4).	M1 A1 [2]
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$ Putting $x = 0$ in C or their C. $y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc $y = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \dfrac{2 \pm \sqrt{48}}{2}\right\}$ Attempt to use formula or a method of completing the square in order to find $y = ...$ So, $y = 1 \pm 2\sqrt{3}$ $1 \pm 2\sqrt{3}$	M1 A1 aef M1 A1 cao cso [4] 8
(a)	**Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks.** **Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.** M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x \pm 2)^2 \pm \alpha}$, $\alpha \neq 0$ or $\underline{(y \pm 1)^2 \pm \beta}$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.	
(b)	M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14. __Note:__ $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0. __Alternative:__ M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark. $(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.	
(c)	1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working. 2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise. 2nd A1: Need exact pair in simplified surd form of $\{y =\}\ 1 \pm 2\sqrt{3}$. This mark is also cso. Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$. Any incorrect working in (c) gets penalised the final accuracy mark. So, __beware:__ incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0. __Special Case for setting $y = 0$:__ Award SC: M0A0M1A0 for an attempt at applying the formula $x = \dfrac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \dfrac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15}\right\}$ Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. __Special Case__: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.	

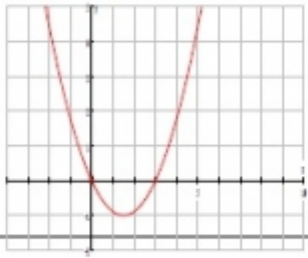
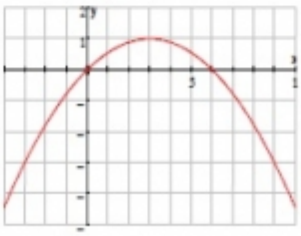
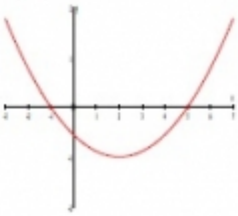
Question 4

Question Number	Scheme		Marks
(a)	$\{V =\}\ 2x^2y = 81$	$2x^2y = 81$	B1 oe
	$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$		
	$y = \dfrac{81}{2x^2} \Rightarrow L = 12x + 4\left(\dfrac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L = 12x + \dfrac{162}{x^2}$ **AG**	Correct solution only. **AG.**	A1 cso
			[3]
(b)	$\dfrac{dL}{dx} = 12 - \dfrac{324}{x^3}\ \left\{= 12 - 324x^{-3}\right\}$	Either $12x \to 12$ or $\dfrac{162}{x^2} \to \dfrac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\dfrac{dL}{dx} =\right\} 12 - \dfrac{324}{x^3} = 0 \Rightarrow x^3 = \dfrac{324}{12};= 27 \Rightarrow x = 3$	$L' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value"	M1;
		$x = \sqrt[3]{27}$ or $x = 3$	A1 cso
	$\{x = 3,\}\ L = 12(3) + \dfrac{162}{3^2} = 54$ (cm)	Substitute candidate's value of $x (\neq 0)$ into a formula for L.	ddM1
		54	A1 cao
			[6]
(c)	$\{\text{For } x = 3\},\ \dfrac{d^2L}{dx^2} = \dfrac{972}{x^4} > 0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\dfrac{972}{x^4}$ and > 0 and conclusion.	A1 **[2]**
			11
(a)	B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. Otherwise, candidates can use any symbol or letter in place of y. M1: Making y the subject of their formula and substituting this into a correct expression for L. A1: Correct solution only. Note that the answer is given.		
(b)	**Note you can mark parts (b) and (c) together.** 2nd M1: Setting their $\dfrac{dL}{dx} = 0$ and "candidate's ft *correct* power of x = a value". The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities. $L' = 0$ can be implied by $12 = \frac{324}{x^3}$. 2nd A1: $x^3 = 27 \Rightarrow x = \pm 3$ scores A0. 2nd A1: can be given for no value of x given but followed through by correct working leading to $L = 54$. 3rd M1: Note that this method mark is dependent upon the two previous method marks being awarded.		
(c)	M1: for attempting correct ft second derivative and <u>considering its sign</u>. A1: Correct second derivative of $\dfrac{972}{x^4}$ (need not be simplified) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. **Note:** 2 marks can be scored from a wrong value of x, no value of x found or from not substituting in the value of their x into L''. Gradient test or testing values either side of their x scores M0A0 in part (c). **Throughout this question allow confused notation such as $\dfrac{dy}{dx}$ for $\dfrac{dL}{dx}$.**		

Question 5

Question Number	Scheme	Marks
(a)	$p=7.5$	B1
		(1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	
		(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Question 6

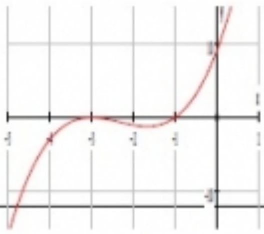
Question Number	Scheme		Marks
(a)		Shape $\cup$ through (0, 0) (3, 0) (1.5, –1)	B1 B1 B1 (3)
(b)		Shape $\cap$ (0, 0) and (6, 0) (3, 1)	B1 B1 B1 (3)
(c)		Shape $\cup$, <u>not</u> through (0, 0) Minimum in 4th quadrant $(-p, 0)$ and $(6-p, 0)$ $(3-p, -1)$	M1 A1 B1 B1 (4) **10**

<u>Notes</u>

(a) B1: U shaped parabola through origin
B1: (3,0) stated or 3 labelled on x axis
B1: (1.5, -1) or equivalent e.g. (3/2, -1)
(b) B1: Cap shaped parabola in any position

B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis
B1: (3,1) shown
(c) M1: U shaped parabola not through origin
A1: Minimum in 4th quadrant (depends on M mark having been given)
B1: Coordinates stated or shown on x axis
B1: Coordinates stated
Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)

Question 7

Question Number	Scheme	Marks
(a)	 Shape (cubic in this orientation) **Touching** x-axis at **–3** **Crossing** at **–1** on x-axis Intersection at **9** on y-axis	B1 B1 B1 B1 (4)
(b)	$y=(x+1)(x^2+6x+9)=x^3+7x^2+15x+9$ or equiv. (possibly unsimplified) Differentiates their polynomial correctly – may be unsimplified $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+14x+15$ **(*)**	B1 M1 A1 cso (3)
(c)	At $x=-5$: $\frac{\mathrm{d}y}{\mathrm{d}x}=75-70+15=20$ At $x=-5$: $y=-16$ $y-(\text{"}-16\text{"})=\text{"}20\text{"}(x-(-5))$ or $y=$ "$20x$" $+ c$ with (-5, -"16") used to find c $y=20x+84$	B1 B1 M1 A1 (4)
(d)	Parallel: $3x^2+14x+15=\text{"}20\text{"}$ $(3x-1)(x+5)=0 \quad x=\ldots$ $x=\frac{1}{3}$	M1 M1 A1 (3) **14**
	<u>Notes</u> (a) Crossing at –3 is B0. Touching at –1 is B0 (b) M: This needs to be correct differentiation here A1: Fully correct simplified answer. (c) M: If the –5 and "-16" are the wrong way round or – omitted the M mark can still be given if a correct formula is seen, (e.g. $y-y_1=m(x-x_1)$) otherwise M0. m should be numerical and not 0 or infinity and should not have involved negative reciprocal. (d) 1st M: Putting the derivative expression equal to their value for gradient 2nd M: Attempt to solve quadratic (see notes) This may be implied by correct answer.	

Question 8

Question Number	Scheme		Marks
(a)	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20\ \{= 0\} \Rightarrow (x-5)(x+4)\ \{= 0\} \Rightarrow x = ...$	Attempt to solve a *resulting* quadratic to give $x =$ their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft [4]
(b)	$\left\{\int(-x^2 + 2x + 24)\,dx\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x}\ \{+c\}$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe cao
			[7]
			11

Question Number	Scheme	Marks
(a)	1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x = ...$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x = 5$ and $x = -4$. 2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give *both correct ft* y-values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$). <u>Note:</u> For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow$ eg. $(-4, 9)$ and $(5, 0)$, award B1 isw. If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark. <u>Special Case</u>: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram. <u>Note:</u> SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$.	
(b)	Note: Do not give marks for working in part (b) which would be creditable in part (a). 1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1st A1 at least two out of three terms correctly integrated. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Area of triangle $= \frac{1}{2}$(their x_2 – their x_1)(their y_2) or Area of triangle $= \int_{x_1}^{x_2} x + 4\ \{dx\}$. Where $x_1 =$ their -4, $x_2 =$ their 5 and $y_2 =$ their y usually found in part (a). 4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve > Area under triangle. 3rd A1: 121.5 or $\frac{243}{2}$ oe cao.	

Question Number	Scheme	Marks
Aliter (b) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4)\,dx$ — 3rd M1: Uses integral of $(x + 4)$ with correct ft limits. 4th M1: Uses "curve" – "line" function with correct ft limits.	
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+ c\}$ — M: $x^n \to x^{n+1}$ for any one term.	M1
	A1 at least two out of three terms	A1ft
	Correct answer (Ignore $+ c$).	A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (\ldots\ldots) - (\ldots\ldots)$ — Substitutes 5 and -4 (or *their limits* from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$	
	See above working to decide to award 3rd M1 mark here:	M1
	See above working to decide to award 4th M1 mark here:	M1
	So area of R is $= 121.5$ — 121.5	A1 oe **cao**
		[7] **11**
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $20 \to 20x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x\right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Uses the integral of $(x + 4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) {where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9)$.} This mark is usually found in the first line of the candidate's working in part (b). 4th M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b). Allow $\int_{-4}^{5} (-x^2 + 2x + 24) - x + 4\,\{dx\}$ for this method mark. 3rd A1: 121.5 oe **cao.** **<u>Note: SPECIAL CASE for this alternative method</u>** Area of $R = \int_{-4}^{5} (x^2 - x - 20)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x\right]_{-4}^{5} = \left(\frac{125}{3} - \frac{25}{2} - 100\right) - \left(-\frac{64}{3} - 8 + 80\right)$ The working so far would score SPEICAL CASE M1A1A1M1M1M0A0. The candidate may then go on to state that $= \left(-70\frac{5}{6}\right) - \left(50\frac{2}{3}\right) = -\frac{243}{2}$ If the candidate then multiplies their answer by -1 then they would gain the 4th M1 and 121.5 would gain the final A1 mark.	

Question Number	Scheme		Marks
Aliter (a) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow y = -(y-4)^2 + 2(y-4) + 24$	Eliminating x correctly.	B1
	$y^2 - 9y\ \{=0\} \Rightarrow y(y-9)\ \{=0\} \Rightarrow y = ...$	Attempt to solve a resulting quadratic to give $y =$ their values.	M1
	So, $y = 0, 9$	Both $y = 0$ and $y = 9$.	A1
	So corresponding y-values are $x = -4$ and $x = 5$.	See notes below.	B1ft
			[4]
	2nd B1ft: For correctly substituting their values of y in equation of line or parabola to give ***both correct ft*** x-values.		
(b)	**Alternative Methods for obtaining the M1 mark for use of limits:**		

There are two alternative methods can candidates can apply for finding "162".

Alternative 1:

$$\int_{-4}^{0}(-x^2 + 2x + 24)\,dx + \int_{0}^{5}(-x^2 + 2x + 24)\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{0} + \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{0}^{5}$$

$$= (0) - \left(\frac{64}{3} + 16 - 96\right) + \left(-\frac{125}{3} + 25 + 120\right) - (0)$$

$$= \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162$$

Alternative 2:

$$\int_{-4}^{6}(-x^2 + 2x + 24)\,dx - \int_{5}^{6}(-x^2 + 2x + 24)\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{6} - \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{5}^{6}$$

$$= \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(\frac{64}{3} + 16 - 96\right)\right\} - \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(-\frac{125}{3} + 25 + 120\right)\right\}$$

$$= \left\{(108) - \left(-58\frac{2}{3}\right)\right\} - \left\{(108) - \left(103\frac{1}{3}\right)\right\}$$

$$= \left(166\frac{2}{3}\right) - \left(4\frac{2}{3}\right) = 162$$

Question 9

Question Number	Scheme		Marks
	Note: A similar scheme would apply for T&I for candidates using their a and their r. So,... 1st M1: For attempting to find one of the correct S_n's either side (but next to) 1000. 2nd M1: For one of these S_n's correct for their a and their r. (You may need to get your calculators out!) 3rd M1: For attempting to find both of the correct S_n's either side (but next to) 1000. A1: Cannot be gained for wrong a and/or r. **Trial & Improvement Cumulative Approach:** A similar scheme to T&I will be applied here: 1st M1: For getting as far as the cumulative sum of 13 terms. 2nd M1: (1) S_{13} = awrt 999.7 or truncated 999. 3rd M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1) S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$. **Trial & Improvement Method:** for $(0.75)^n < \frac{6}{256} = 0.0234375$ 3rd M1: For evidence of examining both $n = 13$ and $n = 14$. Eg: $(0.75)^{13}\{= 0.023757...\}$ and $(0.75)^{14}\{= 0.0178179...\}$ A1: $n = 14$ **Any misreads,** $S_n > 10000$ etc, please escalate up to your Team Leader.		
	(a) $3\sin(x+45^\circ) = 2$; $0 \le x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x$; $0 \le x < 2\pi$		
(a)	$\sin(x+45^\circ) = \frac{2}{3}$, so $(x+45^\circ) = 41.8103...$ $(\alpha = 41.8103...)$	$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x + 45^\circ = \{138.1897..., 401.8103...\}$	$x + 45^\circ$ = either "180 – their α" or "360° + their α" (α could be in radians).	M1
	and $x = \{93.1897..., 356.8103...\}$	Either awrt 93.2° or awrt 356.8°	A1
		Both awrt 93.2° and awrt 356.8°	A1
			[4]
(b)	$2(1-\cos^2 x) + 2 = 7\cos x$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	$2\cos^2 x + 7\cos x - 4 = 0$	Correct 3 term, $2\cos^2 x + 7\cos x - 4\ \{= 0\}$	A1 oe
	$(2\cos x - 1)(\cos x + 4)\ \{= 0\}$, $\cos x = ...$	Valid attempt at solving and $\cos x = ...$	M1
	$\cos x = \frac{1}{2}$, $\{\cos x = -4\}$	$\cos x = \frac{1}{2}$ (See notes.)	A1 cso
	$x = 60^0$		B1
	$x = 300^0$		B1 ft
			[6] 10

Question Number	Scheme	Marks
(a)	1[st] M1: can also be implied for $x =$ awrt -3.2 2[nd] M1: for $x + 45^\circ =$ either "180 – their α" or "360° + their α". This can be implied by later working. The candidate's α could also be in radians. **Note that this mark is not** for $x =$ either "180 – their α" or "360° + their α". __Note:__ Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°. __Note:__ Candidates who apply the following incorrect working of $3\sin(x + 45^\circ) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0. If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360$. **Working in Radians:** Note the answers in radians are $x =$ awrt 1.6, awrt 6.2 If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) **No working:** Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x$ {and not x} = {awrt 93.2, awrt 356.8}	

Question Number	Scheme	Marks
(b)	1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1st M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. **Note:** 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1st B1: for either $\frac{\pi}{3}$ or awrt 1.05^c 2nd B1: for either $\frac{5\pi}{3}$ or awrt 5.24^c or can be ft from 2π – their β or 360° – their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$. **Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.) **Answers from no working:** $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. **No working:** You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. **For candidates using trial & improvement, please forward these to your Team Leader.**	

Question 10

(a)	$10.6^2 + 9.2^2 - 2 \times 10.6 \times 9.2 \times \cos 68°$ o.e.	**M1**	
	QR = 11.1(3…)	**A1**	
	$\frac{\sin 68}{\text{their QR}} = \frac{\sin Q}{9.2}$ or $\frac{\sin R}{10.6}$ o.e.	**M1**	Or correct use of Cosine Rule
	Q = 50.01..° or R = 61.98..°	**A1**	2 s.f. or better
	bearing = 174.9 to 175°	**B1**	

Question 11

Q11	Scheme	Marks
(a)	Use of cosine rule $4^2 = 2.5^2 + 3.5^2 - 2(2.5)(3.5)\cos B$ $\cos B = \frac{1}{7}$	M1 A1 A1
(b)	Use of identity $\sin^2 x + \cos^2 x = 1$ $\sin B = \sqrt{1 - \cos^2 x}$ $= \sqrt{1 - \left(\frac{1}{7}\right)^2}$ $= \sqrt{\frac{48}{49}}$ $= \frac{4\sqrt{3}}{7}$	M1 A1 A1