Pure Mathematics 2 Practice Paper M14 MARK SCHEME

Question 1

Q1	Scheme	Marks
a	$\frac{dy}{dx} = 6x^2 + 24x - 24$	M1
	$\frac{d^2y}{dx^2} = 12x + 24$	M1
	States that $\frac{d^2y}{dx^2} = 12x + 24$ for all values $-5 \le x \le -3$ and concludes *C* is concave over the given interval.	B1
		(3)
b	Point of inflection occurs when $\frac{d^2y}{dx^2} = 0$	M1
	$12x + 24 = 0$ $x = -2$	A1
	Substitutes *x* = −2 into $y = 2x^3 + 12x^2 - 24x - 3$ obtaining *y* = 83 (-2 , 83)	A1
		(3)

Question 2

Question Number	Scheme		Marks
(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $=160$	M1: Use of a correct S_∞ formula A1: 160	M1A1
	Accept correct answer only (160)		
			[2]
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around 7/8) A1: awrt 127.8	M1A1
	T & I in (b) requires all 12 terms to be calculated correctly for M1 and A1 for awrt 127.8		
			[2]
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working. (condone missing brackets around 7/8)(Allow =, <, >, ≥, ≤) but see note below.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $+\left(\frac{7}{8}\right)^N$ oe (Allow =, <, >, ≥, ≤) but see note below. **Dependent on the previous M1**	dM1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the power law of logarithms or takes logs base 0.875 correctly to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ (Allow =, <, >, ≥, ≤) but see note below.	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but not $N > 44$	A1 cso
	An incorrect <u>**inequality**</u> statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			[4]
			Total 8
	<u>**Trial & Improvement Method in (c):**</u>		
	1st M1: Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		
	2nd M1: Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		
	3rd M1: For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		
	A1: $N = 44$ cso		
	Answer of $N = 44$ only with no working scores no marks		

Question 3

Question Number	Scheme	Marks
(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\dfrac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)}$	
	$=\dfrac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1, (y)\geqslant 1$ or $(y)<4, (y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\dfrac{(x+1)}{(x-2)}=x$ $\quad \dfrac{x+1}{x-2}=\dfrac{2x+1}{x-1}$ $\quad \dfrac{2x+1}{x-1}=x$ $\quad \dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0 \Rightarrow x=\ldots$	A1, dM1
	$a=\dfrac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4) (10 marks)

(a)

B1 $x^2+x-6=(x+3)(x-2)$ This can occur anywhere in the solution.

M1 For combining the two fractions with a common denominator. The denominator must be correct for their fractions and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

Accept $\dfrac{x}{x+3}+\dfrac{3(2x+1)}{x^2+x-6}=\dfrac{x(x^2+x-6)+3(2x+1)(x+3)}{(x+3)(x^2+x-6)}$

Condone $\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x\times x-2}{(x+3)(x-2)}+\dfrac{3(2x+1)}{(x+3)(x-2)}$

A1 A correct intermediate form of $\dfrac{\text{simplified quadratic}}{\text{simplified quadratic}}$

Accept $\dfrac{x^2+4x+3}{(x+3)(x-2)}, \dfrac{x^2+4x+3}{x^2+x-6}$, OR $\dfrac{x^3+7x^2+15x+9}{(x+3)(x^2+x-6)} \rightarrow \dfrac{(x+1)(x+3)\cancel{(x+3)}}{\cancel{(x+3)}(x^2+x-6)}$

As in question one they can score this mark having 'invisible' brackets on line 1.

A1* Further factorises and cancels (which may be implied) to complete the proof to reach the given answer $=\dfrac{(x+1)}{(x-2)}$. All aspects including bracketing must be correct. If a cubic is formed then it needs to be correct.

(b)

B1 States either end of the range. Accept either $y<4$, $y\leqslant 4$ or $y>1$, $y\geqslant 1$ with or without the y's.

B1 Correct range. Accept $1<y<4$, $1<g<4$, $y>1$ and $y<4$, $(1,4)$, $1<\text{Range}<4$, even $1<f<4$,

Do not accept $1<x<4$, $1<y\leq 4$, $[1,4)$ etc.

Special case, allow B1B0 for $1<x<4$

(c)

M1 Attempting to set $g(x)=x$, $g^{-1}(x)=x$ or $g(x)=g^{-1}(x)$ or $g^2(x)=x$.

If $g^{-1}(x)$ has been used then a full attempt must have been made to make x the subject of the formula. A full attempt would involve cross multiplying, collecting terms, factorising and ending with division.

As a result, it must be in the form $g^{-1}(x)=\dfrac{\pm 2x\pm 1}{\pm x\pm 1}$

Accept as evidence $\dfrac{(x+1)}{(x-2)}=x$ OR $\dfrac{x+1}{x-2}=\dfrac{\pm 2x\pm 1}{\pm x\pm 1}$ OR $\dfrac{\pm 2x\pm 1}{\pm x\pm 1}=x$ OR $\dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$

A1 $x^2-3x-1=0$ or exact equivalent. The =0 may be implied by subsequent work.

dM1 For solving a 3TQ=0. It is dependent upon the first M being scored. Do not accept a method using factors unless it clearly factorises. Allow the answer written down awrt 3.30 (from a graphical calculator).

A1 a or $x=\dfrac{3+\sqrt{13}}{2}$. Ignore any reference to $\dfrac{3-\sqrt{13}}{2}$

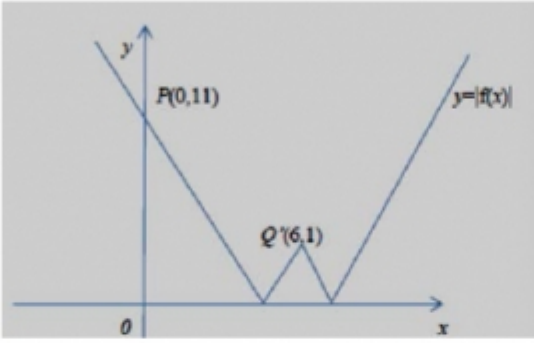
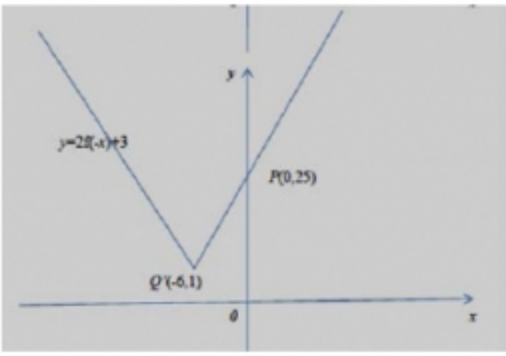
Question 4

Question Number	Scheme		Marks
	$\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \dfrac{(-4)(-4-1)}{2!}(kx)^2 + \ldots\right\}$		
(a)	**Either** $(-4)k = -6$ **or** $(1+kx)^{-4} = 1 + (-4)(kx)$	**see notes**	M1
	leading to $k = \dfrac{3}{2}$	$k = \dfrac{3}{2}$ or 1.5 or $\dfrac{6}{4}$	A1
			[2]
(b)	$\dfrac{(-4)(-5)}{2}(k)^2$	**Either** $\dfrac{(-4)(-5)}{2!}$ **or** $(k)^2$ **or** $(kx)^2$	M1
		Either $\dfrac{(-4)(-5)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(kx)^2$	M1
	$\left\{A = \dfrac{(-4)(-5)}{2!}\left(\dfrac{3}{2}\right)^2\right\} \Rightarrow A = \dfrac{45}{2}$	$\dfrac{45}{2}$ or 22.5	A1
			[3]
			5

Question Notes

Note	**In this question ignore part labelling and mark part (a) and part (b) together.**	
	Note	Writing down $\left\{(1+kx)^{-4}\right\} = 1 + (-4)(kx) + \dfrac{(-4)(-4-1)}{2!}(kx)^2 + \ldots$ gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1
(a)	**M1**	Award M1 for • **either** writing down $(-4)k = -6$ or $4k = 6$ • **or** expanding $(1+kx)^{-4}$ to give $1 + (-4)(kx)$ • **or** writing down $(-4)kx = -6$ or $(-4k) = -6x$ or $-4kx = -6x$
	A1	$k = \dfrac{3}{2}$ or 1.5 or $\dfrac{6}{4}$ **from no incorrect sign errors.**
	Note	The M1 mark can be implied by a candidate writing down the correct value of k.
	Note	Award M1 for writing down $4k = 6$ and then A1 for $k = 1.5$ (or equivalent).
	Note	Award M0 for $4k = -6$ (if there is no evidence that $(1+kx)^{-4}$ expands to give $1 + (-4)(kx) + \ldots$)
	Note	$1 + (-4)(kx)$ leading to $(-4)k = 6$ leading to $k = \dfrac{3}{2}$ is M1A0.
(b)	**M1**	For either $\dfrac{(-4)(-4-1)}{2!}$ **or** $\dfrac{(-4)(-5)}{2!}$ **or** 10 **or** $(k)^2$ **or** $(kx)^2$
	M1	**Either** $\dfrac{(-4)(-4-1)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(kx)^2$ **or** $\dfrac{(-4)(-5)}{2!}(\text{their } k)^2$ **or** $10k^2$
	Note	Candidates are allowed to use 2 instead of 2!
	A1	Uses $k = 1.5$ to give $A = \dfrac{45}{2}$ or 22.5
	Note	$A = \dfrac{90}{4}$ which has not been simplified is A0.
	Note	Award A0 for $A = \dfrac{45}{2}x^2$.
	Note	Allow A1 for $A = \dfrac{45}{2}x^2$ followed by $A = \dfrac{45}{2}$
	Note	$k = -1.5$ leading to $A = \dfrac{45}{2}$ or 22.5 is A0.

Question 5

Question Number	Scheme	Marks
(a)	 'W' Shape (0, 11) and (6, 1)	B1 B1 (2)
(b)	 'V' shape (-6,1) (0,25)	B1 B1 B1 (3)
(c)	One of $a = 2$ or $b = 6$ $a = 2$ and $b = 6$	B1 B1 (2)
		(7 marks)

(a)

B1 A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = f(|x|)$ would score this mark.

B1 A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

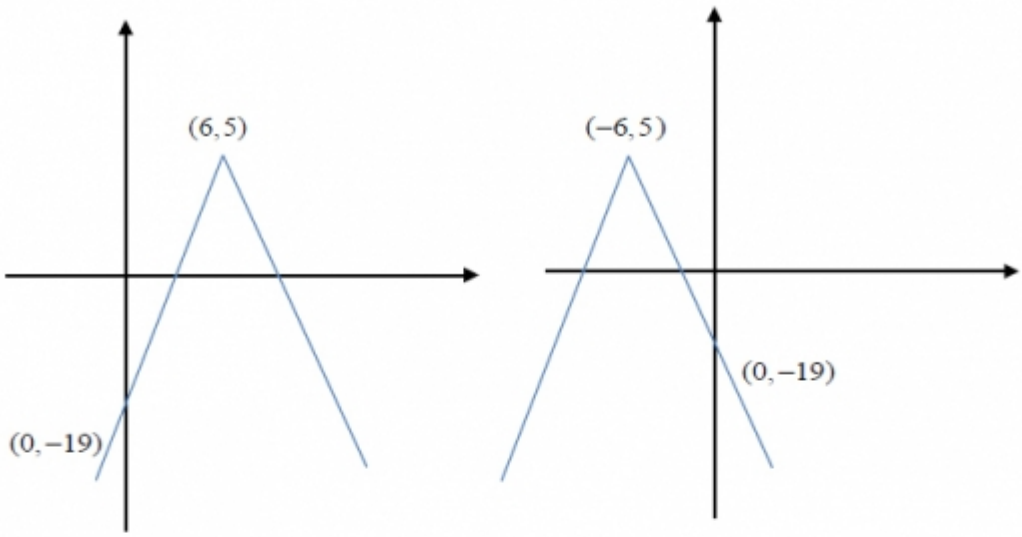
(b)

B1 Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.

Special case: A candidate who mistakenly sketches $y = -2f(x) + 3$ or $y = -2f(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0



(c)

B1 Either states $a = 2$ **or** $b = 6$.
This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x-6|-1$ or $y = 2|x-..|-1$. If they are both stated and written, the stated answer takes precedence.

B1 States both $a = 2$ **and** $b = 6$
This can be implied by the candidate stating that $y = 2|x-6|-1$
If they are both stated and written, the stated answer takes precedence.

Question 6

Question Number	Scheme	Marks
(a)	$\mathrm{f}(x) = \dfrac{4x+1}{x-2}, \quad x > 2$	
	Applies $\dfrac{vu' - uv'}{v^2}$ to get $\dfrac{(x-2)\times 4 - (4x+1)\times 1}{(x-2)^2}$	M1A1
	$= \dfrac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\dfrac{-9}{(x-2)^2} = -1 \Rightarrow x = ..$	M1
	$(5,7)$	A1,
		(2)
		(5 marks)
Alt 1.(a)	$\mathrm{f}(x) = \dfrac{4x+1}{x-2} = 4 + \dfrac{9}{x-2}$	
	Applies chain rule to get $\mathrm{f}'(x) = A(x-2)^{-2}$	M1
	$= -9(x-2)^{-2} = \dfrac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $f(x)=\frac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=..., v'=..$ followed by $\frac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\frac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A, B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=..., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $f(x)=\frac{4x+1}{x-2}=..+\frac{..}{x-2}$ before applying the chain rule to get $f'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\frac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\frac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\frac{4}{(x-2)}-\frac{4x+1}{(x-2)^2}=\frac{4(x-2)-(4x+1)}{(x-2)^2}=\frac{-9}{(x-2)^2}$

(b)

M1 Sets $\frac{-9}{(x-2)^2}=-1$ and proceeds to $x=....$

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x=5$ would also score this mark following $\frac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x=5$

A1 (5, 7) or $x=5$, $y=7$. Ignore any reference to $x=-1$ (and $y=1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

Question 7

Question Number	Scheme	Marks
(a)	$x=8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right)=\pi$	B1*
		(1)
(b)	$\frac{dx}{dy}=8\tan 2y+16y\sec^2(2y)$	M1A1A1
	At P $\frac{dx}{dy}=8\tan 2\frac{\pi}{8}+16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right)=\{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi}=\frac{1}{8+4\pi}$, accept $y-\frac{\pi}{8}=0.049(x-\pi)$	M1A1
	$\Rightarrow(8+4\pi)y=x+\frac{\pi^2}{2}$	A1
		(6)
		(7 marks)

(a)

B1* Either sub $y=\frac{\pi}{8}$ into $x=8y\tan(2y)\Rightarrow x=8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right)=\pi$

Or sub $x=\pi$, $y=\frac{\pi}{8}$ into $x=8y\tan(2y)\Rightarrow\pi=8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right)=\pi\times1=\pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y=\frac{\pi}{8}\Rightarrow x=\pi\tan\left(\frac{\pi}{4}\right)=\pi$

Or $\pi=\pi\times\tan\left(\frac{\pi}{4}\right)=\pi$ ✓

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical } \frac{dx}{dy}}$,

Watch for negative reciprocals which is M0

If the form $y = mx + c$ is used it must be a full method to find a 'numerical' value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

Accept a mixture of decimals and π's for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

Question 8

Question Number	Scheme	Marks
(a)	$\text{cosec}\,2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	M1
	$= \dfrac{1 + \cos 2x}{\sin 2x}$	M1
	$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$	
	$= \dfrac{2\cos^2 x}{2\sin x \cos x}$	M1 A1
	$= \dfrac{\cos x}{\sin x} = \cot x$	A1*
		(5)
(b)	$\text{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$	
	$\cot(2\theta \pm ..°) = \sqrt{3}$	M1
	$2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$	dM1, A1
	$2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$	dM1
	$\theta = 102.5°$	A1
		(5)
		(10 marks)

(a)

M1 Writing $\operatorname{cosec}2x = \frac{1}{\sin 2x}$ **and** $\cot 2x = \frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p \times q}{s \times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x \cos x}$ or $\frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x} = \cot x$ or $\frac{1}{\tan x} = \cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x} = \cot x$

The proof cannot rely on expressions like $\cot = \frac{\cos}{\sin}$ (with missing x's) for the final A1

(b)

M1 Attempt to use the solution to part (a) with $2x = 4\theta + 10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°) = \sqrt{3}$

Watch for attempts which start $\cot \alpha = \sqrt{3}$. The method mark here is not scored until the α has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°) = \sqrt{3}$ where θ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan.. = \frac{1}{\cot..}$ and

$\arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$ to solve $2\theta \pm ...^\circ = 30^\circ \Rightarrow \theta = ..$

A1 $\theta = 12.5^\circ$ or exact equivalent. Condone answers such as $x = 12.5^\circ$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.
Accept $2\theta \pm ... = 180 + PV^\circ \Rightarrow \theta = ..^\circ$

A1 $\theta = 102.5^\circ$ or exact equivalent. Condone answers such as $x = 102.5^\circ$
Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.
If radians appear they could just lose the answer marks. So for example

$2\theta \pm ... = \frac{\pi}{6}(0.524) \Rightarrow \theta = ..$is M1dM1A0 followed by

$2\theta \pm ... = \pi + '\frac{\pi}{6}' \Rightarrow \theta = ..$ dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...^\circ) = \sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.
$\cot(4\theta + \beta^\circ) = \sqrt{3} \Rightarrow 4\theta + \beta = 30^\circ \Rightarrow \theta = ..$ is M0M0A0 where $\beta = 5^\circ \text{or} 10^\circ$
$\Rightarrow 4\theta + \beta = 210^\circ \Rightarrow \theta = ..$ can score M1A1 Special case.
If $\beta = 5^\circ, \theta = 51.25$ If $\beta = 10^\circ, \theta = 50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5
BUT $\cot(2\theta \pm 5^\circ) = \sqrt{3} \Rightarrow \theta = 12.5^\circ, 102.5^\circ$ scores all available marks.

Question Number	Scheme	Marks
(a)Alt 1	$\operatorname{cosec} 2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{1}{\tan 2x}$	1^{ST} M1
	$= \dfrac{1}{2\sin x\cos x} + \dfrac{1-\tan^2 x}{2\tan x}$	
	$= \dfrac{\tan x + (1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $= \dfrac{2\tan x + 2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$	2^{nd} M1
	$= \dfrac{\tan x + \sin x\cos x - \tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \dfrac{\tan x + \sin x\cos x - \tan x\sin^2 x}{2\sin x\cos x\tan x}$	
	$= \dfrac{\tan x(1-\sin^2 x) + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \dfrac{\tan x\cos^2 x + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \dfrac{\sin x\cos x + \sin x\cos x}{2\sin x\cos x\tan x}$	
	$= \dfrac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe	3^{rd} M1A1
	$= \dfrac{1}{\tan x} = \cot x$	A1* (5)
(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\operatorname{cosec} 2x + \cot 2x = \cot x \Leftrightarrow \dfrac{1}{\sin 2x} + \dfrac{1}{\tan 2x} = \dfrac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x\tan x + \sin 2x\tan x = \sin 2x\tan 2x$	2^{nd} M1
	$\Leftrightarrow \dfrac{2\tan x}{1-\tan^2 x}\times\tan x + 2\sin x\cancel{\cos x}\times\dfrac{\sin x}{\cancel{\cos x}} = 2\sin x\cos x\times\dfrac{2\tan x}{1-\tan^2 x}$	
	$\Leftrightarrow \dfrac{2\tan^2 x}{1-\tan^2 x} + 2\sin^2 x = \dfrac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x) \Leftrightarrow 2\tan^2 x + 2\sin^2 x(1-\tan^2 x) = 4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x - 2\sin^2 x\tan^2 x = 2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x) = 2\sin^2 x$	3^{rd} M1
	$\div 2\tan^2 x \Leftrightarrow 1-\sin^2 x = \cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

Question 9

Question Number	Scheme		Marks
	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$, $\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $8\pi h + 16\pi$	M1 A1
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\}$ $(8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow \right\}$ $\frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$	$\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	M1 oe
	When $h = 6$, $\left\{\frac{dh}{dt} = \right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{= \frac{80\pi}{64\pi}\right\}$ $\frac{dh}{dt} = \underline{1.25}\ (\text{cm s}^{-1})$	**dependent on the previous M1 see notes** 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	dM1 A1 oe [5] 5
	<u>Alternative Method for the first M1A1</u> Product rule: $\left\{\begin{array}{ll} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{array}\right\}$ $\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$	$\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $4\pi(h+4) + 4\pi h$	M1 A1

		Question Notes
	M1	An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$. **Can be simplified or un-simplified.**
	A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
	Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
	Note	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
	M1	$\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
	Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
	Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = \mathbf{80\pi t}$ **or 80k** or **80πt or 80k** $\div$ Candidate's $\frac{dV}{dh}$
	dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
	A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
	Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
	Note	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

Question 10

Question Number	Scheme	Marks
(a)	$P=\dfrac{800e^0}{1+3e^0},=\dfrac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\dfrac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\dfrac{dP}{dt}=\dfrac{(1+3e^{0.1t})\times800\times0.1e^{0.1t}-800e^{0.1t}\times3\times0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\dfrac{dP}{dt}=\dfrac{(1+3e)\times80e-240e^2}{(1+3e)^2}=\dfrac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}=\dfrac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\dfrac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub $t = 0$ into P **and** use $e^0 = 1$ in at least one of the two cases. Accept $P = \dfrac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P = \dfrac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t} = B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t} = D$

A1 $e^{0.1t} = 5$ or $e^{-0.1t} = 0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$It could be implied by $t =$ awrt 16.1

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{0.1t} = k\mathrm{e}^{0.1t}$ and NOT $kt\mathrm{e}^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800\mathrm{e}^{0.1t}, v = 1+3\mathrm{e}^{0.1t}, u' = p\mathrm{e}^{0.1t}, v' = q\mathrm{e}^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form $\frac{(1+3\mathrm{e}^{0.1t})\times p\mathrm{e}^{0.1t} - 800\mathrm{e}^{0.1t}\times q\mathrm{e}^{0.1t}}{(1+3\mathrm{e}^{0.1t})^2}$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{0.1t} = k\mathrm{e}^{0.1t}$

For the chain rule look for

$$P = \frac{800\mathrm{e}^{0.1t}}{1+3\mathrm{e}^{0.1t}} = \frac{800}{\mathrm{e}^{-0.1t}+3} \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}t} = 800\times\left(\mathrm{e}^{-0.1t}+3\right)^{-2}\times -0.1\mathrm{e}^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{(1+3\mathrm{e}^{0.1t})\times 800\times 0.1\mathrm{e}^{0.1t} - 800\mathrm{e}^{0.1t}\times 3\times 0.1\mathrm{e}^{0.1t}}{(1+3\mathrm{e}^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{\mathrm{d}P}{\mathrm{d}t}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{\mathrm{d}P}{\mathrm{d}t}$ was correct

A1 $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{80\mathrm{e}}{(1+3\mathrm{e})^2}$ or equivalent such as $\frac{\mathrm{d}P}{\mathrm{d}t} = 80\mathrm{e}(1+3\mathrm{e})^{-2}, \frac{80\mathrm{e}}{1+6\mathrm{e}+9\mathrm{e}^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800\mathrm{e}^{0.1t}}{1+3\mathrm{e}^{0.1t}} \Rightarrow -27 = \mathrm{e}^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800\mathrm{e}^{0.1t}}{1+3\mathrm{e}^{0.1t}} \Rightarrow -27 = \mathrm{e}^{0.1t} \Rightarrow \mathrm{e}^{0.1t}\ /\ \mathrm{e}^{x}$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Question 11

Question Number	Scheme		Marks
	$x = 4\cos\left(t + \dfrac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**Main Scheme** $x = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right)$	$\cos\left(t + \dfrac{\pi}{6}\right) \to \cos t\cos\left(\dfrac{\pi}{6}\right) \pm \sin t\sin\left(\dfrac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\dfrac{\sqrt{3}}{2}\right)\cos t - \left(\dfrac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 * [3]
(a)	**Alternative Method 1** $x = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right)$	$\cos\left(t + \dfrac{\pi}{6}\right) \to \cos t\cos\left(\dfrac{\pi}{6}\right) \pm \sin t\sin\left(\dfrac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\dfrac{\sqrt{3}}{2}\right)\cos t - \left(\dfrac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 * [3]
(b)	**Main Scheme** $\left(\dfrac{x + y}{2\sqrt{3}}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \dfrac{(x + y)^2}{12} + \dfrac{y^2}{4} = 1$		
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	A1 [2]
(b)	**Alternative Method 1** $(x + y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x + y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1 [2]
(b)	**Alternative Method 2** $(x + y)^2 = 12\cos^2 t$ As $12\cos^2 t + 12\sin^2 t = 12$ then $(x + y)^2 + 3y^2 = 12$		M1, A1 [2]
			5

	Question Notes	
(a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **Awarding the dM1 mark which is dependent on the first method mark**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3, b = 12$, but **<u>does not provide a correct proof</u>**.
	Note Note	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3, b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Question 12

Question Number	Scheme	Marks
(a)	$R=\sqrt{20}$	B1
	$\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'4+5R^2'=104$	B1ft
(ii)	$3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt } 0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt } 2.46$	M1A1
		(3)
		(9 marks)

(a)

B1 Accept $R=\sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R=\pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is R

M1 for sight of $\tan\alpha=\pm\frac{4}{2}, \tan\alpha=\pm\frac{2}{4}$. Condone $\sin\alpha=4, \cos\alpha=2\Rightarrow\tan\alpha=\frac{4}{2}$

If R is found first only accept $\sin\alpha=\pm\frac{4}{R}, \cos\alpha=\pm\frac{2}{R}$

A1 $\alpha=$ awrt 1.107. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if R was incorrect allow for the numerical value of their $'4+5R^2'$.

Allow a tolerance of 1 dp on decimal R's.

(b)(ii)

M1 Using $3\theta\pm$ their$'1.107'=\frac{\pi}{2}\Rightarrow\theta=..$

Accept $3\theta\pm$ their$'1.107'=(2n+1)\frac{\pi}{2}\Rightarrow\theta=..$ where n is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their}'1.107')=\frac{\pi}{2}\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-$ their$'63.4°'=90°\Rightarrow\theta=$

Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta\pm$ their$'1.107'=2\pi\Rightarrow\theta=...$

Accept $3\theta\pm$ their$'1.107'=n\pi\Rightarrow\theta=..$ where n is an integer, including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their}'1.107')=2\pi\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-$ their$'63.4°'=360°\Rightarrow\theta=$ but

Do not allow mixed units in this question

A1 $\theta=$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.

Question 13

Question Number	Scheme		Marks
(i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\ \ \alpha \neq 0, \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\,\{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\,\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\,\{+c\}\}$	*{Ignore subsequent working}*.	[2]
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y \sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec} 2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^x\,\{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ or $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,dy = \int e^x\,dx$	$\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^x\,\{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^x \to e^x$ *as part of solving their DE.*	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^0 + c$ or $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
			[7]
			12

		Question Notes
(i)	M1	Integration by parts is applied in the form $\pm\alpha x e^{4x} - \int \beta e^{4x}\{dx\}$, where $\alpha \neq 0, \beta > 0$. (must be in this form).
	A1	$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\{dx\}$ or equivalent.
	A1	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$ with/without $+c$. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{dv}{dx} = e^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	M1	$\pm\lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without $+c$. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	B1	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x$ **or** $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y \cos y$ **or** $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y$, $\mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y$, $\alpha \neq 0, \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) **or** integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ **or** $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

<u>Alternative Method 2 (Using integration by parts twice)</u>

$\int \sin 2y \sin y \, dy = \int e^x \, dx$		B1 oe
	Applies integration by parts **twice** to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	M2
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]