Pure Mathematics 2 Practice Paper M16 MARK SCHEME

Question 1

Question Number	Scheme	Notes	Marks
(a)	John; arithmetic series, $a=60$, $d=15$.		
	$60+75+90=225^*$ or $S_3=\frac{3}{2}(120+(3-1)(15))=225^*$	Finds and adds the first 3 terms or uses sum of 3 terms of an AP and obtains the printed answer, with no errors.	B1 *
	Beware: **The 12th term of the sequence is 225 also so look out for** $60+(12-1)\times 15=225$. **This is B0.**		
			[1]
(b)	$t_9=60+(n-1)15=(£)180$	M1: Uses $60+(n-1)15$ with $n=8$ or 9 A1: (£)180	M1 A1
	Listing: M1: Uses $a=60$ and $d=15$ to select the 8th or 9th term (allow arithmetic slips) A1: (£)180 (Special case (£)165 **only** scores M1A0)		
			[2]
(c)	$S_n=\frac{n}{2}(120+(n-1)(15))$ or $S_n=\frac{n}{2}(60+60+(n-1)(15))$	Uses correct formula for sum of n terms with $a=60$ and $d=15$ (must be a correct formula but ignore the value they use for n or could be in terms of n)	M1
	$S_n=\frac{12}{2}(120+(12-1)(15))$	Correct numerical expression	A1
	$=(£)1710$	cao	A1
	Listing: M1: Uses $a=60$ and $d=15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: (£)1710		
			[3]
(d)	$3375=\frac{n}{2}(120+(n-1)(15))$	Uses correct formula for sum of n terms with $a=60$, $d=15$ and puts $=3375$	M1
	$6750=15n(8+(n-1))\Rightarrow 15n^2+105n=6750$	**Correct three term** quadratic. E.g. $6750=105n+15n^2$, $3375=\frac{15}{2}n^2+\frac{105}{2}n$ This may be implied by equations such as $6750=15n(n+7)$ or $3375=\frac{15}{2}(n^2+7n)$	A1
	$n^2+7n=25\times 18^*$	Achieves the printed answer with no errors but must see the 450 or 450 in factorised form or e.g. 6750, 3375 in factorised form i.e. an intermediate step.	A1*
			[3]
(e)	$n=18\Rightarrow$ Aged 27	M1: Attempts to solve the given quadratic or states $n=18$ A1: Age $=27$ or just 27	M1 A1
	Age $=27$ only scores both marks (i.e. $n=18$ need not be seen)		
	Note that (e) is not hence so allow valid attempts to solve the given equation for M1		
			[2]
			11 marks

n	1	2	3	4	5	6	7	8	9
u_n	60	75	90	105	120	135	150	165	180
S_n	60	135	225	330	450	585	735	900	1080
Age	10	11	12	13	14	15	16	17	18
n	10	11	12	13	14	15	16	17	18
u_n	195	210	225	240	255	270	285	300	315
S_n	1275	1485	1710	1950	2205	2475	2760	3060	3375
Age	19	20	21	22	23	24	25	26	27

Question 2

Question	Scheme	Marks
(a)	$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6}$ quotient x^2+3	A1
	$\underline{x^4+x^3-6x^2}$	
	$3x^2+7x-6$	M1 A1
	$\underline{3x^2+3x-18}$	
	$4x+12$	
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(5)
(b)	$\text{f}'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $\text{f}'(x=3)=2\times3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{\text{f}'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3,\text{f}(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(10 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$\begin{array}{r l} & x^2(+..x)+A \\ x^2+x-6 \big) & x^4+x^3-3x^2+7x-6 \\ & \underline{x^4+x^3-6x^2} \\ & \\ & \overline{\quad\quad (Cx)+D} \end{array}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $(x^3-2x^2+3x-2)\div(x-2)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right) \equiv \text{Their Quadratic Quotient} + \frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\dfrac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\dfrac{B}{x-2} \to 2x \pm \dfrac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\dfrac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\dfrac{Rx+S}{x \pm T}$ using the quotient rule oe.

A1ft $x^2+A+\dfrac{B}{x-2} \to 2x-\dfrac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\dfrac{B}{x-2}$ only

M1 Subs $x=3$ into their $f'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\dfrac{1}{\text{their } f'(3)}$ and the point must be $(3, f(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-f(3)=-\dfrac{1}{f'(3)}(x-3)$ or $(y-f(3))\times -f'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as $c=$

A1 cso $y-16=-\dfrac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2 \Rightarrow A-6=-3, \quad x \Rightarrow A+B=7, \quad \text{const} \Rightarrow -6A+3B=-6$ $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve

$$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x\pm 3)$$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of A and B.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$f'(x)=\dfrac{(x^2+x-6)(4x^3+3x^2-6x+7)-(x^4+x^3-3x^2+7x-6)(2x+1)}{(x^2+x-6)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and

achieves an expression of the form $f'(x)=\dfrac{(x^2+x-6)(..x^3........)-(x^4+x^3-3x^2+7x-6)(..x..)}{(x^2+x-6)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=(x^2+x-6)^{-1}$

Note that this can score full marks from a partially solved part (a) where $f(x)\equiv x^2+3+\dfrac{4x+12}{x^2+x-6}$

Question 3

Question	Scheme	Marks
(a)	$y=\dfrac{4x}{(x^2+5)} \Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{4(x^2+5)-4x\times 2x}{(x^2+5)^2}$	M1A1
	$\Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{20-4x^2}{(x^2+5)^2}$	M1A1
		(4)
(b)	$\dfrac{20-4x^2}{(x^2+5)^2}<0 \Rightarrow x^2>\dfrac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5},\, x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

(a)M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=4x$ and $v=x^2+5$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx$ followed by their $\dfrac{vu'-uv'}{v^2}$

If the rule is neither quoted nor implied only accept expressions of the form

$\dfrac{A(x^2+5)-4x\times Bx}{(x^2+5)^2}, A,B>0$ You may condone missing (invisible) brackets

Alternatively uses the **product rule** with $u(/v)=4x$ and $v(/u)=(x^2+5)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx(x^2+5)^{-2}$ followed by their $vu'+uv'$. If the rule is neither quoted nor implied only accept expressions of the form $A(x^2+5)^{-1}\pm 4x\times Bx(x^2+5)^{-2}$

A1 $\mathrm{f}'(x)$ correct (unsimplified). For the product rule look for versions of $4(x^2+5)^{-1}-4x\times 2x(x^2+5)^{-2}$

M1 Simplifies to the form $\mathrm{f}'(x)=\dfrac{A+Bx^2}{(x^2+5)^2}$ oe. This is not dependent so could be scored from $\dfrac{v'u-u'v}{v^2}$

When the product rule has been used the A of $A(x^2+5)^{-1}$ must be adapted.

A1 CAO. Accept exact equivalents such as $(\mathrm{f}'(x))=\dfrac{4(5-x^2)}{(x^2+5)^2}$, $-\dfrac{4x^2-20}{(x^2+5)^2}$ or $\dfrac{-4(x^2-5)}{x^4+10x^2+25}$

Remember to isw after a correct answer

(b)

M1 Sets their numerator either $=0$, <0, $\leqslant 0$, >0, $\geqslant 0$ and proceeds to at least **one** value for x

For example $20-4x^2 \geqslant 0 \Rightarrow x \geqslant \sqrt{5}$ will be M1 dM0 A0.

It cannot be scored from a numerator such as 4 or indeed $20+4x^2$

dM1 Achieves **two** critical values for their numerator $=0$ and chooses the outside region

Look for $x<$ smaller root, $x>$ bigger root. Allow decimals for the roots.

Condone $x \leqslant -\sqrt{5}, x \geqslant \sqrt{5}$ and expressions like $-\sqrt{5}>x>\sqrt{5}$

If they have $4x^2-20<0$ following an incorrect derivative they should be choosing the inside region

A1 Allow $x<-\sqrt{5}, x>\sqrt{5}$ $\quad x<-\sqrt{5}$ or $x>\sqrt{5}$ $\quad \{x:-\infty<x<-\sqrt{5}\cup\sqrt{5}<x<\infty\}$ $|x|>\sqrt{5}$

Do not allow for the A1 $x<-\sqrt{5}$ and $x>\sqrt{5}$. $\sqrt{5}<x<-\sqrt{5}$ or $\{x:-\infty<x<-\sqrt{5}\cap\sqrt{5}<x<\infty\}$ but you may isw following a correct answer.

Question 4

Question	Scheme	Marks
(a)	$2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$	B1
	$\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv \dfrac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow 3\cot x = \operatorname{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)
(a)alt 1	$2\cot 2x + \tan x \equiv \dfrac{2\cos 2x}{\sin 2x} + \tan x$	B1
	$\equiv 2\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x} + \dfrac{\sin x}{\cos x}$	M1
	$\equiv \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} \equiv \dfrac{\cos^2 x}{\sin x \cos x}$	M1
	$\equiv \dfrac{\cos x}{\sin x}$	
	$\equiv \cot x$	A1*
(a)alt 2	$2\cot 2x + \tan x \equiv 2\dfrac{(1-\tan^2 x)}{2\tan x} + \tan x$	B1M1
	$\equiv \dfrac{2}{2\tan x} - \dfrac{2\tan^2 x}{2\tan x} + \tan x$ or $\dfrac{(1-\tan^2 x) + \tan^2 x}{\tan x}$	
	$\equiv \dfrac{2}{2\tan x} = \cot x$	M1A1*

Alt (b)	$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow \dfrac{3\cos x}{\sin x} = \dfrac{1}{\sin^2 x} - 2$	
	$\left(\times \sin^2 x\right) \Rightarrow 3\sin x \cos x = 1 - 2\sin^2 x$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2x = \cos 2x$	M1A1
	$\Rightarrow \tan 2x = \dfrac{2}{3} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$.

For this to be scored the expression must be in either $\sin x$ and $\cos x$ or just $\tan x$.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*

If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x / \tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The $= 0$ may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt 3.30, -0.30

Be aware that $\cot x = \dfrac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for x in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi \leqslant x \leqslant \pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to x=..using the correct order of operations.

Question 5

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td></td><td>$2x^2y + 2x + 4y - \cos(\pi y) = 17$</td><td></td><td></td></tr>
<tr><td>(a) Way 1</td><td>$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\} \left(\underline{\underline{4xy + 2x^2\frac{dy}{dx}}}\right) \underline{+ 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx}} = 0$</td><td></td><td>M1 A1 B1</td></tr>
<tr><td></td><td>$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$</td><td></td><td>dM1</td></tr>
<tr><td></td><td>$\left\{\frac{dy}{dx} = \right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$</td><td>Correct answer or equivalent</td><td>A1 cso</td></tr>
<tr><td></td><td></td><td></td><td>[5]</td></tr>
<tr><td>(b)</td><td>At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)}$ $\left\{= \frac{-8}{22 + \pi}\right\}$</td><td>Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$</td><td>M1</td></tr>
<tr><td></td><td>$m_N = \frac{22 + \pi}{8}$</td><td>Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N
Can be implied by later working</td><td>M1</td></tr>
<tr><td></td><td>• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$
• $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$
$\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$
Cuts x-axis $\Rightarrow y = 0$
$\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$</td><td>$y - \frac{1}{2} = m_N(x - 3)$ or
$y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$
with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.</td><td>dM1</td></tr>
<tr><td></td><td>So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\} x = \frac{3\pi + 62}{\pi + 22}$</td><td>$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$</td><td>A1 o.e.</td></tr>
<tr><td></td><td></td><td></td><td>[4]</td></tr>
<tr><td></td><td></td><td></td><td>9</td></tr>
<tr><td>(a) Way 2</td><td colspan="2">$\left\{\frac{\cancel{dx}}{\cancel{dy}} \times\right\} \left(\underline{\underline{4xy\frac{dx}{dy} + 2x^2}}\right) \underline{+ 2\frac{dx}{dy} + 4 + \pi\sin(\pi y)} = 0$</td><td>M1 A1 B1</td></tr>
<tr><td></td><td colspan="2">$\frac{dx}{dy}(4xy + 2) + 2x^2 + 4 + \pi\sin(\pi y) = 0$</td><td>dM1</td></tr>
<tr><td></td><td>$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$</td><td>Correct answer or equivalent</td><td>A1 cso</td></tr>
<tr><td></td><td colspan="2"></td><td>[5]</td></tr>
</table>

		Question Notes	
(a)	Note	Writing down *from no working* • $\frac{dy}{dx} = \frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or $\frac{4xy+2}{-2x^2-4-\pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx} = \frac{4xy+2}{2x^2+4+\pi\sin(\pi y)}$ scores M1A0B1M1A0	
	Note	Few candidates will write $4xy\,dx + 2x^2 dy + 2dx + 4dy + \pi\sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or equivalent. This should get full marks.	

		Question Notes Continued
(a) Way 1	M1	Differentiates implicitly to include either $2x^2\frac{dy}{dx}$ or $4y \rightarrow 4\frac{dy}{dx}$ or $-\cos(\pi y) \rightarrow \pm\lambda\sin(\pi y)\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1	$2x + 4y - \cos(\pi y) = 17 \rightarrow 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$
	Note	$4xy + 2x^2\frac{dy}{dx} + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} \rightarrow 2x^2\frac{dy}{dx} + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = -4xy - 2$ will get 1st A1 (implied) as the $"=0"$ can be implied by the rearrangement of their equation.
	B1	$2x^2y \rightarrow 4xy + 2x^2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	dM1	**Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are *at least two terms* in $\frac{dy}{dx}$. ie. $\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + \ldots = \ldots$
	Note	Writing down an extra $\frac{dy}{dx} = \ldots$ and then including it in their factorisation is fine for dM1.
	Note	**Final A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from correct solution.
(a)	Way 2	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	1st M1	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$. E.g. $"-4xy" \rightarrow "-6"$ in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}$, $y = 3$.
	3rd M1	*is dependent on the first M1.*
	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 2nd M1 3rd M1** for $\frac{\frac{1}{2}}{3-x} = \frac{-1}{\text{their } m_T}$
	Note	We can accept $\sin\pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2nd M1 mark. But, $\sin\pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3rd M1 mark. The 3rd M1 can be accessed for terms containing $\pi\sin\left(\frac{\pi}{2}\right)$.

Question 6

Question	Scheme	Marks
(i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x = \text{awrt } 0.9463$ 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5) (10 marks)
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx} = 0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x} \equiv \tan 4x$, moves from $\tan 4x = C, C \neq 0$ using correct order of operations to $x = ...$ Accept $x =$ awrt 0.16 (radians) $x =$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta + \alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x + \text{awrt } 0.93)$ or $5\sin(4x - \text{awrt } 0.64)$ before using the correct order of operations to $x = ...$

Similarly candidates who square $3\cos 4x - 4\sin 4x = 0$ then use a Pythagorean identity should proceed from either $\sin 4x = \frac{3}{5}$ or $\cos 4x = \frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x =$ awrt 0.9463.

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dy}{dx}} = \pm A\sin 2y\cos 2y \pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy} = 2\sin 2y \times 2\cos 2y = (4\sin 2y\cos 2y)$ or

$1 = 2\sin 2y \times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression.

You may just see a statement such as $4\sin 2y\cos 2y = 2\sin 4y$ which is fine.

Candidates who write $\cancel{\frac{dy}{dx}} = A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dy}{dx}} = \frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx} = 1 \big/ \frac{dx}{dy}$ **for their expression in** y. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = 2\text{cosec}\,4y$ is condoned for the M1

If $\frac{dx}{dy} = a + b$ do not allow $\frac{dy}{dx} = \frac{1}{a} + \frac{1}{b}$

A1 $\frac{dy}{dx} = \frac{1}{2}\text{cosec}\,4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as to how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x = \sin^2 2y \Rightarrow x = \pm\frac{1}{2} \pm \frac{1}{2}\cos 4y$ from $\cos 4y = \pm 1 \pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}} = \sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}} = Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y = \text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}} = N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}} \times x^{-0.5}$

Question 7

Question	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-\text{their }'59.5°'$	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$	M1
	$\theta=\text{awrt}\ 86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt}\ 26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their }26.6°)=K$, $|K|\leqslant 1$

A1 $\cos(\theta\pm\text{their }26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt }0.507)$. Can be implied by $(\theta\pm\text{their }26.6°)=\text{awrt }59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their }26.6°=360°-\text{their }59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leqslant\theta<360°$

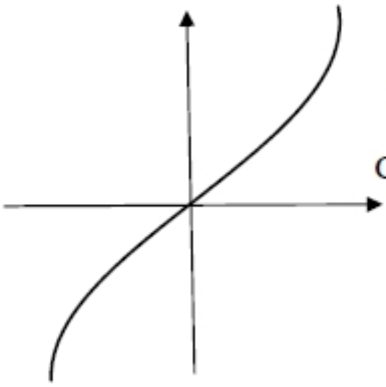
(c)

M1 $\theta-\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$
Alternatively $-\theta+\text{their }26.6°=-\text{their }59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their }26.57°=\text{their }59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt }0.58$ and $\theta_2=\text{awrt }4.78$ (c) $\theta_3=\text{awrt }1.50$. Require 2 dp accuracy

Question 8

Question	Scheme	Marks
(a)	 Correct position or curvature	M1
	Correct position and curvature	A1
		(2)
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\frac{\pi}{3}$	M1
	$\Rightarrow (x+1)=\sin\left(-\frac{\pi}{3}\right)$	
	$\Rightarrow x=-1-\frac{\sqrt{3}}{2}$	dM1A1
		(3)
		(5 marks)

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject

Accept $\arcsin(x+1)=\pm\frac{\pi}{3}$ or even $g(x+1)=\pm\frac{\pi}{3}$. Condone $\frac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\frac{\pi}{3}\right)$ and making x the subject.

Accept for this mark $\Rightarrow x=\pm\frac{\sqrt{3}}{2}\pm 1$. Accept decimal such as -1.866

Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\frac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer

Be careful with single fractions. $-\frac{2-\sqrt{3}}{2}$ and $\frac{-2+\sqrt{3}}{2}$ are incorrect but $-\frac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\frac{\pi}{3}$ to $60°$ and work in degrees for all marks

Question 9

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** *Some evidence* of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y =$ awrt 8.7	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4, y = 5\sqrt{3}$	A1
			[2]
			6

		Question Notes
(a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}$, $\cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]

Question 10

Question Number	Scheme	Notes	Marks
	(i) $\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,\ x=4\sin^2\theta$		
(i) **Way 1**	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y}+\frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A\neq 0,\ B\neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 cao
			[6]
(ii) (a) **Way 1**	$\{x=4\sin^2\theta \Rightarrow\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta}=8\sin\theta\cos\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta}=4\sin2\theta$ or $\mathrm{d}x=8\sin\theta\cos\theta\,\mathrm{d}\theta$		B1
	$\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}.\,4\sin2\theta\ \{\mathrm{d}\theta\}$		M1
	$=\int \underline{\underline{\tan\theta}}.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \underline{\underline{\tan\theta}}.\,4\sin2\theta\ \{\mathrm{d}\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to \pm K\tan\theta$ or $\pm K\left(\frac{\sin\theta}{\cos\theta}\right)$	$\underline{\underline{\text{M1}}}$
	$=\int 8\sin^2\theta\,\mathrm{d}\theta$	$\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1
	$3=4\sin^2\theta$ or $\frac{3}{4}=\sin^2\theta$ or $\sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$ $\{x=0 \to \theta=0\}$	Writes down a correct equation involving $x=3$ leading to $\theta=\frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]
(ii) (b)	$=\{8\}\int\left(\frac{1-\cos2\theta}{2}\right)\mathrm{d}\theta\quad \left\{=\int(4-4\cos2\theta)\mathrm{d}\theta\right\}$	Applies $\cos2\theta=1-2\sin^2\theta$ to their integral. (See notes)	M1
	$=\{8\}\left(\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right)\quad \{=4\theta-2\sin2\theta\}$	For $\pm\alpha\theta\pm\beta\sin2\theta,\ \alpha,\beta\neq 0$	M1
		$\sin^2\theta \to \left(\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right)$	A1
	$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6}-\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right)-(0+0)\right)$		
	$=\frac{4}{3}\pi-\sqrt{3}$	"two term" exact answer of e.g. $\frac{4}{3}\pi-\sqrt{3}$ or $\frac{1}{3}(4\pi-3\sqrt{3})$	A1 o.e.
			[4]
			15

		Question Notes
(i)	1st M1	Writing $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y} + \dfrac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied *for writing down* either $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{-2}{y} + \dfrac{\text{their } B}{(3y+2)}$ or $\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{\text{their } A}{y} + \dfrac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\dfrac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\dfrac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ or $\dfrac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
(ii)(a)	1st M1	Substitutes $x = 4\sin^2\theta$ and their $\mathrm{d}x$ $\left(\text{from their correctly rearranged } \dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\displaystyle\int \sqrt{\left(\dfrac{x}{4-x}\right)}\,\mathrm{d}x$
	Note	$\mathrm{d}x \neq \lambda\,\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$
	Note	Allow substituting $\mathrm{d}x = 4\sin 2\theta$ for the 1st M1 after a correct $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or $\mathrm{d}x = 4\sin 2\theta\,\mathrm{d}\theta$
	2nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\dfrac{x}{4-x}\right)}$ to give $\pm K\tan\theta$ or $\pm K\left(\dfrac{\sin\theta}{\cos\theta}\right)$
	Note	Integral sign is not needed for this mark.
	1st A1	Simplifies to give $\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$
	2nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \dfrac{\pi}{3}$ and no incorrect work seen regarding limits
	Note	Allow 2nd B1 for $x = 4\sin^2\left(\dfrac{\pi}{3}\right) = 3$ **and** $x = 4\sin^2 0 = 0$
	Note	Allow 2nd B1 for $\theta = \sin^{-1}\left(\sqrt{\dfrac{x}{4}}\right)$ followed by $x = 3,\ \theta = \dfrac{\pi}{3};\ x = 0,\ \theta = 0$

(ii)(b)	M1	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2\theta$ **E.g.:** $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ or $K\sin^2\theta = K\left(\dfrac{1-\cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0,\ \beta \neq 0$ (can be simplified or un-simplified).
	1st A1	Integrating $\sin^2\theta$ to give $\dfrac{1}{2}\theta - \dfrac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only. Can be implied by $k\sin^2\theta$ giving $\dfrac{k}{2}\theta - \dfrac{k}{4}\sin 2\theta$ or $\dfrac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.
	2nd A1	A *correct solution in part (ii)* leading to a "two term" exact answer of e.g. $\dfrac{4}{3}\pi - \sqrt{3}$ or $\dfrac{8}{6}\pi - \sqrt{3}$ or $\dfrac{4}{3}\pi - \dfrac{2\sqrt{3}}{2}$ or $\dfrac{1}{3}(4\pi - 3\sqrt{3})$
	Note	A decimal answer of 2.456739397… (without a correct **exact** answer) is A0.
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1st three marks (i.e. M1M1A1) in part (b).
	Note	If they incorrectly obtain $\displaystyle\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$) then the final A1 is available for a correct solution in part (ii)(b).

	Scheme	Notes	Marks
(i) **Way 2**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$		
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 6 = 2A \Rightarrow A = 3$	At least one of their $A = 3$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 3$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]
(i) **Way 3**	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$		
	$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$	See notes	M1
	$y=0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$	At least one of their $A = \frac{5}{2}$ **or** their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$	Both their $A = \frac{5}{2}$ **and** their $B = -\frac{15}{2}$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy$ $= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$; $M \neq 0, A \neq 0, B \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$	$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Scheme	Notes	
(i) **Way 4**	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\displaystyle = \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\displaystyle\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \;\Rightarrow\; 4 = 2A \;\Rightarrow\; A = 2$	At least one of their $A = 2$ or their $B = -6$	A1
	$y = -\frac{2}{3} \;\Rightarrow\; 4 = -\frac{2}{3}B \;\Rightarrow\; B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$ $\displaystyle = \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0,\ B \neq 0,\ C \neq 0$	M1
		At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]

	Alternative methods for B1M1M1A1 in (ii)(a)		
(ii)(a) **Way 2**	$\displaystyle\left\{x = 4\sin^2\theta \Rightarrow\right\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in Way 1	B1
	$\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$	As before	M1
	$\displaystyle = \int\sqrt{\frac{\sin^2\theta}{(1-\sin^2\theta)}}\,.\,8\cos\theta\sin\theta\ \{\mathrm{d}\theta\}$		
	$\displaystyle = \int \frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\,.\,8\sqrt{(1-\sin^2\theta)}\sin\theta\ \{\mathrm{d}\theta\}$		
	$\displaystyle = \int \sin\theta\,.\,8\sin\theta\ \{\mathrm{d}\theta\}$	Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out	M1
	$\displaystyle = \int 8\sin^2\theta\,\mathrm{d}\theta$	$\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso
(ii)(a) **Way 3**	$\displaystyle\left\{x = 4\sin^2\theta \Rightarrow\right\}\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$	As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta,\quad 4 - x = 2 + 2\cos 2\theta$		
	$\displaystyle\int\sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		M1
	$\displaystyle = \int \frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2+2\cos 2\theta}}\,.\,\frac{\sqrt{2-2\cos 2\theta}}{\sqrt{2-2\cos 2\theta}}4\sin 2\theta\ \{\mathrm{d}\theta\} = \int \frac{2-2\cos 2\theta}{\sqrt{4-4\cos^2 2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$		
	$\displaystyle = \int \frac{2-2\cos 2\theta}{2\sin 2\theta}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\} = \int 2(2-2\cos 2\theta).\ \{\mathrm{d}\theta\}$	Correct method leading to $\sin 2\theta$ being cancelled out	M1
	$\displaystyle = \int 8\sin^2\theta\,\mathrm{d}\theta$	$\displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$ including $\mathrm{d}\theta$	A1 cso

Question 11

Question Number	Scheme	Notes	Marks
	$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 1**	$\int \frac{1}{x}\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$	Separates variables as shown. $\mathrm{d}x$ and $\mathrm{d}t$ should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) **Way 2**	$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,\mathrm{d}x$	**Either** $\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{2}{5x}$ **or** $t = \int -\frac{2}{5x}\,\mathrm{d}x$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = \ldots$ **or** $\pm\alpha\ln px;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) **Way 3**	$\int_{60}^{x} \frac{1}{x}\,\mathrm{d}x = \int_{0}^{t} -\frac{5}{2}\,\mathrm{d}t$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

(b)	$20 = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda\mathrm{e}^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda\mathrm{e}^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449\ldots \text{(days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in$ □ $, t > 0$)	dM1
	$\Rightarrow t = 632.8006\ldots = 633$ (to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 cso
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}\ \frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) **Way 5**	$\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}\quad t = \int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

		Question Notes
(a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,dx = \int -\frac{1}{2}\,dt$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{e^{-5t}}$ or $x = \frac{60}{\sqrt{e^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = e^{-\frac{5}{2}t} + 60 \rightarrow x = 60e^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = e^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60e^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply cso for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0