Pure Mathematics 2 Practice Paper M12 MARK SCHEME

Question 1

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(x) = x^2 + \dfrac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$		
	$\mathrm{f}(x) = x^2 + \dfrac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$\mathrm{f}'(x) = 2x - \dfrac{3}{8}x^{-\frac{3}{2}} - 3\ \{+\ 0\}$	M1: $x^n \to x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$\mathrm{f}(4) = -2.625 = -\dfrac{21}{8} = -2\dfrac{5}{8}$ *or* $4^2 + \dfrac{3}{4\sqrt{4}} - 3\times4 - 7$	$\mathrm{f}(4) = -2.625$ A correct evaluation of f(4) or a correct numerical expression for f(4). This can be implied by a correct answer below but in all other cases, f(4) must be seen explicitly evaluated or as an expression.	
	$\mathrm{f}'(4) = 4.953125 = \dfrac{317}{64} = 4\dfrac{61}{64}$	Attempt to insert $x = 4$ into their $\mathrm{f}'(x)$. Not dependent on the first M but must be what they think is $\mathrm{f}'(x)$.	M1
	$\alpha_2 = 4 - \left(\dfrac{"-2.625"}{"4.953125"}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454\ldots \quad \left(= \dfrac{1436}{317} = 4\frac{168}{317}\right)$		
	$= 4.53\ \ (2\,\mathrm{dp})$	4.53 cso	A1 cao
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \dfrac{f(4)}{f'(4)} = 4.53$ can score full marks.		

Question 2

Question Number	Scheme	Marks
	$a_1 = 3,\ a_{n+1} = 2a_n - c,\ \ n \geq 1,\ c$ is a constant	
(a)	$\{a_2 =\}\ 2\times3 - c$ or $2(3) - c$ or $6 - c$	B1
		[1]
(b)	$\{a_3 =\}\ 2\times(\text{"}6-c\text{"}) - c$	M1
	$= 12 - 3c$ (*)	A1 cso
		[2]
(c)	$a_4 = 2\times(\text{"}12-3c\text{"}) - c \qquad \{= 24 - 7c\}$	M1
	$\left\{\sum_{i=1}^{4} a_i =\right\}\ 3 + (6-c) + (12-3c) + (24-7c)$	M1
	$\text{"}45 - 11c\text{"} \geq 23$ or $\text{"}45 - 11c\text{"} = 23$	M1
	$c \leq 2$ or $2 \geq c$	A1 cso
		[4]
		7
	Notes	
(a)	The answer to part (a) cannot be recovered from candidate's working in part (b) or part (c). Once the candidate has achieved the correct result you can ignore subsequent working in this part.	
(b)	**M1:** For a correct substitution of *their* a_2 *which must include term(s) in* c into $2a_2 - c$ giving a result for a_3 in terms of only c. Candidates must use correct bracketing for this mark. **A1:** for correct solution only. No incorrect working/statements seen. (**Note:** the answer is given!)	
(c)	**1st M1:** For a correct substitution of a_3 *which must include term(s) in* c into $2a_3 - c$ giving a result for a_4 in terms of only c. Candidates must use correct bracketing (can be implied) for this mark. **2nd M1:** for an attempt to sum their a_1, a_2, a_3 and a_4 only. **3rd M1:** for their sum (of 3 or 4 or 5 consecutive terms) $=$ or $\geq$ or > 23 to form a linear inequality or equation in c. **A1:** for $c \leq 2$ or $2 \geq c$ from a correct solution only. **Beware:** $-11c \geq -22 \Rightarrow c \geq 2$ is A0. **Note:** $45 - 11c \geq 23 \Rightarrow -11c \leq -22 \Rightarrow c \leq 2$ would be A0 cso. **Note:** Applying either $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ is 2nd M0, 3rd M0. **Note:** If a candidate gives a numerical answer in part (a); they will then get M0A0 in part (b); but if they use the printed result of $a_3 = 12 - 3c$ they could potentially get M0M1M1A0 in part (c) **Note:** If a candidate only adds numerical values (not in terms of c) in part (c) then they could potentially get only M0M0M1A0. **Note:** For the 3rd M1 candidates will usually sum a_1, a_2, a_3 and a_4 or a_2, a_3 and a_4 or a_2, a_3, a_4 and a_5 or a_1, a_2, a_3, a_4 and a_5	

Question 3

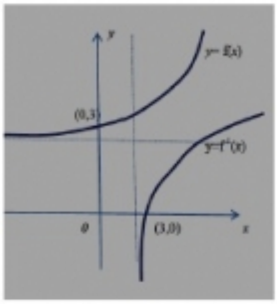
Question Number	Scheme	Marks
	Boy's Sequence: 10, 15, 20, 25, ...	
(a)	$\{a=10, d=5 \Rightarrow T_{15}=\}\ a+14d=10+14(5);=80$ or $0.1+14(0.05);=£0.80$	M1; A1 [2]
(b)	$\{S_{60}=\}\ \frac{60}{2}[2(10)+59(5)]$	M1 A1
	$=30(315)=9450$ or $£94.50$	A1 [3]
	Boy's Sister's Sequence: 10, 20, 30, 40, ...	
(c)	$\{a=10, d=10 \Rightarrow S_m=\}\ \frac{m}{2}(2(10)+(m-1)(10))$ $\left(\text{or } \frac{m}{2}\times 10(m+1) \text{ or } 5m(m+1)\right)$	M1 A1
	63 or $6300=\frac{m}{2}(2(10)+(m-1)(10))$	dM1
	$6300=\frac{m}{2}(10)(m+1)$ or $12600=10m(m+1)$	
	$1260=m(m+1)$	
	$35\times 36=m(m+1)$ (*)	A1 cso [4]
(d)	$\{m=\}\ 35$	B1 [1]
		10

Notes

(a)	**M1:** for using the formula $a+14d$ with either a or d correct. **A1:** for 80 or 80p or £0.80 or £0.80p and apply ISW. Otherwise, £80 or 0.80 or 0.80p would be A0. Award M0 if candidate applies $a+59d$. Listing the first 15 terms and highlighting that the 15th term is 80 or listing 15 terms with the final 15th term aligned with 80 will then be awarded all two marks of M1A1. Writing down 80 with no working is M1A1.
(b)	**M1:** for use of correct $\frac{60}{2}[2(10)+59(5)]$ or $\frac{15}{2}(2(10)+14(5))$ with $a=10,\ d=5$ and $n=60$ or $a=10,\ d=5$ and $n=15$. If a candidate uses $\frac{n}{2}(a+l)$ with $n=60$ or 15, there must be a full method of finding or stating l as either $a+59d\ (=305)$ or $a+14d\ (=80)$, respectively. **1st A1:** for a correct expression for S_{60}. ie. $\frac{60}{2}[2(10)+59(5)]$ or $\frac{60}{2}[2(0.1)+59(0.05)]$ or $\frac{60}{2}[10+305]$ or $\frac{60}{2}[0.10+3.05]$. This mark can be implied by later working. **2nd A1:** for 9450 or 9450p or £94.50 and apply ISW. Otherwise, £9450 or 94.50 without £ sign is A0. **Note:** the bracketing error of $\frac{60}{2}2(10)+59(5)$ is A0 unless recovered from later working. Adding together the first 60 terms to obtain 9450 will then be awarded all three marks of M1A1A1.

(c)	**1st M1:** for correct use of S_m formula with one of a or d correct. **1st A1:** for a correct expression for S_m. Eg: $\frac{m}{2}(2(10)+(m-1)(10))$ or $\frac{m}{2}\times 10(m+1)$ or $5m(m+1)$ **2nd M1:** for forming a suitable equation using 63 or 6300 and their S_m. **Dependent on 1st M1.** **2nd A1cso:** for *reaching the printed result* with no incorrect working seen. Long multiplication is not necessary for the final accuracy mark. **Note:** $\frac{m}{2}(2(10)+(m-1)(10))=630$ and not either 6300 or 63 is dM0. **Beware:** Some candidates will try and fudge the result given on the question paper. **Notes for awarding 2nd A1** Going from $m(m+1)=1260$ straight to $m(m+1)=35\times 36$ is 2nd A1. Going from $m(m+1)=$ some factor decomposition of 6300 straight to $m(m+1)=35\times 36$ is 2nd A1. Going from $10m(m+1)=12600$ straight to $m(m+1)=35\times 36$ is 2nd A0. Going from $m(m+1)=\frac{6300}{5}$ straight to $m(m+1)=35\times 36$ is 2nd A0. ***Alternative: working in an different letter, say n or p.*** **M1A1:** for $\frac{n}{2}(2(10)+(n-1)(10))$ (although mixing letters eg. $\frac{n}{2}(2(10)+(m-1)(10))$ is M0A0). **dM1:** for 63 or $6300=\frac{n}{2}(2(10)+(n-1)(10))$ Leading to $6300=\frac{n}{2}(10)(n+1) \Rightarrow 1260=n(n+1) \Rightarrow 35\times 36=n(n+1)$ The candidate then needs to write either $35\times 36=m(m+1)$ or $m\equiv n$ or $m=n$ to gain the final A1.
(d)	**B1:** for 35 only.

Question 4

Question Number	Scheme	Marks
	(a) f(x)>2	B1 (1)
	(b) $\text{fg}(x) = e^{\ln x} + 2, = x + 2$	M1,A1 (2)
	(c) $e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1A1 (4)
	(d) Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$	M1
	$\text{f}^{-1}(x) = \ln(x-2), \quad x > 2.$	A1 , **B1ft** (3)
	(e) 	
	Shape for f(x)	B1
	(0, 3)	B1
	Shape for $\text{f}^{-1}(x)$	B1
	(3, 0)	B1 (4)
		(14 marks)

(a) B1 Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1 For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0
A1 Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks

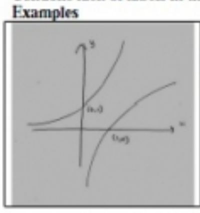
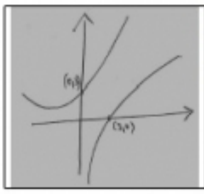
(c) M1 Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = ...$
A1 $e^{2x+3} = 4$
M1 Takes ln's both sides, $2x + 3 = \ln ..$ and proceeds to x=….
A1 $x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**
M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0
A1 $\text{f}^{-1}(x) = \ln(x-2)$ or y= $\ln(x-2)$ or y= $\ln|x-2|$ There must be some form of bracket
B1ft Either x>2, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
Do not accept y>2 or $\text{f}^{-1}(x)$>2.

(e) B1 Shape for y=e^x. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

B1 Shape for y=ln$x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects y=e^x
B1 (3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect
	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question 5

Question Number	Scheme	Marks
	(a)(i) $\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
	(ii) $\frac{dy}{dx} = \frac{(2x-1)^5\times -10 - (1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \Rightarrow \frac{dx}{dy} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5)
		(11 marks)

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form
$\ln(3x)\times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{B}{x}, \quad A, B > 0$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work
$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x} = (\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}) = x^{-\frac{1}{2}}(\frac{1}{2}\ln 3x + 1)$
Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm 10 - (1-10x)\times C(2x-1)^4}{(2x-1)^{10\text{ or }7\text{ or }25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx} = \frac{(2x-1)^5\times -10 - (1-10x)\times 5(2x-1)^4\times 2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan 2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan 2y$ as $\frac{3\sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy} = 6\sec^2 2y$ or implicitly to get $1 = 6\sec^2 2y\frac{dy}{dx}$
Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y\times 6\cos 2y - 3\sin 2y\times -2\sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx} = f(y)$, or changes the subject of their implicit
. differential to achieve a similar result $\frac{dy}{dx} = f(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use
$\sec^2 2y = 1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x. $\frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{dy}{dx} = \frac{3}{18+2x^2}$ or $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
	(a)(ii) Alt using the product rule	
	Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'.	
	See (a)(i) for rules on how to apply	
	$(2x-1)^{-5}\times -10 + (1-10x)\times -5(2x-1)^{-6}\times 2$	M1A1
	Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	A1
		(3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2	
	Rearrange $x = 3\tan 2y$ to $y = \frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate**	M1A1
	Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $= \frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	M1, A2
		(5)

Question 6

Question Number	Scheme	Marks
	(a $\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{3}{2}} \times \frac{1}{x} dx$	M1 A1
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{1}{2}} \, dx$	
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}} \quad (+C)$	M1 A1 (4)
	(b $\left[\frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}} \ln 8 - \frac{4}{9}4^{\frac{3}{2}}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$	M1
	$= (16 \ln 2 - \ldots) - \ldots$ Using or implying $\ln 2^n = n \ln 2$	M1
	$= \frac{46}{3}\ln 2 - \frac{28}{9}$	A1 (3)
		[11]

Question 7

Question Number	Scheme	Marks
	(a) $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$	B1 B1
		(2)
	(b) $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}\cos^2\theta}$	M1
	$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$	M1A1*
		(4)
	(c) $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$	M1
	$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	A1,A1
		(3)
		(9 marks)

Note (a) and (b) can be scored together

(a) B1 One term correct. Eg. writes $4\operatorname{cosec}^2 2\theta$ as $\dfrac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\operatorname{cosec}^2\theta$ as $\dfrac{1}{\sin^2\theta}$. Accept terms like $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{\cos^2\theta}{\sin^2\theta}$. The question merely asks for an expression in $\sin\theta$ and $\cos\theta$

B1 A fully correct expression in $\sin\theta$ and $\cos\theta$. Eg. $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ **Accept equivalents**

Allow a different variable say x's instead of θ's but do not allow mixed units.

b) M1 Attempts to combine their expression in $\sin\theta$ and $\cos\theta$ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1-\cos^2\theta = \sin^2\theta$

M1 Cancels correctly by $\sin^2\theta$ terms and replaces $\dfrac{1}{\cos^2\theta}$ with $\sec^2\theta$

A1* Cso. This is a given answer. All aspects must be correct

IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER

c) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order .
Similarly accept $\tan^2\theta = 3, \sin^2\theta = \dfrac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\dfrac{1}{2}$

A1 Obtains one correct answer usually $\theta = \dfrac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ Do not award if there are extra solutions inside the range.
Ignore solutions outside the range.

Question 8

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)
	(b) At $x=0 \quad \frac{dy}{dx} = 3$	B1
	Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=….,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx} = Ae^{x\sqrt{3}}\sin 3x + e^{x\sqrt{3}} \times \pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\frac{dy}{dx} = 0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in $\sin 3x$ and $\cos 3x$

A1 Achieves either $\tan 3x = -\sqrt{3}$ or $\tan 3x = -\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{9}$ or $3x = \frac{-\pi}{3} \Rightarrow x = \frac{-\pi}{9}$ but not $x = \arctan(\frac{-\sqrt{3}}{3})$

A1 CSO $x = \frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of 3 for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'} = \frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y = -\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3} = \frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$(\sqrt{12})\sin(3x + \frac{\pi}{3}) = 0$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x + \frac{\pi}{3}) = 0$ or $(\sqrt{12})\cos(3x - \frac{\pi}{6}) = 0$

M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x + \frac{\pi}{3} = 0$ or π or $2\pi \Rightarrow x = ...$

A1 Cao $x = \frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\sqrt{3}\sin 3x = -3\cos 3x \Rightarrow \cos^2(3x) = \frac{1}{4}$ or $\sin^2(3x) = \frac{3}{4}$	A1
	$x = \frac{1}{3}\arccos(\pm\sqrt{\frac{1}{4}})$ oe	M1
	$x = \frac{2\pi}{9}$	A1

Question 9

Question Number	Scheme	Marks
	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $(1 = A)$	M1
	$x \to \frac{1}{3}$ $1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) \mathrm{d}x$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

Question 10

Question Number	Scheme		Marks	
	(a) $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ *	cso	B1	(1)
	(b) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$		M1	
	At $x = 8$ $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025 \quad (\text{cm s}^{-1})$	2.5×10^{-4}	A1	(2)
	(c) $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$		B1	
	$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x\left(\frac{0.048}{3x^2}\right)$		M1	
	At $x = 8$ $\frac{dS}{dt} = 0.024 \quad (\text{cm}^2\,\text{s}^{-1})$		A1	(3)
				[6]

Question 11

Question Number	Scheme	Marks
	(a) $R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = \text{(awrt)}73.7^0$	M1A1
		(3)
	(b) $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$	M1
	$2x + their\,'\alpha' = 60^0$	A1
	$2x + their\,'\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$	M1
	$x = awrt\ 113.1^0, 173.1^0$	A1A1
		(5)
	(c) Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x\cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$	
	$= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
	(d) $14\cos^2 x - 48\sin x\cos x = R\cos(2x + \alpha) + 7$	
	Maximum value = 'R' + 'c'	M1
	= 32 cao	A1
		(2)
		(12 marks)

(a) B1 Accept 25, awrt 25.0, $\sqrt{625}$. Condone ± 25

M1 For $\tan\alpha = \pm\frac{24}{7}$ $\tan\alpha = \pm\frac{7}{24}$ $\sin\alpha = \pm\frac{24}{their\ R}$, $\cos\alpha = \pm\frac{7}{their\ R}$

A1 $\alpha = \text{(awrt)}73.7^0$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 Achieving $2x + their\ \alpha = 60^{(0)}$. This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}/-6.85^{(0)}/-6.9^{(0)}$

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\frac{360 \pm \text{'their' principal value} \pm \text{'their'}\ \alpha}{2}$

A1 $x = awrt\ 113.1^0/113.2^0$ OR $173.1^0/173.2^0$.

A1 $x = awrt\ 113.1^0$ AND 173.1^0. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

(c) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x\cos x$ in expression.

Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm\cos 2x \pm 1$

A1 Cao $= 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept a=7, b=-24, c=7

(d) M1 This mark is scored for adding their R to their c

A1 cao 32

Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 The correct principal value of $\frac{\pi}{3}$ or awrt 1.05 radians. Accept $60^{(0)}$

This can be implied by awrt – 0.12 radians or awrt or 1.97 radians or awrt 3.02 radians

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\frac{2\pi \pm \text{'their' principal value} \pm \text{'their'}\ \alpha}{2}$ Do not allow mixed units.

A1 $x = awrt\ 1.97$ OR 3.02.

A1 $x = awrt\ 1.97$ AND 3.02. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range