Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M12

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

$$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$$

A root α of the equation f $(x) = 0$ lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to f (x) to obtain a second approximation to α. Give your answer to 2 decimal places. (5)

(Total 5 marks)

Question 2

A sequence of numbers a_1, a_2, a_3 … is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2 (1)

(b) Show that $a_3 = 12 - 3c$ (2)

Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c (4)

(Total 7 marks)

Question 3

A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15 (2)

(b) Calculate the total amount he saves over the 60 week period. (3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$ (4)

(d) Hence write down the value of m. (1)

(Total 10 marks)

Question 4

The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f. (1)

(b) Find fg(x), giving your answer in its simplest form. (2)

(c) Find the exact value of x for which f $(2x + 3) = 6$ (4)

(d) Find f^{-1}, the inverse function of f, stating its domain. (3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. (4)

(Total 14 marks)

Question 5

(a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3tan2y$ find $\frac{dy}{dx}$ in terms of x. (5)

(Total 11 marks)

Question 6

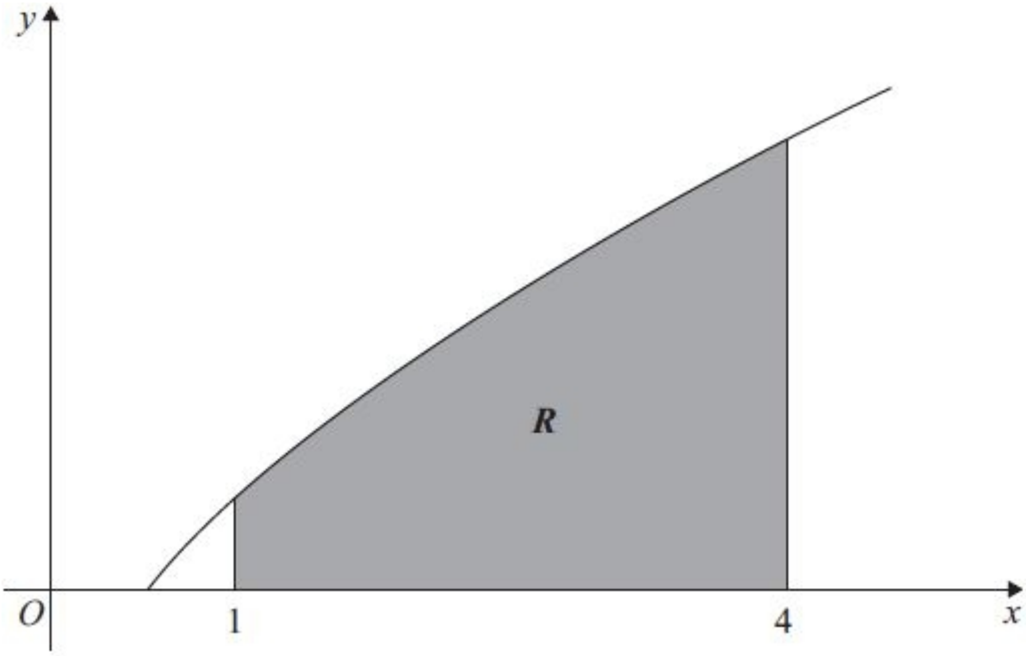

Figure 3

(a) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$. (4)

(b) Hence find the exact area of R, giving your answer in the form $a\ln 2 + b$, where a and b are exact constants. (3)

(Total 7 marks)

Question 7

(a) Express $4\text{cosec}^2 2\theta - \text{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$. (2)

(b) Hence show that

$$4\text{cosec}^2 2\theta - \text{cosec}^2\theta = \sec^2\theta$$ (4)

(c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\text{cosec}^2 2\theta - \text{cosec}^2\theta = 4$$

giving your answers in terms of π. (3)

(Total 9 marks)

Question 8

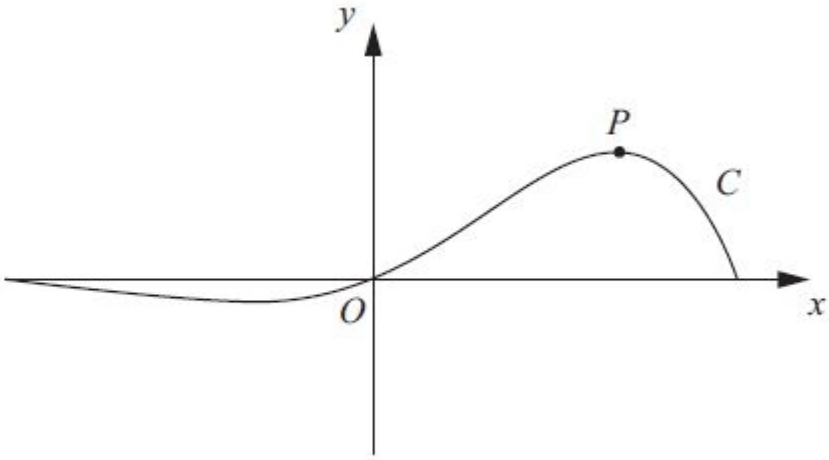

Figure 1

Figure 1 shows a sketch of the curve *C* which has equation

$$y = e^{x\sqrt{3}} \sin 3x \,, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the *x* coordinate of the turning point *P* on *C*, for which $x > 0$
Give your answer as a multiple of π. **(6)**

(b) Find an equation of the normal to *C* at the point where $x = 0$ **(3)**

(Total 9 marks)

Question 9

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants *A*, *B* and *C*. **(4)**

b) Hence find $\int f(x)\,dx.$

(ii) Find $\int_1^2 f(x)\,dx$, leaving your answer in the form $a + \ln b$,
where *a* and *b* are constants. **(6)**

(Total 10 marks)

Question 10

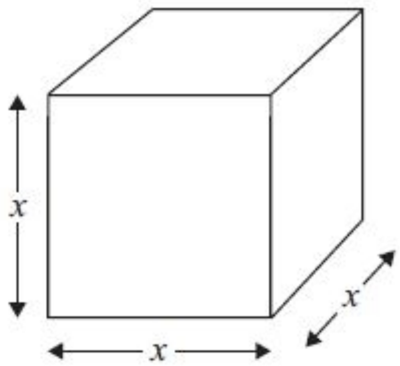

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm,

and the volume of the cube is V cm^3.

(a) Show that $\frac{dV}{dx} = 3x^2$ **(1)**

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\frac{dx}{dt}$, when $x = 8$ **(2)**

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$ **(3)**

(Total 6 marks)

Question 11

$f(x) = 7\cos 2x - 24\sin 2x$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the value of R and the value of α. **(3)**

(b) Hence solve the equation

$7\cos 2x - 24\sin 2x = 12.5$

for $0 \le x < 180°$, giving your answers to 1 decimal place. **(5)**

(c) Express $14\cos^2 x - 48\sin x \cos x$ in the form $a\cos 2x + b\sin 2x + c$,
where a, b, and c are constants to be found. **(2)**

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$14\cos^2 x - 48\sin x \cos x$ **(2)**

(Total 12 marks)

TOTAL FOR PAPER IS 100 MARKS