Pure Mathematics 2 Practice Paper M13 MARK SCHEME

Question 1

Question Number	Scheme		Marks
(a)	$600 = 200 + (N-1)20 \Rightarrow N = \ldots$	Use of 600 with a **correct** formula in an attempt to find N. A correct formula could be implied by a correct answer.	M1
	$N = 21$	cso	A1
	Accept correct answer only.		
	$600 = 200 + 20N \Rightarrow N = 20$ is M0A0 (wrong formula) $\frac{600-200}{20} = 20 \therefore N = 21$ is M1A1 (correct formula implied)		
	Listing: All terms must be listed up to 600 and 21 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.		
			(2)
(b)	**Look for an AP first:**		
	$S = \frac{21}{2}(2\times200+20\times20)$ or $\frac{21}{2}(200+600)$ or $S = \frac{20}{2}(2\times200+19\times20)$ or $\frac{20}{2}(200+580)$ (= 8400 or 7800)	M1: Use of correct sum formula with their **integer** $n = N$ or $N-1$ from part (a) where $3 < N < 52$ and $a = 200$ and $d = 20$. A1: Any correct un-simplified numerical expression with $n = 20$ or $n = 21$ (No follow through here)	M1A1
	Then for the constant terms:		
	$600\times(52-"N")(= 18600)$	M1: $600\times k$ where k is an integer and $3 < k < 52$ A1: A correct un-simplified follow through expression with their k consistent with n so that $n + k = 52$	M1A1ft
	So total is 27000	Cao	A1
	Note that for the constant terms, they may correctly use an AP sum with $d = 0$.		
	There are no marks in (b) for just finding S_{52}		
			(5)
			[7]
	If they obtain $N = 20$ in (a) (0/2) and then in (b) proceed with, $S = \frac{20}{2}(2\times200+19\times20)+32\times600 = 7800+19\ 200 = 27\ 000$ allow them to 'recover' and score full marks in (b) Similarly If they obtain $N = 22$ in (a) (0/2) and then in (b) proceed with, $S = \frac{21}{2}(2\times200+20\times20)+31\times600 = 8400+18\ 600 = 27\ 000$ allow them to 'recover' and score full marks in (b)		

Question 2

Question Number	Scheme		Marks
(a)	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\frac{1}{2}\right)x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + ...\right)\times\left(1+\left(-\frac{1}{2}\right)(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2 + ...\right)$	See notes	M1 A1 A1
	$= \left(1+\frac{1}{2}x - \frac{1}{8}x^2 + ...\right)\times\left(1+\frac{1}{2}x + \frac{3}{8}x^2 + ...\right)$		
	$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2 + ...$	See notes	M1
	$= 1 + x + \frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
(b)	$\sqrt{\left(\frac{1+(\frac{1}{26})}{1-(\frac{1}{26})}\right)} = 1 + \left(\frac{1}{26}\right) + \frac{1}{2}\left(\frac{1}{26}\right)^2$		M1
	ie: $\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$		B1
	so, $\sqrt{3} = \frac{7025}{4056}$	$\frac{7025}{4056}$	A1 cao
			[3] 9

Notes for Question

(a)

B1: $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ or $\sqrt{(1+x)}(1-x)^{-\frac{1}{2}}$ seen or implied. (Also allow $\left((1+x)(1-x)^{-1}\right)^{\frac{1}{2}}$).

M1: Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1 + ... + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{-\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(-\frac{1}{2}\right)(-x)$ or $+\left(-\frac{1}{2}\right)(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ or $1 + ... + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$

Also allow: $1 + ... + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x)^2$ for M1.

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

Note: Candidates can give decimal equivalents when expanding out their binomial expansions.

M1: Multiplies out to give 1, exactly two terms in x and exactly three terms in x^2.

A1: Candidate achieves the result on the exam paper. Make sure that their working is sound.

Special Case: Award SC FINAL M1A1 for ***a correct*** $\left(1+\frac{1}{2}x-\frac{1}{8}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$ multiplied out with no errors to give either $1+x+\frac{3}{8}x^2+\frac{1}{4}x^2-\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{4}x^2$ or $1+\frac{1}{2}x+\frac{5}{8}x^2+\frac{1}{2}x-\frac{1}{8}x^2$ leading to the correct answer of $1+x+\frac{1}{2}x^2$.

Notes for Question Continued			
(a) ctd	**Note:** If a candidate writes down either $(1+x)^{\frac{1}{2}} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots$ **or** $(1-x)^{-\frac{1}{2}} = 1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots$ with no working then you can award 1st M1, 1st A1. **Note:** If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.		
(b)	**M1:** Substitutes $x=\frac{1}{26}$ into **both** sides of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ and $1+x+\frac{1}{2}x^2$ **B1:** For sight of $\sqrt{\frac{27}{25}}$ (or better) and $\frac{1405}{1352}$ or equivalent fraction Eg: $\frac{3\sqrt{3}}{5}$ and $\frac{1405}{1352}$ or $0.6\sqrt{3}$ and $\frac{1405}{1352}$ or $\frac{3\sqrt{3}}{5}$ and $1\frac{53}{1352}$ or $\sqrt{3}$ and $\frac{5}{3}\left(\frac{1405}{1352}\right)$ are fine for B1. **A1:** $\frac{7025}{4056}$ **or any equivalent fraction**, eg: $\frac{14050}{8112}$ or $\frac{182650}{105456}$ etc. **Special Case:** Award SC: M1B1A0 for $\sqrt{3}\approx 1.732001972..$ or truncated 1.732001 or awrt 1.732002. Note that $\frac{7025}{4056}=1.732001972...$ and $\sqrt{3}=1.732050808...$		
Aliter (a) Way 2	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}=\sqrt{\frac{(1+x)(1-x)}{(1+x)(1-x)}}=\sqrt{\frac{(1-x^2)}{(1-x)^2}}=\right\}=(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	$(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	B1
	$=\left(1+\left(\frac{1}{2}\right)(-x^2)+\ldots\right)\times\left(1+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2+\ldots\right)$	See notes	M1A1A1
	$=\left(1-\frac{1}{2}x^2+\ldots\right)\times(1+x+x^2+\ldots)$		
	$=1+x+x^2-\frac{1}{2}x^2$	See notes	M1
	$=1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
Aliter (a) Way 2	**B1:** $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ seen or implied. **M1:** Expands $(1-x^2)^{\frac{1}{2}}$ to give both terms simplified or un-simplified, $1+\left(\frac{1}{2}\right)(-x^2)$ **or** expands $(1-x)^{-1}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+(-1)(-x)$ or $\ldots+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2$ or $1+\ldots+\frac{(-1)(-2)}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1:** Multiplies out to give 1, exactly one term in x and exactly two terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.		

Notes for Question Continued		
Aliter (a) Way 3	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} = \right\} = (1+x)(1-x^2)^{-\frac{1}{2}}$ $\qquad (1+x)(1-x^2)^{-\frac{1}{2}}$	B1
	$= (1+x)\left(1+\frac{1}{2}x^2+\ldots\right)$ Must follow on from above.	M1A1A1
	$= 1+x+\frac{1}{2}x^2$	dM1A1
	Note: The final M1 mark is dependent on the previous method mark for Way 3.	
Aliter (a) Way 4	**Assuming the result on the Question Paper. (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**	
	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \frac{\sqrt{(1+x)}}{\sqrt{(1-x)}} = 1+x+\frac{1}{2}x^2\right\} \Rightarrow (1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+\ldots \left\{= 1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right\}$, $(1-x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+\ldots \left\{= 1-\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right\}$	M1A1A1
	$\text{RHS} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right)$	
	$= 1-\frac{1}{2}x-\frac{1}{8}x^2+x-\frac{1}{2}x^2+\frac{1}{2}x^2$ See notes	M1
	$= 1+\frac{1}{2}x-\frac{1}{8}x^2$	
	So, $\text{LHS} = 1+\frac{1}{2}x-\frac{1}{8}x^2 = \text{RHS}$	A1 *
		[6]

B1: $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$ seen or implied.

M1: For Way 4, this M1 mark is dependent on the first B1 mark.

Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(\frac{1}{2}\right)(-x)$ or $+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

M1: For Way 4, this M1 mark is dependent on the first B1 mark.
Multiplies out RHS to give 1, exactly two terms in x and exactly three terms in x^2.

A1: Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both the LHS and RHS of $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$.

Question 3

Question Number	Scheme	Marks
(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	$ff(0) = f(5), \ = 3$	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to (d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4$ oe	A1 (5)

Notes for Question

(a)

B1 Correct range. Allow $0 \leqslant f(x) \leqslant 10$, $0 \leqslant f \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0,10]$

Allow $f(x) \geqslant 0$ and $f(x) \leqslant 10$ but not $f(x) \geqslant 0$ or $f(x) \leqslant 10$

Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at x=0

Possible ways to score this mark are f(0)=5, f(5) $0 \to 5 \to \ldots$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $g^{-1}(x) = \dfrac{5x-4}{3+x}$ $y = \dfrac{5x-4}{3+x}$

Accept alternatives such as $y = \dfrac{4-5x}{-3-x}$ and $y = \dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\dfrac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ..$

A1 Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$

B1 $x = 6$ and may be given if there is no working

M1 Full method to obtain other value from line $y = 5 - 2.5x$

$5 - 2.5x = 4 \Rightarrow x = \ldots$.

Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2-x}{4}(oe) \Rightarrow x = ..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $gf(x) = 16$ with a linear $f(x)$. The order of gf(x) must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$

B1 $x = 6$ and may be given if there is no working

M1 Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x = 0.4, \dfrac{2}{5}$ or equivalent

Question 4

Question Number	Scheme	Marks
(a)	$x^2 + 4xy + y^2 + 27 = 0$	
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \times\right\}$ $\quad \underline{2x} + \left(\underline{4y + 4x\frac{dy}{dx}}\right) + \underline{2y\frac{dy}{dx} = 0}$	M1 $\underline{\text{A1}}$
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$ $\left\{= \frac{-x-2y}{2x+y}\right\}$	A1 cso oe
		(4)
(b)	$4x + 2y = 0$	M1
	$y = -2x$ $\quad\Big\vert\quad$ $x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$ $\quad\Big\vert\quad$ $\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$ $\quad\Big\vert\quad$ $-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$ $\quad\Big\vert\quad$ $y^2 = 36$	dM1*
	$x = -3$ $\quad\Big\vert\quad$ $y = 6$	A1
	When $x = -3$, $y = -2(-3)$ $\quad\Big\vert\quad$ When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$ $\quad\Big\vert\quad$ $x = -3$	A1 cso
		[7]
	Notes for Question	
(a)	**M1**: Differentiates implicitly to include either $4x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).	
	A1: $(x^2) \to (\underline{2x})$ and $\left(\ldots + y^2 + 27 = 0 \to \underline{+ 2y\frac{dy}{dx} = 0}\right)$.	
	Note: If an extra term appears then award A0.	
	Note: The "$= 0$" can be implied by rearrangement of their equation.	
	i.e.: $2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx}$ leading to $4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$ will get A1 (implied).	
	B1: $4y + 4x\frac{dy}{dx}$ or $4\left(y + x\frac{dy}{dx}\right)$ or equivalent	
	dM1: An attempt to factorise out $\frac{dy}{dx}$ as long as there are at least two terms in $\frac{dy}{dx}$.	
	ie. $\ldots + (4x + 2y)\frac{dy}{dx} = \ldots$ or $\ldots + 2(2x + y)\frac{dy}{dx} = \ldots$	
	Note: This mark is dependent on the previous method mark being awarded.	
	A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x-2y}{2x+y}$	
	cso: If the candidate's solution is not completely correct, then do not give this mark.	

Notes for Question Continued	
(b)	**M1:** Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe. **A1:** Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only). The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$. **M1*:** Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0,\ \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable. **dM1*:** leading to at least either $x^2 = A,\ A > 0$ or $y^2 = B,\ B > 0$ **Note:** This mark is dependent on the previous method mark (M1*) being awarded. **A1:** For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only). **ddM1*** Substitutes their value of x into $y = \pm\lambda x$ to give $y =$ value or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value. **Alternatively,** substitutes their value of y into $x = \pm\mu y$ to give $x =$ value or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value **Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded. **A1:** $(-3, 6)$ **cso**. **Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.** **Note:** $x = -3$ followed later in working by $y = 6$ is fine for A1. **Note:** $y = 6$ followed later in working by $x = -3$ is fine for A1. **Note:** $x = -3$, 3 followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$ **Note:** Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ { or even $(6, -3)$ }. **Note:** Candidates who set ***the numerator*** or ***the denominator*** of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0. **Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b). Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

Question 5

Question Number	Scheme	Marks
(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \frac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x° = \frac{1}{3}\tan 40°$ * cao	A1 *
		(4)
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$	M1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	A1
		(4)
		[8 marks]
Alt 1 (a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \frac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \frac{1}{3}\tan 40$	A1,A1
Alt 2 (a)	$2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$	
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \frac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \frac{1}{3}\tan 40$	A1,A1

Notes for Question

(a)

M1 Expand both expressions using $\cos(x+50) = \cos x \cos 50 - \sin x \sin 50$ and $\sin(x+40) = \sin x \cos 40 + \cos x \sin 40$. Condone a missing bracket on the lhs.

The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.

Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.

Below is an example of M1M1 with incorrect sign on left hand side

$2\cos x \cos 50 + 2\sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$

$\Rightarrow 2\cos 50 + 2\tan x \sin 50 = \tan x \cos 40 + \sin 40$

This is independent of the first mark.

A1 $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$

Accept for this mark $\tan x =$ awrt $0.28\ldots$ as long as M1M1 has been achieved.

A1* States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ leading to showing

$$\tan x = \frac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \frac{\sin 40}{3\cos 40} = \frac{1}{3}\tan 40$$

This is a given answer and all steps above must be shown. The line above is acceptable.

Do not allow from $\tan x =$ awrt $0.28\ldots$

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3}\tan 40$

A1 Solves to find one solution of θ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of θ. It must be a full method but can be implied by any correct answer.

Accept $\theta = \dfrac{180 + their\,\alpha}{2}, (or) \dfrac{360 + their\,\alpha}{2}, (or) \dfrac{540 + their\,\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.

Ignore any extra solutions outside the range.

Withhold this mark for extras inside the range.

Condone a different variable. Accept $x = 7.8, 97.8, 187.8, 277.8$

Answers fully given in radians, loses the first A mark.

Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85

Mixed units can only score the first M 1

Question 6

Question Number	Scheme		Marks
	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t \quad$ or $\quad \frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	
	At $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
		Correct value for $\frac{dy}{dx}$ of 1	A1 cao cso
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 cso isw
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2] 9

Notes for Question

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t, \quad \frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

	Notes for Question Continued	
(b)	**M1:** Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1:** Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form** $y = \mathrm{f}(x)$. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1:** Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.	
(c)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1:** Achieves an inclusive upper or lower limit, using acceptable notation. Eg: $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ **B1:** $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$ **Special Case: SC: B1B0** for either $0 < \mathrm{f}(x) < 2$ or $0 < \mathrm{f} < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$. **IMPORTANT: Note that:** Therefore candidates can use either y or f in place of $\mathrm{f}(x)$ **Examples:** $0 \leqslant x \leqslant 2$ is SC: B1B0 — $0 < x < 2$ is B0B0 $x \geqslant 0$ is B0B0 — $x \leqslant 2$ is B0B0 $\mathrm{f}(x) > 0$ is B0B0 — $\mathrm{f}(x) < 2$ is B0B0 $x > 0$ is B0B0 — $x < 2$ is B0B0 $0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0 — $0 < \mathrm{f}(x) \leqslant 2$ is B1B0 $0 \leqslant \mathrm{f}(x) < 2$ is B1B0. — $\mathrm{f}(x) \geqslant 0$ is B1B0 $\mathrm{f}(x) \leqslant 2$ is B1B0 — $\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$ $2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0 — $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0. $\lvert\mathrm{f}(x)\rvert \leqslant 2$ is B1B0 — $\lvert\mathrm{f}(x)\rvert \geqslant 2$ is B0B0 $1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0 — $1 < \mathrm{f}(x) < 2$ is B0B0 $0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0 — $0 < \mathrm{f}(x) < 4$ is B0B0 $0 \leqslant$ Range $\leqslant 2$ is B1B0 — Range is in between 0 and 2 is B1B0 $0 <$ Range < 2 is B0B0. — Range $\geqslant 0$ is B1B0 Range $\leqslant 2$ is B1B0 — Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0. $[0, 2]$ is B1B1 — $(0, 2)$ is SC B1B0	
Aliter (a) Way 2	$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{dy}{dx} = 1$	So B1, B1. So implied M1, A1.

Notes for Question Continued		
Aliter (a) Way 3	$y=\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=x$ — Correct differentiation of their Cartesian equation.	B1ft
	Finds $\frac{dy}{dx}=x$, using the correct Cartesian equation only.	B1
	At $t=\frac{\pi}{6},\ \frac{dy}{dx}=2\sin\left(\frac{\pi}{6}\right)$ — Finds the value of "x" when $t=\frac{\pi}{6}$ and substitutes this into their $\frac{dy}{dx}$	M1
	$=1$ — Correct value for $\frac{dy}{dx}$ of 1	A1
Aliter (b) Way 2	$y=1-\cos 2t=1-(2\cos^2 t-1)$	M1
	$y=2-2\cos^2 t \Rightarrow \cos^2 t=\frac{2-y}{2} \Rightarrow 1-\sin^2 t=\frac{2-y}{2}$	
	$1-\left(\frac{x}{2}\right)^2=\frac{2-y}{2}$	(Must be in the form $y=f(x)$).
	$y=2-2\left(1-\left(\frac{x}{2}\right)^2\right)$	A1
Aliter (b) Way 3	$x=2\sin t \Rightarrow t=\sin^{-1}\left(\frac{x}{2}\right)$	
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ — Rearranges to make t the subject and substitutes the result into y.	M1
	$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 4	$y=1-\cos 2t \Rightarrow \cos 2t=1-y \Rightarrow t=\frac{1}{2}\cos^{-1}(1-y)$	
	So, $x=\pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$ — Rearranges to make t the subject and substitutes the result into y.	M1
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ — $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 5	$\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2+c$ — $\frac{dy}{dx}=x \Rightarrow y=\frac{1}{2}x^2+c$	M1
	Eg: when eg: $t=0$ (nb: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$), $x=0,\ y=1-1=0 \Rightarrow c=0 \Rightarrow y=\frac{1}{2}x^2$ — Full method of finding $y=\frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$	A1
	Note: $\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2$, with no attempt to find c is M1A0.	

Question 7

Question Number	Scheme	Marks
(a)	$R = \sqrt{(7^2 + 24^2)} = 25$	B1
	$\tan\alpha = \frac{24}{7}, \quad \Rightarrow \quad \alpha = \text{awrt } 73.74°$	M1A1
		(3)
(b)	maximum value of $24\sin x + 7\cos x = 25$ so $V_{\min} = \frac{21}{25} = (0.84)$	M1A1
		(2)
(c)	Distance $AB = \frac{7}{\sin\theta}$, with $\theta = \alpha$	M1, B1
	So distance = 7.29m $= \frac{175}{24}$ m	A1
		(3)
(d)	$R\cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$	M1, A1
	$\theta - \alpha = 60 \Rightarrow \theta = .., \theta - \alpha = -60 \Rightarrow \theta = ..$	dM1, dM1
	$\theta = \text{awrt } 133.7, 13.7$	A1, A1
		(6)
		(14 marks)

Notes for Question

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001

M1 For $\tan\alpha = \pm\frac{24}{7}, \tan\alpha = \pm\frac{7}{24}$.

If the value of R is used only accept $\sin\alpha = \pm\frac{24}{R}, \cos\alpha = \pm\frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V = \frac{21}{their\ 'R'}$ NOT - R

A1 Obtains correct answer. $V = \frac{21}{25}$ Accept 0.84

Do not accept if you see incorrect working- ie from $\cos(\theta - \alpha) = -1$ or the minus just disappearing from a previous line.

Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate V by the quotient rule (or similar), set $V'=0$ to find θ and then sub this back into V to find its value.

Notes for Question Continued
(c) M1 Uses the trig equation $\sin\theta = \frac{7}{AB}$ with a numerical θ to find $AB = ...$ B1 Uses $\theta =$ their value of α in a trig calculation involving sin. ($\sin\alpha = \frac{AB}{7}$ is condoned) A1 Obtains answer $\frac{175}{24}$ or awrt 7.29 (d) M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \frac{21}{24\sin\theta + 7\cos\theta}$ to get an equation of the form $R\cos(\theta \pm \alpha) = \frac{21}{1.68}$ or $1.68R\cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \frac{21}{1.68R}$. Follow through on their R and α A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their α. It may be implied by later working. dM1 Obtains one value of θ **in the range** $0 < \theta < 150$ from inverse cos +their α It is dependent upon the first M being scored. dM1 Obtains second angle of θ **in the range** $0 < \theta < 150$ from inverse cos +their α It is dependent upon the first M being scored. A1 one correct answer awrt $\theta = 133.7$ *or* 13.7 1dp A1 both correct answers awrt $\theta = 133.7$ *and* 13.7 1dp. Extra solutions in the range loses the last A1. Answers in radians, lose the first time it occurs. Answers must be to 3dp For your info $\alpha = 1.287, \theta_1 = 2.334, \theta_2 = 0.240$

Question 8

Question Number	Scheme		Marks
(a)	$\{x = u^2 \Rightarrow\}\ \dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$		B1
	$\left\{\int \dfrac{1}{x(2\sqrt{x}-1)}\,dx\right\} = \int \dfrac{1}{u^2(2u-1)}\,2u\,du$		M1
	$= \int \dfrac{2}{u(2u-1)}\,du$		A1 * cso
			[3]
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$ $u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ $u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$	See notes	M1 A1
	So $\int \dfrac{2}{u(2u-1)}\,du = \int \dfrac{-2}{u} + \dfrac{4}{(2u-1)}\,du$	Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda \ln u$ or $\pm\mu \ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$	At least one term correctly followed through	A1 ft
		$-2\ln u + 2\ln(2u-1)$.	A1 cao
	So, $[-2\ln u + 2\ln(2u-1)]_1^3$		
	$= (-2\ln 3 + 2\ln(2(3)-1)) - (-2\ln 1 + 2\ln(2(1)-1))$	Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$		
	$= 2\ln\left(\dfrac{5}{3}\right)$	$2\ln\left(\dfrac{5}{3}\right)$	A1 cso cao
			[7] 10

Notes for Question

(a)	**B1:** $\dfrac{dx}{du} = 2u$ or $dx = 2u\,du$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$ or $du = \dfrac{dx}{2\sqrt{x}}$ **M1:** A full substitution producing an integral in u only (including the du) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "dx" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1). **A1*:** leading to the result printed on the question paper (including the du). (Integral sign is needed).
(b)	**M1:** Writing $\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)}$ or writing $\dfrac{1}{u(2u-1)} \equiv \dfrac{P}{u} + \dfrac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q). **A1:** Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign). **M1:** Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of $\pm\lambda \ln u$ or $\pm\mu \ln(2u-1)$ or $\pm\mu \ln\left(u - \frac{1}{2}\right)$ **A1ft:** At least one term correctly followed through from their A or from their B (or their P and their Q). **A1:** $-2\ln u + 2\ln(2u-1)$

Notes for Question Continued

(b) ctd	**M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the

correct way round.

Note: If a candidate just writes $(-2\ln 3 + 2\ln(2(3)-1))$ oe , this is ok for M1.

A1: $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a = 5, b = 3$).

Important note: Award M0A0M1A1A0 for a candidate who writes

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$$

AS EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.**

Important note: Award M0A0M0A0A0 for a candidate who writes down either

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1) \quad \textbf{or} \quad \int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Important note: Award M1A1M1A1A1 for a candidate who writes down

$$\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Note: In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of

M1A0 M1A1ftA0 M1A0.

Question 9

Question Number	Scheme	Marks
(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\quad \left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 (2)
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* (4)
(c)	$\frac{d^2y}{dx^2} = \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} \quad = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to (a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question

(a)

M1 Uses the chain rule to get $A\sec 3y \sec 3y \tan 3y = \left(A\sec^2 3y \tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y \sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

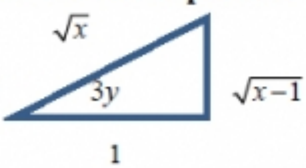
(b)

M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept 

$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$

Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question Continued

Example 1- Scores 0 marks in part (b)

$$\frac{dx}{dy}=6\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{dx}{dy}=2\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\left(\cancel{\frac{d^2y}{dx^2}}\right)=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \left(\cancel{\frac{d^2y}{dx^2}}\right)=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{d^2y}{dx^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and

v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\cancel{\frac{d^2y}{dx^2}} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{d^2y}{dx^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) **Using Quotient and Chain rules Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving

$u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$\left(\cancel{\frac{d^2y}{dx^2}}\right) = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$

A1 Correct un simplified expression $\cancel{\frac{d^2y}{dx^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{d^2y}{dx^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}=\frac{1}{(36x^3-36x^2)^{\frac{1}{2}}}=(36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Question 10

Question Number	Scheme	Marks
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120-\theta), \quad \theta \leqslant 100$	
(a)	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$ or $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta = \int \mathrm{d}t$	B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$ See notes	M1 A1; M1 A1
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$ See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$	
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$ $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$ $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100}$ ***or...*** $\lambda t = \ln 100 - \ln(120-\theta)$ $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$ $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta}$	dddM1
	$100\mathrm{e}^{-\lambda t} = 120-\theta$ or $(120-\theta)\mathrm{e}^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t}$ leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	A1 *
		[8]
(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100\mathrm{e}^{-0.01t}$	M1
	$\Rightarrow 100\mathrm{e}^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$ $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$ Uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$	
	$t = 160.94379\ldots = 161$ (s) (nearest second) awrt 161	A1
		[3] 11

	Notes for Question	
(a)	**B1:** Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. *Either*: **M1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$ *or* $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant. **A1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$ *or* $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$, **M1:** $\int \lambda\,\mathrm{d}t \to \lambda t$ *or* $\int 1\,\mathrm{d}t \to t$ **A1:** $\int \lambda\,\mathrm{d}t \to \lambda t + c$ or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation. **IMPORTANT:** $+c$ can be on either side of their equation for the 2nd A1 mark. **M1:** Substitutes $t=0$ AND $\theta=20$ in an integrated or changed equation containing c (or A or $\ln A$). **Note** that this mark can be implied by the correct value of c. {Note that $-\ln 100 = -4.60517...$}. **dddM1:** Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded. **A1*:** This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either: (1): $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100\mathrm{e}^{-\lambda t} = 120-\theta \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$ or (2): $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)\mathrm{e}^{\lambda t} = 100 \Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t} \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$ is required for A1. **Note:** $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).	
(b)	**M1:** Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working. **dM1:** Candidate uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to $t = ...$ **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b). **A1:** awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).	
Aliter (a) Way 2	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$	B1
	$-\ln(120-\theta) = \lambda t + c$ See notes	M1 A1; M1 A1
	$-\ln(120-\theta) = \lambda t + c$ $\ln(120-\theta) = -\lambda t + c$ $120-\theta = A\mathrm{e}^{-\lambda t}$ $\theta = 120 - A\mathrm{e}^{-\lambda t}$	
	$\{t=0, \theta=20 \Rightarrow\}$ $20 = 120 - A\mathrm{e}^{0}$	M1
	$A = 120-20 = 100$	
	So, $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	dddM1 A1 *
		[8]

Notes for Question Continued

(a)

B1M1A1M1A1: Mark as in the original scheme.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A.

dddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.

Note: This mark is dependent on all three previous method marks being awarded.

Note: $\ln(120-\theta) = -\lambda t + c$ leading to $120-\theta = e^{-\lambda t} + e^{c}$ or $120-\theta = e^{-\lambda t} + A$, would be dddM0.

A1*: Same as the original scheme.

Note: The jump from $\ln(120-\theta) = -\lambda t + c$ to $120-\theta = Ae^{-\lambda t}$ ***with no incorrect working*** is condoned in part (a).

Aliter (a) Way 3

$\displaystyle\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,d\theta = \int \lambda\,dt\right\}$		B1
$-\ln\lvert\theta-120\rvert = \lambda t + c$	*Modulus required for 1st A1.*	M1 A1
$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0)+c$	*Modulus not required here!*	M1 A1
$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$		M1

then either...

$-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$

$-\lambda t = \ln\left\lvert\dfrac{\theta-120}{100}\right\rvert$

or...

$\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$

$\lambda t = \ln\left\lvert\dfrac{100}{\theta-120}\right\rvert$

As $\theta \leqslant 100$

either: $-\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$, $e^{-\lambda t} = \dfrac{120-\theta}{100}$

or: $\lambda t = \ln\left(\dfrac{100}{120-\theta}\right)$, $e^{\lambda t} = \dfrac{100}{120-\theta}$

Understanding of modulus is required here! **dddM1**

either: $100e^{-\lambda t} = 120-\theta$

or: $(120-\theta)e^{\lambda t} = 100$, $\Rightarrow 120-\theta = 100e^{-\lambda t}$

leading to $\theta = 120 - 100e^{-\lambda t}$ **A1 ***

[8]

B1: Mark as in the original scheme.

M1: Mark as in the original scheme ignoring the modulus.

A1: $\displaystyle\int \frac{1}{120-\theta}\,d\theta \rightarrow -\ln\lvert\theta-120\rvert$. ***(The modulus is required here).***

M1A1: Mark as in the original scheme.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus.

dddM1: Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. **Note:** This mark is dependent on all three previous method marks being awarded.

A1: Mark as in the original scheme.

Notes for Question Continued		
Aliter (a) Way 4	***Use of an integrating factor***	
	$\frac{d\theta}{dt} = \lambda(120-\theta) \Rightarrow \frac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\frac{d}{dt}(e^{\lambda t}\theta) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t=0, \theta=20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1