Pure Mathematics 2 Practice Paper M15 MARK SCHEME

Question 1

	Scheme	Marks
1	Assume there exists a rational number $\frac{a}{b}$ such that $\frac{a}{b}$ is the greatest positive rational number.'	**B1**
	Consider a number $\frac{a}{b}+1$, which is greater than $\frac{a}{b}$	**M1**
	Simplifying gives $\frac{a}{b}+1 \equiv \frac{a}{b}+\frac{b}{b} \equiv \frac{a+b}{b}$ But $\frac{a+b}{b}$ is a rational number	**M1**
	This contradicts the assumption that there exists a greatest positive rational number therefore there is not a greatest positive rational number.	**B1**

Question 2

Question Number	Scheme	Marks
(a)	In triangle OCD **complete method** used to find angle COD so:	
	Either $\cos COD = \dfrac{8^2+8^2-7^2}{2\times 8\times 8}$ or uses $\angle COD = 2\times \arcsin\frac{3.5}{8}$ oe so $\angle COD =$	M1
	($\angle COD = 0.9056(331894)$) $= 0.906$ (3sf) * accept awrt 0.906	A1 * (2)
(b)	Uses $s = 8\theta$ for any θ in radians or $\dfrac{\theta}{360}\times 2\pi\times 8$ for any θ in degrees	M1
	$\theta = \dfrac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta (= awrt\ 2.24)$ and Perimeter $= 23 + (16\times\theta)$	M1
	accept awrt 40.9 (cm)	A1 (3)
(c)	Either Way 1: (Use of Area of two sectors + area of triangle)	
	Area of triangle $= \frac{1}{2}\times 8\times 8\times \sin 0.906$ (or 25.1781155 accept awrt 25.2)or $\frac{1}{2}\times 8\times 7\times \sin 1.118$ or $\frac{1}{2}\times 7\times h$ after h calculated from correct Pythagoras or trig.	M1
	Area of sector $= \frac{1}{2}8^2 \times "1.117979732"$ (or 35.77535142 accept awrt 35.8)	M1
	Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 (3)
	Or Way 2: (Use of area of semicircle – area of segment)	
	Area of semi-circle $= \frac{1}{2}\times\pi\times 8\times 8$ (or 100.5)	M1
	Area of segment $= \frac{1}{2}8^2 \times ("0.906" - \sin "0.906")$ (or 3.807)	M1
	So area required = awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 (3) [8]

Notes

(a) **M1**: Either use correctly quoted cosine rule – may quote as $7^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\alpha \Rightarrow \alpha = ...$
Or split isosceles triangle into two right angled triangles and use arcsin or longer methods using Pythagoras and arcos (i.e. $\pi - 2\times \arccos\frac{3.5}{8}$). **There are many ways of showing this result.**

Must conclude that $\angle COD =$

A1*: (NB this is a given answer) If any errors or over-approximation is seen this is A0. It needs correct work **leading to stated answer** of 0.906 or awrt 0.906 for A1. The cosine of COD is equal to 79/128 or awrt 0.617. Use of 0.62 (2sf) does not lead to printed answer. They may give 51.9 in degrees then convert to radians. This is fine.

The minimal solution $7^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\alpha \Rightarrow \alpha = ...0.906$ (with no errors seen) can have M1A1 but errors rearranging result in M1A0

(b) **M1**: Uses formula for arc length with $r = 8$ and any angle i.e. $s = 8\theta$ if working in rads or $s = \dfrac{\theta}{360}\times 2\pi\times 8$ in degrees

(If the formula is quoted with r the 8 may be implied by the value of their $r\theta$)

M1: Uses angles on straight line (or other geometry) to find angle BOC or AOD and uses Perimeter = 23 + arc lengths BC and AD (may make a slip – in calculation or miscopying)

A1: correct work leading to awrt 40.9 not 40.8 (do not need to see cm) This answer implies M1M1A1

(c) Way 1: **M1**: Mark is given for **correct** statement of area of triangle $\frac{1}{2}\times 8\times 8\times \sin 0.906$ (must use correct angle) or for correct answer (awrt 25.2) Accept alternative correct methods using Pythagoras and ½ base×height

M1: Mark is given for formula for area of sector $\frac{1}{2}8^2 \times "1.117979732"$ with $r = 8$ and their angle BOC or AOD or ($BOC + AOD$) **not** COD. May use $A = \dfrac{\theta}{360}\times\pi\times 8^2$ **if working in degrees**

A1: Correct work leading to awrt 96.7, 96.8 or 96.9 (This answer implies M1M1A1)

NB. Solution may combine the two sectors for part (b) and (c) and so might use $2\times\angle BOC$ rather than $\angle BOC$

Way 2: **M1**: Mark is given for **correct** statement of area of semicircle $\frac{1}{2}\times\pi\times 8\times 8$ or for correct answer 100.5

M1: Mark is given for formula for area of segment $\frac{1}{2}8^2 \times ("0.906" - \sin "0.906")$ with $r = 8$ or 3.81 **A1**: As in Way 1

Question 3

Question Number	Scheme	Marks
(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow f'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow f'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $f'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow f(x)$ is an increasing function as $f'(x)>0$,	M1
	$f'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

(a)

B1 For seeing $x^2-3kx+2k^2=(x-2k)(x-k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$ or

$$2-\frac{(x-5k)(x-k)}{(x-2k)(x-k)}=\frac{2(x-2k)(x-k)-(x-5k)(x-k)}{(x-2k)(x-k)} \qquad \left(=\frac{x^2-k^2}{x^2-3kx+2k^2}\right)$$

A1* Proceeds without any errors (including bracketing) to $=\dfrac{x+k}{(x-2k)}$

(b)

M1 Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$.

If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=x-2k$ with their u', v' and $\frac{vu'-uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $f'(x)=\frac{x-2k-x\pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y=(x+k)(x-2k)^{-1}$ If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=(x-2k)^{-1}$ with their u', v' and $vu'+uv'$

If it is neither stated nor implied only accept expressions of the form

$f'(x)=(x-2k)^{-1}\pm(x+k)(x-2k)^{-2}$

Alternatively writes $y=\frac{x+k}{x-2k}$ as $y=1+\frac{3k}{x-2k}$ and differentiates to $\frac{dy}{dx}=\frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $f'(x)$.

$f'(x)=\frac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$ by quotient rule

$f'(x)=(x-2k)^{-1}-(x+k)(x-2k)^{-2}$ by product rule

and $f'(x)=\frac{-3k}{(x-2k)^2}$ by the third method

A1 cao $f'(x)=\frac{-3k}{(x-2k)^2}$. Allow $f'(x)=\frac{-3k}{x^2-4kx+4k^2}$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $f'(x)=\frac{-Ck}{(x-2k)^2}$, look for f(x) is an increasing function as $f'(x)$ / gradient >0

Accept a version that states as $k<0\Rightarrow -Ck>0$ hence increasing

If in part (b) $f'(x)=\frac{(+)Ck}{(x-2k)^2}$, look for f(x) is an decreasing function as $f'(x)$ / gradient <0

Similarly accept a version that states as $k<0\Rightarrow (+)Ck<0$ hence decreasing

A1 Must have $f'(x)=\frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

Question 4

Question Number	Scheme		Marks
(a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$		
	$=2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$	See notes below!	
	$=2+\frac{5}{4}x;-\frac{25}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\left\{x=\frac{1}{10}\Rightarrow(4+5(0.1))^{\frac{1}{2}}=\sqrt{4.5}=\sqrt{\frac{9}{2}}=\frac{3}{\underline{\underline{\sqrt{2}}}}=\frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$		
	$=\frac{3}{2}\sqrt{2}$	$\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
			[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\underline{\underline{\sqrt{2}}}}=2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$	See notes	M1
	So, $\frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\underline{\underline{\sqrt{2}}}}=\frac{543}{256}$		
	yields, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	$\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ or $\frac{256}{181}$ etc.	A1 oe
			[2] 8

	Question 1 Notes	
(a)	B1	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and where $k\neq 1$.
	A1	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with consistent (kx).
	Note	(kx), $k\neq 1$, must be consistent (on the RHS, not necessarily on the LHS) in a candidate's expansion.

(a) ctd.	Note	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(5x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+...\right]$ because (kx) is not consistent.
	Note	**Incorrect bracketing:** $2\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x^2}{4}\right)+...\right]$ is B1M1A0 unless recovered.
	A1	$2+\frac{5}{4}x$ (simplified fractions) or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	SC	If a candidate *would otherwise score* 2nd A0, 3rd A0 then allow Special Case 2nd A1 for either SC: $2\left[1+\frac{5}{8}x; ...\right]$ or SC: $2\left[1+ ... -\frac{25}{128}x^2+...\right]$ or SC: $\lambda\left[1+\frac{5}{8}x-\frac{25}{128}x^2+...\right]$ or SC: $\left[\lambda+\frac{5\lambda}{8}x-\frac{25\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, OR SC: for $2+\frac{10}{8}x-\frac{50}{128}x^2+...$ (i.e. for not simplifying their correct coefficients.)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{5x}{4}\right)^2+...\right]$, where $k=-\frac{5}{4}$ and not $\frac{5}{4}$ and achieve $2-\frac{5}{4}x-\frac{25}{64}x^2+...$ will get B1M1A1A0A1
	Note	Ignore extra terms beyond the term in x^2.
	Note	You can ignore subsequent working following a correct answer.
(b)	B1	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e. (Ignore how $k=\frac{3}{2}$ is found.)
(c)	M1	Substitutes $x=\frac{1}{10}$ or 0.1 into their binomial expansion found in part (a) which must contain both an x term and an x^2 term (or even an x^3 term) and equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b), where k is a numerical value.
	Note	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}$ = awrt 2.121
	Note	M1 *can be implied* by $\frac{1}{k}\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	Note	M1 *cannot be implied* by $(k)\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	A1	$\frac{181}{128}$ or any equivalent fraction, eg: $\frac{362}{256}$ or $\frac{543}{384}$. Also allow $\frac{256}{181}$ or any equivalent fraction.
	Note	Also allow A1 for $p=181, q=128$ or $p=181\lambda, q=128\lambda$ or $p=256, q=181$ or $p=256\lambda, q=181\lambda$, where $\lambda\in\mathbb{Z}^+$
	Note	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c).
	Note	Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b).
	Note	Award M1 A1 for the correct answer from no working.

(a)	**Alternative methods for part (a)**	
	Alternative method 1: Candidates can apply an alternative form of the binomial expansion.	
	$\left\{(4+5x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\frac{1}{2}\right)(4)^{-\frac{1}{2}}(5x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(4)^{-\frac{3}{2}}(5x)^2$	
	B1	$(4)^{\frac{1}{2}}$ or 2
	M1	Any two of three (un-simplified) terms correct.
	A1	All three (un-simplified) terms correct.
	A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(5x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(5x)^2$ without further working is B0M0A0.

Alternative Method 2: Maclaurin Expansion $f(x) = (4+5x)^{\frac{1}{2}}$

$f''(x) = -\frac{25}{4}(4+5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1
$f'(x) = \frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	$\pm a(4+5x)^{-\frac{1}{2}};\ a \neq \pm 1$	M1
	$\frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	A1 oe
$\left\{\therefore f(0) = 2,\ f'(0) = \frac{5}{4} \text{ and } f''(0) = -\frac{25}{32}\right\}$		
So, $f(x) = 2 + \frac{5}{4}x; - \frac{25}{64}x^2 + \ldots$		A1; A1

Question 5

Question Number	Scheme	Marks
(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times -2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0 \Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0 \Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)e^{-2x} \Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e},\quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1
		(6)
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	(1)
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times \pm Ae^{-2x}+\left(Bx\pm Cx^2\right)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and uses **the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$

Condone issues in the signs of the last two terms for the method mark

Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times Ce^{2x}}{\left(e^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3 \Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$

Condone errors on coefficients and signs

A1 A correct (unsimplified form) of the answer
$g'(x)=\left(x^2-x^3\right)\times\text{-}2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$ by one use of the product rule
or $g'(x)=x^2\times\text{-}2e^{-2x}+2xe^{-2x}-x^3\times\text{-}2e^{-2x}-3x^2\times e^{-2x}$ using the first alternative
or $g'(x)=2x(1-x)e^{-2x}+x^2\times-1\times e^{-2x}+x^2(1-x)\times-2e^{-2x}$ using the product rule on 3 terms
or $g'(x)=\dfrac{e^{2x}\left(2x-3x^2\right)-\left(x^2-x^3\right)\times 2e^{2x}}{\left(e^{2x}\right)^2}$ using the quotient rule.

A1 Writes $g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$. You do not need to see $f(x)$ stated and award even if a correct $g'(x)$ is followed by an incorrect $f(x)$. If the f(x) is not simplified at this stage you need to see it simplified later for this to be awarded.

(b) Note: The last mark in e-pen has been changed from a 'B' to an A mark

M1 For setting their f(x) =0. The = 0 may be implied by subsequent working.
Allow even if the candidate has failed to reach a 3TC for f(x).
Allow for $f(x)\geqslant 0$ or $f(x)\leqslant 0$ as they can use this to pick out the relevant sections of the curve

M1 For solving their 3TC = 0 by ANY correct method.
Allow for division of x or factorising out the x followed by factorisation of 3TQ. Check first and last terms of the 3TQ. Allow for solutions from either $f(x)\geqslant 0$ or $f(x)\leqslant 0$
Allow solutions **from the cubic equation** just appearing from a Graphical Calculator

A1 $x=\dfrac{1}{2},2$. Correct answers from a correct $g'(x)$ would imply all 3 marks so far in (b)

dM1 Dependent upon both previous M's being scored. For substituting their **two** (non zero) values of x into g(x) to find both y values. Minimal evidence is required $x=..\Rightarrow y=..$ is OK.

A1 Accept decimal answers for this mark. $g\left(\dfrac{1}{2}\right)=\dfrac{1}{8e}$=awrt 0.046 AND $g(2)=-\dfrac{4}{e^4}$=awrt -0.073

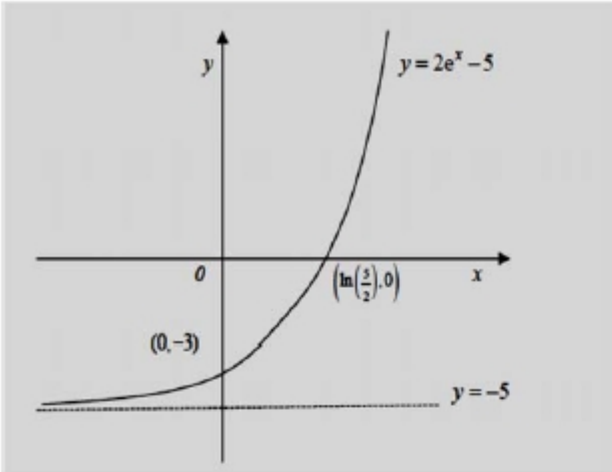
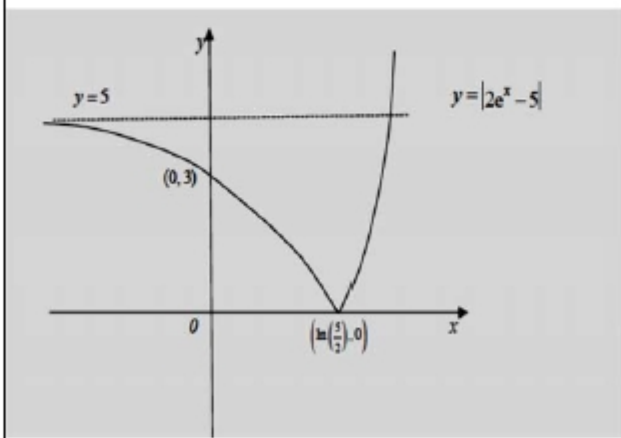
A1 CSO Allow $-\dfrac{4}{e^4}\leqslant \text{Range}\leqslant\dfrac{1}{8e}$, $-\dfrac{4}{e^4}\leqslant y\leqslant\dfrac{1}{8e}$, $\left[-\dfrac{4}{e^4},\dfrac{1}{8e}\right]$. Condone $y\geqslant-\dfrac{4}{e^4}$ $y\leqslant\dfrac{1}{8e}$

Note that the question states hence and part (a) must have been used for all marks. Some students will just write down the answers for the range from a graphical calculator.
Seeing just $-\dfrac{4}{e^4}\leqslant g(x)\leqslant\dfrac{1}{8e}$ or $-0.073\leqslant g(x)\leqslant 0.046$ special case 100000.
They know what a range is!

(c)

B1 If the candidate states 'NOT ONE TO ONE' then accept unless the explicitly link it to $g^{-1}(x)$.
So accept 'It is not a one to one function'. 'The function is not one to one' '$g(x)$ is not one to one'
If the candidate states 'IT IS MANY TO ONE' then accept unless the candidate explicitly links it to $g^{-1}(x)$. So accept 'It is a many to one function.' 'The function is many to one' '$g(x)$ is many to one'
If the candidate states 'IT IS ONE TO MANY' then accept unless the candidate explicitly links it to $g(x)$
Accept an explanation like " one value of x would map/ go to more than one value of y"
Incorrect statements scoring B0 would be $g^{-1}(x)$ is not one to one, $g^{-1}(x)$ is many to one and $g(x)$ is one to many.

Question 6

Question Number	Scheme		Marks
(ai)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
		$y=-5$	B1
			(3)
(aii)		Shape inc cusp	B1ft
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$	B1ft
		$y=5$	B1ft
			(3)
(b)	$x \geqslant \ln\left(\frac{5}{2}\right)$		B1 ft **(1)**
(c)	$2e^x-5=-2 \Rightarrow (x)=\ln\left(\frac{3}{2}\right)$		M1A1
	$(x)=\ln\left(\frac{7}{2}\right)$		B1
			(3)
			(10 marks)

(a)(i)

B1 For an exponential (growth) shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1 Intersections with the axes at $\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$.

Allow $\ln\left(\frac{5}{2}\right)$ and -3 being marked on the correct axes.

Condone $\left(0,\ln\left(\frac{5}{2}\right)\right)$ and $(-3,0)$ being marked on the x and y axes respectively.

Do not allow $\left(\ln\left(\frac{5}{2}\right),0\right)$ appearing as awrt (0.92, 0) for this mark unless seen elsewhere. Allow if seen in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then **the ones on the curve take precedence.**

B1 Equation of the asymptote given as $y=-5$. Note that the curve must appear to have an asymptote at $y=-5$, not necessarily drawn. It is not enough to have -5 marked on the axis or indeed $x=-5$. An extra asymptote **with an equation** gets B0

(a)(ii)

B1ft For **either** the correct shape **or** a reflection of their curve from (a)(i) in the x- axis. For this to be scored it must have appeared both above and below the x - axis. The shape must be correct including the cusp. The curve to the lhs of the cusp must appear to have the correct curvature

B1ft Score for both intersections or follow through on both the intersections given in part (a)(i), including decimals, as long as the curve appeared both above and below the x - axis. See part (a) for acceptable forms

B1ft Score for an asymptote of $y=5$ or follow through on an asymptote of $y=-C$ from part (a)(i). Note that the curve must appear to have an asymptote at $y=C$ but do not penalise if the first mark in (a)(ii) has been withheld for incorrect curvature on the lhs.

(b)

B1ft Score for $x \geqslant \ln\left(\frac{5}{2}\right)$, $x \geqslant$ awrt 0.92 or follow through on the x intersection in part (a)

(c)

M1 Accept $2e^x-5=-2$ or $-2e^x+5=2 \Rightarrow x=..\ln(..)$

Allow squaring so $\left(2e^x-5\right)^2=4 \Rightarrow e^x=..$ and $.. \Rightarrow x=\ln(..), \ln(..)$

A1 $x=\ln\left(\frac{3}{2}\right)$ or exact equivalents such as $x=\ln 1.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 0.405

B1 $x=\ln\left(\frac{7}{2}\right)$ or exact equivalents such as $x=\ln 3.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 1.25

If both answers are given in decimals and there is no working $x=$ awrt 1.25, 0.405 award SC 100

Question 7

Question Number	Scheme	Marks
(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \dfrac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\dfrac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \dfrac{dx}{dy}=24\pi \quad (=75.4)$ / $\dfrac{dy}{dx}=\dfrac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$ $\Rightarrow 0.5x^{-0.5}\dfrac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=\left(16y^2-8y\sin 2y+\sin^2 2y\right)$ $\Rightarrow 1=32y\dfrac{dy}{dx}-8\sin 2y\dfrac{dy}{dx}-16y\cos 2y\dfrac{dy}{dx}+4\sin 2y\cos 2y\dfrac{dy}{dx}$ Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	M1A1

(a)

B1 $p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form

$A(4y - \sin 2y)(B \pm C\cos 2y)$, $A, B, C \neq 0$ on the right hand side

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x = (16y^2 - 8y\sin 2y + \sin^2 2y) \Rightarrow \frac{dx}{dy} = Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y$ $P,Q,R,S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4 - Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$ or $\frac{dy}{dx} = \frac{1}{2(4y - \sin 2y)(4 - 2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.

In the alternative $\frac{dx}{dy} = 32y - 8\sin 2y - 16y\cos 2y + 4\sin 2y\cos 2y$

M1 Sub $y = \frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = ...$

It is not dependent upon the previous M1 but it must be a changed $x = (4y - \sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } \left({}^{dx}\!/\!_{dy}\right)}(x - \text{their } 4\pi^2)$

Allow for $\left(y - \frac{\pi}{2}\right) \times \text{their numerical } \left({}^{dx}\!/\!_{dy}\right) = (x - \text{their } 4\pi^2)$

Even allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } \left({}^{dx}\!/\!_{dy}\right)}(x - p)$

It is possible to score this by stating the equation $y = \frac{1}{24\pi}x + c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y = mx + c$ and stating the value of 'c'

Or setting $x = 0$ in their $y - \frac{\pi}{2} = \frac{1}{24\pi}(x - 4\pi^2)$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2} - y}{4\pi^2} = \frac{1}{24\pi} \Rightarrow y = ..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y = \frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

Question 8

Question Number	Scheme		Marks
	$y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$		
(a)	$\left\{y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$		
	$e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{\int xe^{\frac{1}{2}x}\,dx\right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$	$\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\},\ \alpha > 0, \beta > 0$	M1
		$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$, with or without dx	A1 (M1 on ePEN)
	$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\ \{+c\}$	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,dx\right\} = 2x^2$	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - xe^{\frac{1}{2}x})\,dx\right\} = \left[2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}\right)$	See notes	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
			[3] 8

		Question Notes
(a)	M1	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	NOTE	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha > 0, \beta > 0$. (must be in this form) with or without dx
	A1	$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$ or equivalent, with or without dx. **Can be un-simplified.**
	A1	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ or equivalent with or without $+c$. **Can be un-simplified.**
	Note	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	SPECIAL CASE: A candidate who uses $u = x$, $\dfrac{dv}{dx} = e^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

(c)	B1	$4x \to 2x^2$ or $\frac{4x^2}{2}$ oe
	M1	Complete method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	5.19378… without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	5.19378… following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.
	Note	5.19378… from no working is M0A0.

Question 9

Question Number	Scheme		Marks
	Note: You can mark parts (a) and (b) together.		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2}\ = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 o.e.
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b"$, $g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ *or* $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ *or* $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ *or* $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3},\ \ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
			[3] 6

Question Number	Scheme		Marks
(b)	**Alternative Method 1 of Equating Coefficients**		
	$y=\dfrac{x^2+ax+b}{x-3} \Rightarrow y(x-3)=x^2+ax+b$		
	$y(x-3)=(4t+3)^2+2(4t+3)-5 = 16t^2+32t+10$		
	$x^2+ax+b=(4t+3)^2+a(4t+3)+b$		
	$(4t+3)^2+a(4t+3)+b = 16t^2+32t+10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $24+4a=32 \Rightarrow a=2$ constant: $9+3a+b=10 \Rightarrow b=-5$	Equates their coefficients in t and finds both $a=\dots$ and $b=\dots$	dM1
		$a=2$ and $b=-5$	A1
			[3]
(b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t=\dfrac{x-3}{4} \Rightarrow\right\}\ y=4\left(\dfrac{x-3}{4}\right)+8+\dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y=x-3+8+\dfrac{10}{x-3} \Rightarrow y=x+5+\dfrac{10}{(x-3)}$ $\underline{\underline{y(x-3)}}=(x+5)(x-3)+10 \Rightarrow x^2+ax+b=\underline{\underline{(x+5)(x-3)+10}}$		dM1
	$\Rightarrow y=\dfrac{x^2+2x-5}{x-3}$ or equating coefficients to give $a=2$ and $b=-5$	Correct algebra leading to $y=\dfrac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$	A1 cso
			[3]

		Question Notes
(a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	M1	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \overline{\overline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\left(\frac{10}{x-3}\right)}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \overline{\overline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\ldots + 8(x-3) + \ldots$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\ldots + 5(x-3) + \ldots$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Question 10

Question Number	Scheme		Marks
(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x\ ,\ x = 1 + 2\sin\theta$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int\sqrt{(3-x)(x+1)}\,\mathrm{d}x \text{ or } \int\sqrt{(3+2x-x^2)}\,\mathrm{d}x\right\}$		
	$= \int\sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda\mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int\sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$ $= \int\sqrt{(4-4\sin^2\theta)}\ 2\cos\theta\,\{\mathrm{d}\theta\}$		
	$= \int\sqrt{(4-4(1-\cos^2\theta))}\ 2\cos\theta\,\{\mathrm{d}\theta\}$ or $\int\sqrt{4\cos^2\theta}\ 2\cos\theta\,\{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1-\sin^2\theta$ see notes	M1
	$= 4\int\cos^2\theta\,\mathrm{d}\theta,\ \{k=4\}$	$4\int\cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1+2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1+2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$	See notes	B1
			[5]
(b)	$\left\{k\int\cos^2\theta\,\{\mathrm{d}\theta\}\right\} = \{k\}\int\left(\frac{1+\cos2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin2\theta,\ \alpha\neq0, \beta\neq0$ or $k(\pm\alpha\theta\pm\beta\sin2\theta)$	M1 (A1 on ePEN)
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^2\theta\mathrm{d}\theta = \left[2\theta + \sin2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 cao cso
			[3] 8

		Question Notes
(a)	**B1**	$\dfrac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta\,d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged } \dfrac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\,dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\,d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}\,dx$ is equal to $4\int \cos^2\theta\,d\theta$ or $\int 4\cos^2\theta\,d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\dfrac{1}{2}$ which then leads to $\theta = -\dfrac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \dfrac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\dfrac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\dfrac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\dfrac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\dfrac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\dfrac{\pi}{6};\ x = 3,\ \theta = \dfrac{\pi}{2}$
(b)	**NOTE**	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\dfrac{1 + \cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A *correct solution in part (b)* leading to a "two term" exact answer. Eg: $\dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$ or $\dfrac{8\pi}{6} + \dfrac{\sqrt{3}}{2}$ or $\dfrac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Question 11

Question Number	Scheme	Marks
(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1 **(4)**
		(9 marks)

(a)

B1 A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **OR** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A\tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1 For getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1-2\sin^2 A$.

Alternatively for getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A/\cos A}{1-\sin^2 A/\cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ **and** factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.
Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.

(b)
M1 For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$
Condone $\tan 2\theta = k$ for this mark only

A1 $\tan\theta = -\frac{1}{3}$

dM1 Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.

A1 $\theta = \text{awrt } 2.820, 5.961$ with no extra solutions within the range. Condone 2.82 for 2.820.

You may condone different/ mixed variables in part (b)

There are some long winded methods. Eg. M1, dM1 applied as in main scheme

$$\Rightarrow (2\cos\theta + 2\sin\theta)^2 = (\cos\theta - \sin\theta)^2 \Rightarrow 4 + 4\sin 2\theta = 1 - \sin 2\theta$$

$$\Rightarrow \sin 2\theta = -\frac{3}{5} \text{ is M1 (for } \sin 2\theta = k\text{) A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ for dM1 (for } \theta = \frac{\arcsin k}{2}\text{) A1}$$

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\cos(\theta - 1.25) = 0 \text{ M1 for..} \cos(\theta - \alpha) = 0, \alpha = \arctan\left(\pm\frac{3}{1} \text{ or } \pm\frac{1}{3}\right) \text{ A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\sin(\theta + 0.32) = 0 \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{10} \Rightarrow \sin\theta = (\pm)\sqrt{\frac{1}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \cos^2\theta = \frac{9}{10} \Rightarrow \cos\theta = (\pm)\sqrt{\frac{9}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

Question Number	Scheme	Marks
Alt I **From RHS**	$\frac{\cos A+\sin A}{\cos A-\sin A} = \frac{\cos A+\sin A}{\cos A-\sin A}\times\frac{\cos A+\sin A}{\cos A+\sin A}$ $=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\frac{1+\sin 2A}{\cos 2A}$ $=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$ $=\sec 2A+\tan 2A$	(Pythagoras) **M1** (Double Angle) **M1** (Single Fraction) **M1** **B1**(Identity), **A1***
Alt II **Both sides**	Assume true $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\times(\cos A-\sin A)\Rightarrow\frac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ **True**	B1 (identity) M1 (single fraction) M1(double angles) M1(Pythagoras)A1*
Alt 111 **Very difficult**	$\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\tan 2A$ $=\frac{1}{\cos 2A}+\frac{2\tan A}{1-\tan^2 A}$ $=\frac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ $=\frac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ $=\frac{1-\frac{\sin^2 A}{\cos^2 A}+2\frac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\frac{\sin^2 A}{\cos^2 A}\right)}$ $\times\cos^2 A=\frac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ $=\frac{\cancel{(\cos^2 A-\sin^2 A)}(1+2\sin A\cos A)}{\cancel{(\cos^2 A-\sin^2 A)}(\cos^2 A-\sin^2 A)}$ Final two marks as in main scheme	(Identity) **B1** (Single fraction) **M1** (Double Angle and in just sin and cos) **M1** **M1A1***

Question 12

Question Number	Scheme		Marks
(a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. cao, aef	A1
			[3]
(b)	$\dfrac{dP}{dt} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,dP = \int \cos 2t\,dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t \quad (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c \quad \{\Rightarrow\ c = -\ln 3 \text{ or } \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right) \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467\ldots$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] 13

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
(b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \ \ (+c)$	As before for…	B1M1A1
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	3[rd] M1
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	4[th] dM1
	$\{t = 0, P = 3 \Rightarrow\}\ \ 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2[nd] M1 mark on ePEN).**	2[nd] M1
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso

	Question Notes	
(a)	M1	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer *cannot* be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2,\ -2A = 2 \Rightarrow A = -1,\ B = 1$

(b)	B1	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	Eg: $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ o.e. are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2)\pm\mu\ln P,\ \lambda\neq 0,\ \mu\neq 0$. Also allow $\pm\lambda\ln(M(P-2))\pm\mu\ln NP$; M,N can be 1.
	Note	Condone $2\ln(P-2)+2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1	Correct result of $\ln(P-2)-\ln P=\frac{1}{2}\sin 2t$ or $2\ln(P-2)-2\ln P=\sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t=0$ and $P=3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta)\pm\mu\ln P=\pm K\sin\delta t+c,\ \lambda,\mu,\beta,K,\delta\neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right)=\frac{1}{2}\sin 2t+c$ followed by $\frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}+\mathrm{e}^{c}$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right)=\frac{1}{2}\sin 2t+c \to \frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t+c} \to \frac{(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}+\mathrm{e}^{c}$ is final M1M0A0
		<u>4th M1 for making *P* the subject</u> **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** (1) M1 for $\frac{3(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3(P-2)=P\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3P-6=P\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow P(3-\mathrm{e}^{\frac{1}{2}\sin 2t})=6$ $\Rightarrow P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ (2) M1 for $\frac{3(P-2)}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3-\frac{6}{P}=\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow 3-\mathrm{e}^{\frac{1}{2}\sin 2t}=\frac{6}{P}\Rightarrow\Rightarrow P=\frac{6}{(3-\mathrm{e}^{\frac{1}{2}\sin 2t})}$ (3) M1 for $\left\{\ln(P-2)+\ln P=\frac{1}{2}\sin 2t+\ln 3\Rightarrow\right\}P(P-2)=3\mathrm{e}^{\frac{1}{2}\sin 2t}\Rightarrow P^2-2P=3\mathrm{e}^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2-1=3\mathrm{e}^{\frac{1}{2}\sin 2t}$ leading to $P=$..
(c)	M1 M1 A1	States $P=4$ or applies $P=4$ Obtains $\pm\lambda\sin 2t=\ln k$ or $\pm\lambda\sin t=\ln k$, where λ and k are numerical values and λ can be 1 anything that rounds to 0.473. *(Do not apply isw here)*
	Note	*Do not apply ignore subsequent working for A1.* (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P=4000$</u>: Without the mention of $P=4$, $\frac{1}{2}\sin 2t=\ln 2.9985$ or $\sin 2t=2\ln 2.9985$ or $\sin 2t=2.1912$... will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t=$ awrt 27.1 will usually imply M1M1A0