Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M16

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225 **(1)**

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday. **(2)**

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday. **(3)**

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

(d) Show that $n^2 + 7n = 25 \times 18$ **(3)**

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time. **(2)**

(Total for question = 11 marks)

Question 2

$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B. **(5)**

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$ **(5)**

(Total for question = 10 marks)

Question 3

$$y = \frac{4x}{x^2 + 5}$$

(a) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form. (4)

(b) Hence find the set of values of *x* for which $\frac{dy}{dx} < 0$ (3)

(Total for question = 7 marks)

Question 4

(a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) (6)

(Total for question = 10 marks)

Question 5

The curve *C* has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*. (5)

The point *P* with coordinates $\left(3, \frac{1}{2}\right)$ lies on *C*.

The normal to *C* at *P* meets the *x*-axis at the point *A*.

(b) Find the *x* coordinate of *A*, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$,

where *a*, *b*, *c* and *d* are integers to be determined. (4)

(Total for question = 9 marks)

Question 6

(i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places. (5)

(ii) Given $x = \sin^2 2y, \quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined. (5)

(Total for question = 10 marks)

Question 7

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + a)$, where R and a are constants, $R > 0$ and $0 < a$ 90° Give the exact value of R and give the value of a to 2 decimal places. (3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place. (5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place. (2)

(Total for question = 10 marks)

Question 8

(a) For $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x \qquad -1 \leq x \leq 1$$ (2)

(b) Find the exact value of x for which

$$3g(x + 1) + \pi = 0$$ (3)

(Total for question = 5 marks)

Question 9

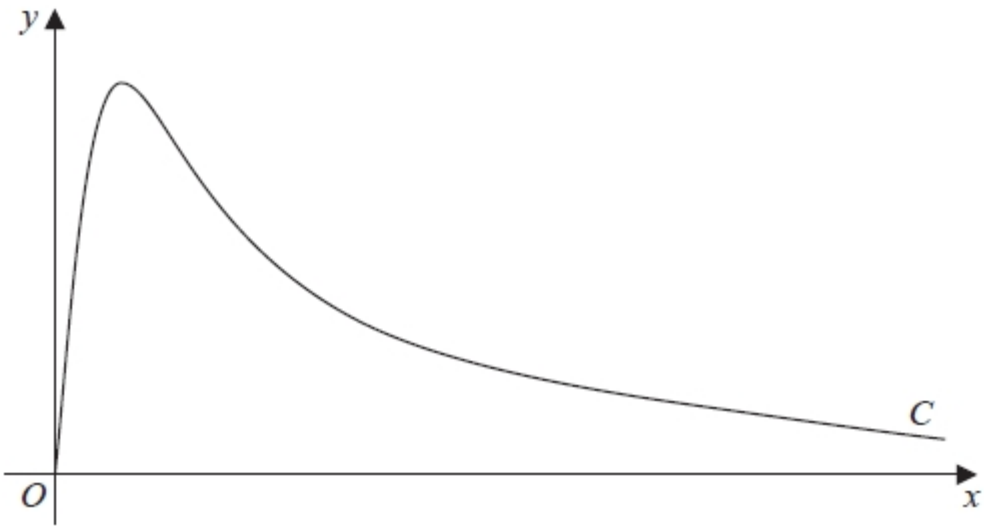

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd. (4)

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q. (2)

(Total for question = 6 marks)

Question 10

(i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y+2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta\,d\theta$$

where λ is a constant to be determined. (5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants. (4)

(Total for question = 15 marks)

Question 11

The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geqslant 0$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form. (4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.

Give your answer to the nearest minute. (3)

(Total for question = 7 marks)

TOTAL FOR PAPER IS 100 MARKS