## Finding Areas using Integration - Edexcel Past Exam Questions MARK SCHEME

### Question 1: June 05 Q5

| Question Number | Scheme | Marks |
|---|---|---|
| | $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x}\, dx$ Attempting parts in the right direction | M1 A1 |
| | $= \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}$ | A1 |
| | $\left[\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}e^2$ | M1 A1 |
| | | (5) |

### Question 2: June 06 Q3

| Question Number | Scheme | Marks |
|---|---|---|
| | Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right) dx$ | |
| | $= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$ Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits. | M1 |
| | $= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$ $-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ | A1 oe. |
| | $= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$ $\underline{12}$ | A1 cao |
| | | [3] |
| | (Answer of 12 with no working scores M0A0A0.) | |

**Question 3: June 06 Q6**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $\left\{\begin{array}{l} u = \ln x \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 \quad \Rightarrow \quad v = \frac{x^2}{2} - x \end{array}\right\}$ | Use of 'integration by parts' formula in the correct direction | M1 |
| | $I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$ | Correct expression | A1 |
| | $= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int \left(\frac{x}{2} - 1\right) dx}$ | An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ... | |
| | $= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)} \quad (+c)$ | integrate; <br> correct integration | M1; <br> A1 |
| | $\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$ | | |
| | $= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$ | Substitutes limits of 3 and 1 and subtracts. | ddM1 |
| | $= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG** | $\frac{3}{2}\ln 3$ | A1 cso |
| | | | **[6]** |

**Question 4: Jan 07 Q8**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (a) | $t=(3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx}=\frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$ | $A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx}=A$ | M1 |
| | ... or $t^2=3x+1 \Rightarrow \underline{2t\frac{dt}{dx}=3}$ | $\underline{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx}=3}$ | A1 |
| | so $\frac{dt}{dx}=\frac{3}{2.(3x+1)^{\frac{1}{2}}}=\frac{3}{2t} \Rightarrow \frac{dx}{dt}=\frac{2t}{3}$ | Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t ... | |
| | $\therefore I=\int e^{\sqrt{(3x+1)}}\,dx = \int e^t\frac{dx}{dt}.dt = \int e^t.\frac{2t}{3}.dt$ | ... and moves on to substitute this into I to convert an integral wrt x to an integral wrt t. | dM1 |
| | $\therefore I=\underline{\int \frac{2}{3}te^t\,dt}$ | $\underline{\int \frac{2}{3}te^t}$ | A1 |
| | change limits:<br>when x = 0, t = 1 & when x = 5, t = 4 | changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$ | B1 |
| | Hence $I=\int_1^4 \frac{2}{3}te^t\,dt$ ; where $a=1, b=4, k=\frac{2}{3}$ | | **[5]** |
| (b) | $\left\{\begin{matrix} u=t & \Rightarrow & \frac{du}{dt}=1 \\ \frac{dv}{dt}=e^t & \Rightarrow & v=e^t \end{matrix}\right\}$ | Let k be any constant for the first three marks of this part. | |
| | $k\int te^t\,dt = k\left(te^t-\int e^t.1\,dt\right)$ | Use of 'integration by parts' formula in the correct direction. | M1 |
| | | Correct expression with a constant factor k. | A1 |

| | | | |
|---|---|---|---|
| | $= k\left(\underline{te^t - e^t}\right) + c$ | <u>Correct integration</u> with/without a constant factor k | A1 |
| | $\therefore \int_1^4 \frac{2}{3}te^t\,dt = \frac{2}{3}\left\{\left(4e^4 - e^4\right) - \left(e^1 - e^1\right)\right\}$ | Substitutes their changed limits into the integrand and subtracts oe. | dM1 **oe** |
| | $= \frac{2}{3}(3e^4) = \underline{2e^4} = 109.1963...$ | either $2e^4$ or awrt 109.2 | A1 |
| | | | **[5]** |
| | | | **15 marks** |

**Question 5: Jan 08 Q7**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (a) | $\left[x = \ln(t+2),\ y = \dfrac{1}{t+1}\right],\quad \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{t+2}$ | Must state $\dfrac{dx}{dt} = \dfrac{1}{t+2}$ | B1 |
| | $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \dfrac{1}{t+1} dx; = \int_0^2 \left(\dfrac{1}{t+1}\right)\left(\dfrac{1}{t+2}\right) dt$ | Area $= \int \dfrac{1}{t+1} dx$. Ignore limits. | M1; |
| | | $\int \left(\dfrac{1}{t+1}\right) \times \left(\dfrac{1}{t+2}\right) dt$. Ignore limits. | A1 **AG** |
| | Changing limits, when:<br>$x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$<br>$x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$ | changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$ | B1 |
| | Hence, $\text{Area}(R) = \int_0^2 \dfrac{1}{(t+1)(t+2)} dt$ | | **[4]** |
| (b) | $\left(\dfrac{1}{(t+1)(t+2)}\right) = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ | $\dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ with $A$ and $B$ found | M1 |
| | $1 = A(t+2) + B(t+1)$ | | |
| | Let $t = -1$, $1 = A(1) \Rightarrow \underline{A = 1}$<br>Let $t = -2$, $1 = B(-1) \Rightarrow \underline{B = -1}$ | Finds both $A$ and $B$ correctly. Can be implied. (See note below) | A1 |
| | $\int_0^2 \dfrac{1}{(t+1)(t+2)} dt = \int_0^2 \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)} dt$ | | |
| | $= [\ln(t+1) - \ln(t+2)]_0^2$ | Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ | dM1 |
| | | Both ln terms correctly ft. | A1 $\sqrt{}$ |
| | $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ | Substitutes *both* limits of 2 and 0 and subtracts the correct way round. | ddM1 |
| | $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$ | $\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1) | A1 aef isw |
| | | | **[6]** |

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1 in (b).

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $x=\ln(t+2)$, $\quad y=\frac{1}{t+1}$ | | |
| (c) | $e^x=t+2 \Rightarrow t=e^x-2$ | Attempt to make $t=$... the subject<br>giving $t=e^x-2$ | M1<br>A1 |
| | $y=\frac{1}{e^x-2+1} \Rightarrow y=\frac{1}{e^x-1}$ | Eliminates $t$ by substituting in $y$<br>giving $y=\frac{1}{e^x-1}$ | dM1<br>A1 |
| | | | [4] |
| ***Aliter*** (c) **Way 2** | $t+1=\frac{1}{y} \Rightarrow t=\frac{1}{y}-1$ or $t=\frac{1-y}{y}$<br>$y(t+1)=1 \Rightarrow yt+y=1 \Rightarrow yt=1-y \Rightarrow t=\frac{1-y}{y}$ | Attempt to make $t=$... the subject<br>Giving either $t=\frac{1}{y}-1$ or $t=\frac{1-y}{y}$ | M1<br>A1 |
| | $x=\ln\left(\frac{1}{y}-1+2\right)$ or $x=\ln\left(\frac{1-y}{y}+2\right)$ | Eliminates $t$ by substituting in $x$ | dM1 |
| | $x=\ln\left(\frac{1}{y}+1\right)$ | | |
| | $e^x=\frac{1}{y}+1$ | | |
| | $e^x-1=\frac{1}{y}$ | | |
| | $y=\frac{1}{e^x-1}$ | giving $y=\frac{1}{e^x-1}$ | A1 |
| | | | [4] |
| (d) | Domain : $\underline{x>0}$ | $\underline{x>0}$ or just $>0$ | B1 |
| | | | [1] |
| | | | **15 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| *Aliter* (c) **Way 3** | $e^x = t + 2 \Rightarrow t + 1 = e^x - 1$ | Attempt to make $t+1 = \ldots$ the subject | M1 |
| | | giving $t + 1 = e^x - 1$ | A1 |
| | $y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$ | Eliminates $t$ by substituting in $y$ | dM1 |
| | | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |
| *Aliter* (c) **Way 4** | $t+1 = \dfrac{1}{y} \Rightarrow t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$ | Attempt to make $t+2 = \ldots$ the subject | M1 |
| | | Either $t+2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$ | A1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$ | Eliminates $t$ by substituting in $x$ | dM1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ | | |
| | $e^x = \dfrac{1}{y} + 1 \Rightarrow e^x - 1 = \dfrac{1}{y}$ | | |
| | $y = \dfrac{1}{e^x - 1}$ | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |

**Question 6: Jan 09 Q2**

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| (a) | $\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$ | | | |
| | $= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$ | *Integrating* $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. | M1 | |
| | | Correct integration. Ignore limits. | A1 | |
| | $= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$ | | | |
| | $= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ | Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round. | M1 | |
| | $= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$ | $\underline{3}$ | $\underline{\text{A1}}$ | |
| | (Answer of 3 with no working scores M0A0M0A0.) | | | (4) |
| (b) | $\text{Volume} = \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 dx}$ | Use of $V = \underline{\pi \int y^2}\,dx$. Can be implied. Ignore limits and $dx$. | B1 | |
| | $= (\pi)\int_0^2 \frac{9}{1+4x}\,dx$ | | | |
| | $= (\pi)\left[\frac{9}{4}\ln\|1+4x\|\right]_0^2$ | $\pm k\ln\|1+4x\|$ | M1 | |
| | | $\frac{9}{4}\ln\|1+4x\|$ | A1 | |
| | $= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$<br>Note that $\ln 1$ can be implied as equal to 0. | Substitutes limits of 2 and 0 and subtracts the correct way round. | dM1 | |
| | So Volume $= \underline{\frac{9}{4}\pi\ln 9}$<br>Note the answer must be a one term exact value. Note, also you can ignore subsequent working here. | $\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$<br>Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0. | A1 oe isw | (5) |
| | | | | [9] |

**Question 7: June 09 Q2**

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| Q (a) | 1.14805 | awrt 1.14805 | B1 | (1) |
| (b) | $A \approx \frac{1}{2} \times \frac{3\pi}{8}( \ldots )$ | | B1 | |
| | $= \ldots \left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$ | 0 can be implied | M1 | |
| | $= \frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$ | ft their (a) | A1ft | |
| | $= \frac{3\pi}{16} \times 15.08202 \ldots \quad = 8.884$ | cao | A1 | (4) |
| (c) | $\int 3\cos\left(\frac{x}{3}\right) \mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ | | M1 A1 | |
| | $= 9\sin\left(\frac{x}{3}\right)$ | | | |
| | $A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$ | cao | A1 | (3) |
| | | | | **[8]** |

**Question 8: Jan 10 Q2**

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| | (a) 1.386, 2.291 | awrt 1.386, 2.291 | B1 B1 | (2) |
| | (b) $A \approx \frac{1}{2} \times 0.5( \ldots )$ | | B1 | |
| | $= \ldots \left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$ | | M1 | |
| | $= 0.25\left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$ | ft their (a) | A1ft | |
| | $= 0.25 \times 29.477 \ldots \approx 7.37$ | cao | A1 | (4) |
| | (c)(i) $\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$ | | M1 A1 | |
| | $= \frac{x^2}{2}\ln x - \int \frac{x}{2} dx$ | | | |
| | $= \frac{x^2}{2}\ln x - \frac{x^2}{4} \ (+C)$ | | M1 A1 | |
| | (ii) $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4 = (8\ln 4 - 4) - \left(-\frac{1}{4}\right)$ | | M1 | |
| | $= 8\ln 4 - \frac{15}{4}$ | | | |
| | $= 8(2\ln 2) - \frac{15}{4}$ | $\ln 4 = 2\ln 2$ seen or implied | M1 | |
| | $= \frac{1}{4}(64\ln 2 - 15)$ | $a = 64, b = -15$ | A1 | (7) |
| | | | | [13] |

**Question 9: Jan 11 Q7**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $x=3 \Rightarrow y=0.1847$ awrt | B1 |
| | $x=5 \Rightarrow y=0.1667$ awrt or $\frac{1}{6}$ | B1 |
| | | (2) |
| (b) | $I \approx \frac{1}{\underline{\underline{2}}}\left[0.2+0.1667+2(0.1847+0.1745)\right]$ | $\underline{\underline{\text{B1}}}$ M1 A1ft |
| | $\approx 0.543$ 0.542 or 0.543 | A1 (4) |
| (c) | $\frac{dx}{du}=2(u-4)$ | B1 |
| | $\int \frac{1}{4+\sqrt{(x-1)}}dx = \int \frac{1}{u}\times 2(u-4)du$ | M1 |
| | $=\int\left(2-\frac{8}{u}\right)du$ | A1 |
| | $=2u-8\ln u$ | M1 A1 |
| | $x=2 \Rightarrow u=5,\ x=5 \Rightarrow u=6$ | B1 |
| | $\left[2u-8\ln u\right]_5^6=(12-8\ln 6)-(10-8\ln 5)$ | M1 |
| | $=2+8\ln\left(\frac{5}{6}\right)$ | A1 |
| | | (8) |
| | | [14] |

**Question 10: June 11 Q4**

| Question Number | Scheme | Marks |
|---|---|---|
| | (a) 0.0333, 1.3596 awrt 0.0333, 1.3596 | B1 B1 **(2)** |
| | (b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ \dots]$ | B1 |
| | $\approx \ \dots \left[0+2(0.0333+0.3240+1.3596)+3.9210\right]$ | M1 |
| | $\approx 1.30$ Accept 1.3 | A1 **(3)** |
| | (c) $u = x^2+2 \Rightarrow \frac{du}{dx} = 2x$ | B1 |
| | $\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,dx$ | B1 |
| | $\int x^3 \ln(x^2+2)\,dx = \int x^2 \ln(x^2+2)x\,dx = \int (u-2)(\ln u)\frac{1}{2}\,du$ | M1 |
| | Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,du$ ✱ cso | A1 **(4)** |
| | (d) $\int (u-2)\ln u\,du = \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u^2}{2}-2u\right)\frac{1}{u}\,du$ | M1 A1 |
| | $= \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u}{2}-2\right)du$ | |
| | $= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right) \quad (+C)$ | M1 A1 |
| | $\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$ | |
| | $= \frac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - ((2-4)\ln 2 - 1 + 4)\right]$ | M1 |
| | $= \frac{1}{2}(2\ln 2+1)$ $\quad \ln 2 + \frac{1}{2}$ | A1 **(6)** **[15]** |