## Finding Areas using Integration - Edexcel Past Exam Questions

1. 



Figure 1

Figure 1 shows the graph of the curve with equation

$$
y=x \mathrm{e}^{2 x}, \quad x \geq 0 .
$$

The finite region $R$ bounded by the lines $x=1$, the $x$-axis and the curve is shown shaded in Figure 1.

Use integration to find the exact value of the area for $R$.
2.


Figure 3

The curve with equation $y=3 \sin \frac{x}{2}, 0 \leq x \leq 2 \pi$, is shown in Figure 1. The finite region enclosed by the curve and the $x$-axis is shaded.

Find, by integration, the area of the shaded region.
3.

Figure 3


Figure 3 shows a sketch of the curve with equation $y=(x-1) \ln x, x>0$.
Given that $I=\int_{1}^{3}(x-1) \ln x \mathrm{~d} x$,
Show, by integration, that the exact value of $\int_{1}^{3}(x-1) \ln x \mathrm{~d} x$ is $\frac{3}{2} \ln 3$.
4.

$$
I=\int_{0}^{5} \mathrm{e}^{\sqrt{ }(3 x+1)} \mathrm{d} x .
$$

(a) Use the substitution $t=\sqrt{ }(3 x+1)$ to show that $I$ may be expressed as $\int_{a}^{b} k t \mathrm{e}^{t} \mathrm{~d} t$, giving the values of $a, b$ and $k$.
(b) Use integration by parts to evaluate this integral, and hence find the value of $I$ correct to 4 significant figures, showing all the steps in your working.
5.


Figure 3

The curve $C$ has parametric equations

$$
x=\ln (t+2), \quad y=\frac{1}{(t+1)}, \quad t>-1 .
$$

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x=\ln 2$ and $x=\ln 4$, is shown shaded in Figure 3.
(a) Show that the area of $R$ is given by the integral

$$
\begin{equation*}
\int_{0}^{2} \frac{1}{(t+1)(t+2)} \mathrm{d} t \tag{4}
\end{equation*}
$$

(b) Hence find an exact value for this area.
(c) Find a cartesian equation of the curve $C$, in the form $y=\mathrm{f}(x)$.
(d) State the domain of values for $x$ for this curve.
6.


Figure 1

Figure 1 shows part of the curve $y=\frac{3}{\sqrt{ }(1+4 x)}$. The region $R$ is bounded by the curve, the $x$ axis, and the lines $x=0$ and $x=2$, as shown shaded in Figure 1 .

Use integration to find the area of $R$.
7.


Figure 1
Figure 1 shows the finite region $R$ bounded by the $x$-axis, the $y$-axis and the curve with equation $y=3 \cos \left(\frac{x}{3}\right), 0 \leq x \leq \frac{3 \pi}{2}$.
(a) Use integration to find the exact area of $R$.
8.


Figure 1

Figure 1 shows a sketch of the curve with equation $y=x \ln x, x \geq 1$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the line $x=4$.
(a) (i) Use integration by parts to find $\int x \ln x \mathrm{~d} x$.
(ii) Hence find the exact area of $R$, giving your answer in the form $\frac{1}{4}(a \ln 2+b)$, where $a$ and $b$ are integers.
9.

$$
I=\int_{2}^{5} \frac{1}{4+\sqrt{ }(x-1)} \mathrm{d} x
$$

Using the substitution $x=(u-4)^{2}+1$, or otherwise, and integrating, find the exact value of $I$.
10.


Figure 2

Figure 2 shows a sketch of the curve with equation $y=x^{3} \ln \left(x^{2}+2\right), x \geq 0$.
The finite region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line $x=\sqrt{ } 2$.
(a) Use the substitution $u=x^{2}+2$ to show that the area of $R$ is

$$
\begin{equation*}
\frac{1}{2} \int_{2}^{4}(u-2) \ln u \mathrm{~d} u \tag{4}
\end{equation*}
$$

(b) Hence, or otherwise, find the exact area of $R$.

