Partial Fractions 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme		Marks
1.	$(2+3x)^{-3} = \underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$	$\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$	B1
	$= \left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\dots\right]$	see notes	M1 A1
	$= \left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\dots}\right]$		
	$= \frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$	See notes below!	
	$= \frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\dots$		A1; A1
			[5] **5**

B1: $\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$ outside brackets or $\underline{\frac{1}{8}}$ as constant term in the binomial expansion.

M1: Expands $(\dots+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+(-3)(kx)$ or $(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2$ or $1+\dots\dots+\frac{(-3)(-4)}{2!}(kx)^2$

or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k \neq 1$ are ok for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx) where $k \neq 1$.

"Incorrect bracketing" $\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{2}\right)+\dots}\right]$ is M1A0 unless recovered.

A1: For $\frac{1}{8}-\frac{9}{16}x$ **(simplified fractions)** or also allow $0.125-0.5625x$.

Allow Special Case A1 for either SC: $\frac{1}{8}\left[1-\frac{9}{2}x;\dots\right]$ or **SC:** $K\left[1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$

(where K can be 1 or omitted), with each term in the [.........] either a simplified fraction or a decimal.

A1: Accept only $\frac{27}{16}x^2-\frac{135}{32}x^3$ or $1\frac{11}{16}x^2-4\frac{7}{32}x^3$ or $1.6875x^2-4.21875x^3$

1. ctd	Candidates who write $= \frac{1}{8}\left[1 + (-3)\left(-\frac{3x}{2}\right) + \frac{(-3)(-4)}{2!}\left(-\frac{3x}{2}\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{2}\right)^3 + \dots\right]$ where $k = -\frac{3}{2}$ and not $\frac{3}{2}$ and achieve $\frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \dots$ will get B1M1A1A0A0. ***<u>Alternative method:</u>*** Candidates can apply an alternative form of the binomial expansion. $(2+3x)^{-3} = (2)^{-3} + (-3)(2)^{-4}(3x) + \frac{(-3)(-4)}{2!}(2)^{-5}(3x)^2 + \frac{(-3)(-4)(-5)}{3!}(2)^{-6}(3x)^3$ **B1:** $\frac{1}{8}$ or $(2)^{-3}$ **M1:** Any two of four (un-simplified) terms correct. **A1:** All four (un-simplified) terms correct. **A1:** $\frac{1}{8} - \frac{9}{16}x$ **A1:** $+\frac{27}{16}x^2 - \frac{135}{32}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{-3}C_0(2)^{-3} + {}^{-3}C_1(2)^{-4}(3x) + {}^{-3}C_2(2)^{-5}(3x)^2 + {}^{-3}C_3(2)^{-6}(3x)^3$ without further working is B0M0A0.

Question 2

Question Number	Scheme		Marks
	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2,\ \ C = 2$	Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.	B1 cso
	$5x+3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 cso
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		**[4]** **4**

Notes for Question

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:
- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Question 3

Question Number	Scheme		Marks
(a)	$\frac{25}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$		
		At least one of "B" or "C" correct.	B1
	$B = 25,\ C = 100$	Breaks up their partial fraction correctly into *three terms* and both "B" $= 25$ and "C" $= 100$. **See notes.**	B1 cso
	$25 \equiv Ax(2x+1) + B(2x+1) + Cx^2$ $x = 0,\quad 25 = B$ $x = -\frac{1}{2},\quad 25 = \frac{1}{4}C \Rightarrow C = 100$ x^2 terms: $0 = 2A + C$ $0 = 2A + 100 \Rightarrow A = -50$ $x^2: 0 = 2A + C,\ x: 0 = A + 2B,$ constant: $25 = B$	Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	leading to $A = -50$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1
	$\left\{\frac{25}{x^2(2x+1)} \equiv -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)}\right\}$		[4]
(b)	$V = \pi\int_1^4 \left(\frac{5}{x\sqrt{(2x+1)}}\right)^2 dx$	For $\pi\int\left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ Ignore limits and dx. Can be implied.	B1
	For their partial fraction		
	$\left\{\int \frac{25}{x^2(2x+1)} dx = \int -\frac{50}{x} + \frac{25}{x^2} + \frac{100}{(2x+1)} dx\right\}$ $= -50\ln x + \frac{25x^{-1}}{(-1)} + \frac{100}{2}\ln(2x+1)\ \{+c\}$	Either $\pm\frac{A}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\frac{B}{x^2} \to \pm bx^{-1}$ or $\frac{C}{(2x+1)} \to \pm c\ln(2x+1)$	M1
		At least two terms correctly integrated	A1ft
		All three terms correctly integrated.	A1ft
	$\left\{\int_1^4 \frac{25}{x^2(2x+1)} dx = \left[-50\ln x - \frac{25}{x} + 50\ln(2x+1)\right]_1^4\right\}$ $= \left(-50\ln 4 - \frac{25}{4} + 50\ln 9\right) - (0 - 25 + 50\ln 3)$	Applies limits of 4 and 1 and subtracts the correct way round.	dM1
	$= 50\ln 9 - 50\ln 4 - 50\ln 3 - \frac{25}{4} + 25$ $= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$		
	So, $V = \frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$		A1 oe
			[6] 10

	Question Notes	
(a)	**BE CAREFUL!** Candidates will assign ***their own*** "A, B and C" for this question.	
	B1	At least one of "B" or "C" are correct.
	B1	Breaks up their partial fraction correctly into three terms **and** both "B" = 25 and "C" = 100.
	Note	If a candidate does not give partial fraction decomposition then: • the 2nd B1 mark can follow from a correct identity.
	M1	Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
	A1	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.
	Note	If a candidate does not give partial fraction decomposition then the final A1 mark can be awarded for a correct "A" if a candidate writes out their partial fractions at the end.
	Note	The correct partial fraction from no working scores B1B1M1A1.
	Note	A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.
	Note	Award SC B1B0M0A0 for $\frac{25}{x^2(2x+1)} \equiv \frac{B}{x^2} + \frac{C}{(2x+1)}$ leading to "B" = 25 **or** "C" = 100
(b)	**B1**	For a correct statement of $\pi\int\left(\frac{5}{x\sqrt{(2x+1)}}\right)^2$ or $\pi\int\frac{25}{x^2(2x+1)}$. Ignore limits and dx. Can be implied.
	Note	**The π can only be recovered later from a correct expression.** **For their partial fraction**, (not $\sqrt{}$their partial fraction), where A, B, C are "their" part (a) constants
	M1	Either $\pm\frac{A}{x} \to \pm a\ln x$ or $\pm\frac{B}{x^2} \to \pm bx^{-1}$ or $\frac{C}{(2x+1)} \to \pm c\ln(2x+1)$.
	Note	$\sqrt{\frac{B}{x^2}} \to \frac{\sqrt{B}}{x}$ which integrates to $\sqrt{B}\ln x$ is **not** worthy of M1.
	A1ft	At least two terms from any of $\pm\frac{A}{x}$ or $\pm\frac{B}{x^2}$ or $\frac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	A1ft	All 3 terms from $\pm\frac{A}{x}$, $\pm\frac{B}{x^2}$ and $\frac{C}{(2x+1)}$ correctly integrated. Can be un-simplified.
	Note	The 1st A1 and 2nd A1 marks in part (b) are both follow through accuracy marks.
	dM1	**Dependent on the previous M mark.** Applies limits of 4 and 1 and subtracts the correct way round.
	A1	Final correct **exact** answer in the form $a + b\ln c$. i.e. either $\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or $50\pi\ln\left(\frac{3}{4}\right) + \frac{75}{4}\pi$ or $50\pi\ln\left(\frac{9}{12}\right) + \frac{75}{4}\pi$ or $\frac{75}{4}\pi - 50\pi\ln\left(\frac{4}{3}\right)$ or $\frac{75}{4}\pi + 25\pi\ln\left(\frac{9}{16}\right)$ etc. Also allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$ or equivalent.
	Note	A candidate who achieves full marks in (a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of B1M1A1A0M1A0 in part (b).
	Note	The π in the volume formula is only required for the B1 mark and the final A1 mark.

(b)	**Alternative method of integration**		
	$V = \pi \int_1^4 \left(\frac{5}{x\sqrt{(2x+1)}} \right)^2 \mathrm{d}x$	B1	For $\pi \int \left(\frac{5}{x\sqrt{(2x+1)}} \right)^2$ Ignore limits and $\mathrm{d}x$. Can be implied.
	$\int \frac{25}{x^2(2x+1)} \mathrm{d}x \; ; \; u = \frac{1}{x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}$ $= \int \frac{-25}{\left(\frac{2}{u}+1\right)} \mathrm{d}u = \int \frac{-25}{\left(\frac{2+u}{u}\right)} \mathrm{d}u = \int \frac{-25u}{(2+u)} \mathrm{d}u = -25 \int \frac{2+u-2}{(2+u)} \mathrm{d}u$		
	$= -25 \int 1 - \frac{2}{(2+u)} \mathrm{d}u = -25\,(u - 2\ln(2+u))$	M1	Achieves $\pm \alpha \pm \frac{\beta}{(k+u)}$ **and** integrates to give either $\pm \alpha u$ or $\pm \beta \ln(k+u)$
		A1	**Dependent on the M mark.** Either $-25u$ or $50\ln(2+u)$
		A1	$-25\,(u - 2\ln(2+u))$
	$\left\{ \int_1^4 \frac{25}{x^2(2x+1)} \mathrm{d}x = \left[-25u + 50\ln(2+u) \right]_1^{\frac{1}{4}} \right\}$ $= \left(-\frac{25}{4} + 50\ln\left(\frac{9}{4}\right) \right) - (-25 + 50\ln 3)$ $= 50\ln\left(\frac{9}{4}\right) - 50\ln 3 - \frac{25}{4} + 25$ $= 50\ln\left(\frac{3}{4}\right) + \frac{75}{4}$	dM1	Applies limits of $\frac{1}{4}$ and 1 in u or 4 and 1 in x in their integrated function and subtracts the correct way round.
	So, $V = \frac{75}{4}\pi + 50\pi \ln\left(\frac{3}{4}\right)$	A1	$\frac{75}{4}\pi + 50\pi\ln\left(\frac{3}{4}\right)$ or allow $\pi\left(\frac{75}{4} + 50\ln\left(\frac{3}{4}\right)\right)$