Pure Mathematics 1 Practice Paper M8 MARK SCHEME

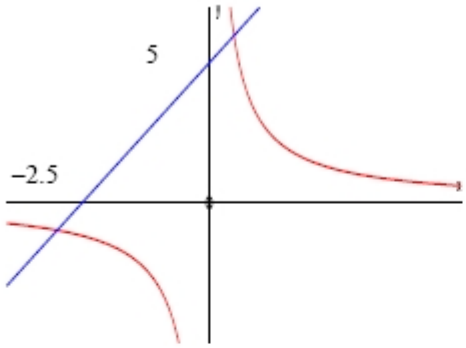
Question 1

Question Number	Scheme	Marks
(a)	$(1+ax)^{10} = 1+10ax.....$	B1
	$+\frac{10\times9}{2}(ax)^2+\frac{10\times9\times8}{6}(ax)^3$	M1
	$+45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$	A1 A1 (4)
(b)	$120a^3 = 2\times45a^2$ $a=\frac{3}{4}$ or equiv. $\left(\text{e.g. } \frac{90}{120}, 0.75\right)$	M1 A1 (2)
		(6 marks)

Question 2

Question Number	Scheme	Marks
(a)	$QR=\sqrt{(7-1)^2+(0-3)^2}$	M1
	$=\sqrt{36+9}$ or $\sqrt{45}$	A1
	$=3\sqrt{5}$ or $a=3$	A1 (3)
(b)	Gradient of QR (or l_1) $=\frac{3-0}{1-7}$ or $\frac{3}{-6}, =-\frac{1}{2}$	M1 A1
	Gradient of l_2 is $-\frac{1}{-\frac{1}{2}}$ or 2	M1
	Equation for l_2 is: $y-3=2(x-1)$ or $\frac{y-3}{x-1}=2$ [or $y=2x+1$]	M1 A1 ft (5)
(c)	P is (0, 1) (allow "$x=0, y=1$" but it must be clearly identifiable as P)	B1 (1)
(d)	$PQ=\sqrt{(1-x_P)^2+(3-y_P)^2}$	M1
	$PQ=\sqrt{1^2+2^2}=\sqrt{5}$	A1
	Area of triangle is $\frac{1}{2}QR\times PQ=\frac{1}{2}3\sqrt{5}\times\sqrt{5}, =\frac{15}{2}$ or 7.5	M1 A1 (4)
		(13 marks)

Question 3

Question Number	Scheme	Marks
(a)		B1 M1 A1 (3)
(b)	$2x+5=\frac{3}{x}$	M1
	$2x^2+5x-3\,[=0]$ or $2x^2+5x=3$	A1
	$(2x-1)(x+3)\,[=0]$	M1
	$x=-3$ or $\frac{1}{2}$	A1
	$y=\frac{3}{-3}$ or $2\times(-3)+5$ or $y=\frac{3}{\frac{1}{2}}$ or $2\times\left(\frac{1}{2}\right)+5$	M1
	Points are $(-3,-1)$ and $(\frac{1}{2},6)$ (correct pairings)	A1 ft (6)
		(9 marks)

Question 4

Question Number	Scheme	Marks
(a)	$\left(x^2+3\right)^2=x^4+3x^2+3x^2+3^2$	M1
	$\frac{\left(x^2+3\right)^2}{x^2}=\frac{x^4+6x^2+9}{x^2}=x^2+6+9x^{-2}$ (*)	A1 cso (2)
(b)	$y=\frac{x^3}{3}+6x+\frac{9}{-1}x^{-1}(+c)$	M1 A1 A1
	$20=\frac{27}{3}+6\times3-\frac{9}{3}+c$	M1
	$c=-4$	A1
	$[y=]\frac{x^3}{3}+6x-9x^{-1}-4$	A1 ft (6)
		(8 marks)

Question 5

Question Number	Scheme	Marks
(a)	$(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$	A1 (4)
(b)	Gradient of radius $=\frac{2}{5}$ (or exact equivalent)	B1
	Gradient of tangent $=\frac{-5}{2}$	M1
	$y-3=\frac{-5}{2}(x-8)$	M1 A1 ft
	$5x+2y-46=0$ or equivalent	A1 (5)
		(9 marks)

Question 6

Question Number	Scheme	Marks
(a)	$\left[\frac{dy}{dx}=\right]3kx^2-2x+1$	M1 A1 (2)
(b)	Gradient of line is $\frac{7}{2}$	B1
	When $x=-\frac{1}{2}$: $3k\times\left(\frac{1}{4}\right)-2\times\left(-\frac{1}{2}\right)+1,=\frac{7}{2}$	M1
	$\frac{3k}{4}=\frac{3}{2}\Rightarrow k=2$	A1 A1 (4)
(c)	$x=-\frac{1}{2}\Rightarrow y=k\times\left(-\frac{1}{8}\right)-\left(\frac{1}{4}\right)-\frac{1}{2}-5,=-6$	M1 A1 (2)
		(8 marks)

Question 7

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx} =\right) 8+2x-3x^2$	M1 A1
	$3x^2-2x-8=0 \quad (3x+4)(x-2)=0 \quad x=2$	A1 cso (3)
(b)	Area of triangle $= \quad \frac{1}{2}\times 2\times 22$	M1 A1
	$\int 10+8x+x^2-x^3\,dx = 10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$	M1 A1 A1
	$\left[10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots \left(=20+16+\frac{8}{3}-4\right)$	M1
	Area of $R = 34\frac{2}{3}-22 = \frac{38}{3}\left(=12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
		(11 marks)

Question 8

Question Number	Scheme	Marks
(a)	$x=\frac{\log 7}{\log 5}$ or $x=\log_5 7$	M1
	1.21	A1 (2)
(b)	$\left(5^x-7\right)\left(5^x-5\right)$	M1 A1
	$\left(5^x=7 \text{ or } 5^x=5\right) \quad x=1.2$ (awrt)	A1 ft
	$x=1$	B1 (4)
		(6 marks)

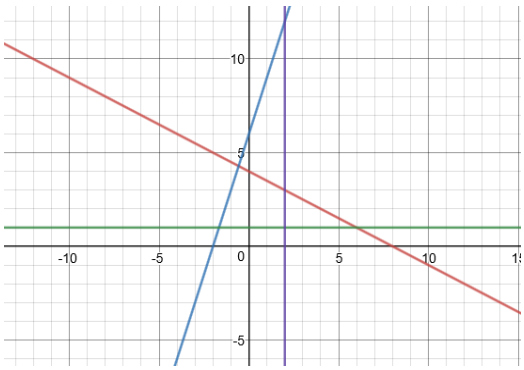
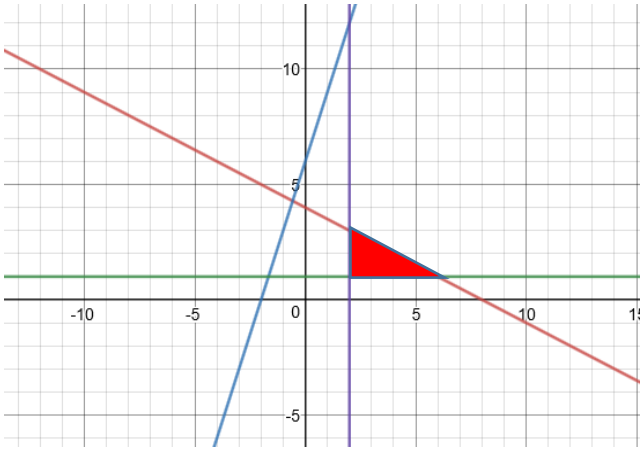
Question 9

Question Number	Scheme	Marks
(a)	45 (α)	B1
	$180-\alpha$, Add 20 (for at least one angle)	M1 M1
	65 155	A1 (4)
(b)	120 or 240 (β):	B1
	$360-\beta$, $360+\beta$	M1 M1
	Dividing by 3 (for at least one angle)	M1
	40 80 160 200 280 320	A1 A1 (6)
		(10 marks)

Question 10

Q	Scheme	Marks
(a)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ $= \begin{pmatrix}-4\\-6\end{pmatrix} + \begin{pmatrix}6\\2\end{pmatrix}$ $= \begin{pmatrix}2\\-4\end{pmatrix}$	M1 A1
(b)	$\overrightarrow{AB} = \sqrt{4^2+6^2} = \sqrt{52}$ $\overrightarrow{AC} = \sqrt{6^2+2^2} = \sqrt{40}$ $\overrightarrow{BC} = \sqrt{2^2+(-4)^2} = \sqrt{20}$ (M1 awarded for any 2 calculated length) Use of cosine rule $(\sqrt{20})^2 = (\sqrt{52})^2 + (\sqrt{40})^2 - 2(\sqrt{52})(\sqrt{40})\cos Q$ $Q = \cos^{-1}(\frac{72}{8\sqrt{130}})$ $Q = 37.87$ (2 d.p)	M1 M1A1 A1
(c)	Use of sine rule Area = $\frac{1}{2} \times \sqrt{52} \times \sqrt{40} \times \sin 37.87 \ldots$ $= 14$ (sq units)	M1 A1

Question 11

Q11	Scheme	Marks
(a)	Drawing the line $2y + x = 8$ Drawing the line $y = 3x + 6$ Drawing the line $y = 1$ Drawing the line $x = 2$ 	M1 M1 M1 M1
(b)	 Area of triangle = $\frac{1}{2} \times 4 \times 2 = 4\ sq\ units$	M1 A1

Question 12

Question Number	Scheme	Marks
(a)	$e^{2x+1} = 2$ $2x = \ln 2$ $x = \frac{1}{2}\ln 2$	M1 A1

Question Number	Scheme	Marks
(b)	$\frac{dy}{dx} = 8e^{2x}$	B1
	$x = \frac{1}{2}(\ln 2 \ ) \quad \Rightarrow \quad \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 \ \right)$	M1
	$y = 16x + \ 8 - 8\ln 2$	A1 (4)
		(6 marks)