

Algebraic Fractions 2 - Edexcel Past Exam Questions **MARK SCHEME**
Question 1

Question Number	Scheme	Marks
	$9x^2 - 4 = (3x - 2)(3x + 2)$ At any stage Eliminating the common factor of $(3x + 2)$ at any stage $\frac{2(3x+2)}{(3x-2)(3x+2)} = \frac{2}{3x-2}$ Use of a common denominator $\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)} - \frac{2(9x^2-4)}{(9x^2-4)(3x+1)} \text{ or } \frac{2(3x+1)}{(3x-2)(3x+1)} - \frac{2(3x-2)}{(3x+1)(3x-2)}$ $\frac{6}{(3x-2)(3x+1)} \text{ or } \frac{6}{9x^2-3x-2}$	B1 B1 M1 A1 (4 marks)

Notes

- B1 For factorising $9x^2 - 4 = (3x - 2)(3x + 2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark
- B1 For eliminating/cancelling out a factor of $(3x + 2)$ at any stage of the answer.
- M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$$\frac{2(3x+2)}{(9x^2-4)(3x+1)} - \frac{2(9x^2-4)}{(9x^2-4)(3x+1)} \quad \text{Only one numerator adapted, separate fractions}$$

$$\frac{2 \times 3x + 1 - 2 \times 3x - 2}{(3x-2)(3x+1)} \quad \text{Invisible brackets, single fraction}$$

A1
$$\frac{6}{(3x-2)(3x+1)}$$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$$\frac{2(3x+2)}{(9x^2-4)} - \frac{2}{(3x+1)} = \frac{2(3x+2)(3x+1) - 2(9x^2-4)}{(9x^2-4)(3x+1)} = \frac{18x+12}{(9x^2-4)(3x+1)} \quad \text{has scored 0,0,1,0 so far}$$

$$= \frac{6(3x+2)}{(3x+2)(3x-2)(3x+1)} \quad \text{is now 1,1,1,0}$$

$$= \frac{6}{(3x-2)(3x+1)} \quad \text{and now 1,1,1,1}$$

Question 2

Question Number	Scheme	Marks
	<p>(a) $\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5) + 4(x+2) - 18}{(x+2)(x^2+5)}$</p> <p>$= \frac{2x(x+2)}{(x+2)(x^2+5)}$</p> <p>$= \frac{2x}{(x^2+5)}$</p>	<p>M1A1</p> <p>M1</p> <p>A1*</p> <p>(4)</p>
	<p>(b) $h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$</p> <p>$h'(x) = \frac{10-2x^2}{(x^2+5)^2}$</p>	<p>M1A1</p> <p>cs0</p> <p>A1</p> <p>(3)</p>
	<p>(c) Maximum occurs when $h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = \dots$</p> <p>$\Rightarrow x = \sqrt{5}$</p>	<p>M1</p> <p>A1</p>
	<p>(c) Maximum occurs when $h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = \dots$</p> <p>$\Rightarrow x = \sqrt{5}$</p>	<p>M1</p> <p>A1</p>
	<p>When $x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5}$</p> <p>Range of $h(x)$ is $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$</p>	<p>M1,A1</p> <p>A1ft</p> <p>(5)</p>
		(12 marks)

- (a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are
- $\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets
- $\frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)}$ Eg 2 An example of an error (on middle term), 1st term has been adapted
- $\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator
- A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.
- $\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$
- Accept if there are three separate fractions with the correct (lowest) common denominator.
Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4(x+2)}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)}$
- Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator
- M1 There must be a single denominator. Terms must be collected on the numerator.
A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'
- A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors
- (b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out $u=..., u'=..., v=..., v'=...$ followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form
- $\frac{(x^2+5) \times A - 2x \times Bx}{(x^2+5)^2}$ where $A, B > 0$
- A1 Correct unsimplified answer $h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$
- A1 $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}, \frac{-2(x^2-5)}{(x^2+5)^2}, \frac{10-2x^2}{(x^2+5)^2}$
- DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0**
- (c) M1 Sets their $h'(x)=0$ and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.
- A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.
- M1 Substitutes their answer into their $h'(x)=0$ in $h(x)$ to determine the maximum value
- A1 Cso-the maximum value of $h(x) = \frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ but not 0.447
- A1ft Range of $h(x)$ is $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0 \leq y \leq \frac{\sqrt{5}}{5}, 0 \leq \text{Range} \leq \frac{\sqrt{5}}{5}, \left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \leq x \leq \frac{\sqrt{5}}{5}, \left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow.

Do not allow $h^{-1}(x)$ to be used for $h'(x)$ in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

- M1 Sets $h(x) = 2x(x^2+5)^{-1}$ and applies the product rule $vu'+uv'$ with terms being $2x$ and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out $u=..., u'=..., v=..., v'=...$ followed by their $vu'+uv'$) then only accept answers of the form

$$(x^2+5)^{-1} \times A + 2x \times \pm Bx(x^2+5)^{-2}$$

- A1 Correct un simplified answer $(x^2+5)^{-1} \times 2 + 2x \times -2x(x^2+5)^{-2}$

- A1 The question asks for $h'(x)$ to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$h'(x) = \frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

Question 3

Question Number	Scheme	Marks
	<p>(a) $x^2 + x - 12 = (x + 4)(x - 3)$</p> <p>Attempt as a single fraction $\frac{(3x+5)(x-3) - 2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\frac{3x+5-2(x+4)}{(x+4)(x-3)}$</p> <p>$= \frac{x-3}{(x+4)(x-3)}, = \frac{1}{(x+4)}$ cao</p>	<p>B1</p> <p>M1</p> <p>A1, A1</p> <p>(4 marks)</p>
Notes for Question		
B1	For correctly factorising $x^2 + x - 12 = (x + 4)(x - 3)$. It could appear anywhere in their solution	
M1	For an attempt to combine two fractions. The denominator must be correct for 'their' fractions. The terms could be separate but one term must have been modified. Condone invisible brackets. Examples of work scoring this mark are;	
	$\frac{(3x+5)(x-3)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ <p>Two separate terms</p>	
	$\frac{3x+5-2x+4}{(x+4)(x-3)}$ <p>Single term, invisible bracket</p>	
	$\frac{(3x+5)}{(x^2+x-12)(x-3)} - \frac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ <p>Separate terms, only one numerator modified</p>	
A1	Correct un simplified answer $\frac{x-3}{(x+4)(x-3)}$	
	If $\frac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\frac{x-3}{x^2+x-12}$ or $\frac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.	
A1	cao $\frac{1}{(x+4)}$	
	Do Not isw in this question.	
	The method of partial fractions is perfectly acceptable and can score full marks	
	$\frac{3x+5}{(x+4)(x-3)} - \frac{2}{x-3} = \frac{1}{x+4} + \frac{2}{x-3} - \frac{2}{x-3} = \frac{1}{x+4}$ <p style="text-align: center;"> $\frac{3x+5}{(x+4)(x-3)}$ B1 $\frac{1}{x+4} + \frac{2}{x-3} - \frac{2}{x-3}$ M1A1 $\frac{1}{x+4}$ A1 </p>	

Question 4

Question Number	Scheme	Marks
	<p>Factorise $4x^2 - 9 = (2x - 3)(2x + 3)$</p> <p>Use of common denominator</p> $\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9} = \frac{3(2x-3) - 1(2x+3) + 6}{(2x+3)(2x-3)}$ $= \frac{4x-6}{(2x+3)(2x-3)}$ $= \frac{2(2x-3)}{(2x+3)(2x-3)} = \frac{2}{2x+3}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p> <p>4 marks</p>
	<p>Alternative where $4x^2 - 9$ is not factorised</p> $\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9} = \frac{3(2x-3)(4x^2-9) - 1(2x+3)(4x^2-9) + 6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ $= \frac{2(2x-3)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)} \text{ or } \frac{(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)} \text{ or } \frac{(2x-3)(8x^2-18)}{(2x+3)(2x-3)(4x^2-9)}$ $= \frac{(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)} \text{ or } \frac{2(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ $= \frac{2}{2x+3}$	<p>M1</p> <p>B1</p> <p>A1</p> <p>A1</p>

B1 For **factorising** $4x^2 - 9$ to $(2x - 3)(2x + 3)$ at any point. Note that this is not scored for combining the terms $(2x - 3)(2x + 3)$ and writing the product as $4x^2 - 9$

M1 Use of common denominator – combines three fractions to form one. The denominator must be correct for their fractions and at least one numerator must have been adapted. Condone missing brackets.

$\frac{16x^3 - 24x^2 - 36x + 54}{(4x^2 - 9)^2}$ is a correct intermediate stage but needs to be factorised and cancelled before A1

Examples of incorrect fractions scoring this mark are: $\frac{3(2x-3) - 2x+3+6}{(2x+3)(2x-3)}$ missing bracket

$\frac{3(4x^2-9) - 4x^2-9+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ denominator correct and at least one numerator has been adapted.

A1 Correct simplified intermediate answer. It must be a CORRECT $\frac{\text{Linear}}{\text{Quadratic}}$ or $\frac{\text{Quadratic}}{\text{Cubic}}$

Accept versions of $\frac{4x-6}{(2x+3)(2x-3)}$ or $\frac{8x^2-18}{(2x+3)(4x^2-9)}$

A1 $\text{cao} = \frac{2}{2x+3}$

Allow recovery from invisible brackets for all 4 marks as the answer is not given.

Question 5

Question Number	Scheme	Marks
(a)	$x^2 - 3kx + 2k^2 = (x - 2k)(x - k)$ $2 - \frac{(x - 5k)(x - k)}{(x - 2k)(x - k)} = 2 - \frac{(x - 5k)}{(x - 2k)} = \frac{2(x - 2k) - (x - 5k)}{(x - 2k)}$ $= \frac{x + k}{(x - 2k)}$	B1 M1 A1* (3)
(b)	Applies $\frac{vu' - uv'}{v^2}$ to $y = \frac{x + k}{x - 2k}$ with $u = x + k$ and $v = x - 2k$ $\Rightarrow f'(x) = \frac{(x - 2k) \times 1 - (x + k) \times 1}{(x - 2k)^2}$ $\Rightarrow f'(x) = \frac{-3k}{(x - 2k)^2}$	M1, A1 A1 (3)
(c)	If $f'(x) = \frac{-3k}{(x - 2k)^2} \Rightarrow f(x)$ is an increasing function as $f'(x) > 0$, $f'(x) = \frac{-3k}{(x - 2k)^2} > 0$ for all values of x as $\frac{\text{negative} \times \text{negative}}{\text{positive}} = \text{positive}$	M1 A1 (2)
		(8 marks)

(a)

B1 For seeing $x^2 - 3kx + 2k^2 = (x - 2k)(x - k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2 - \frac{(x - 5k)}{(x - 2k)} = \frac{2(x - 2k) - (x - 5k)}{(x - 2k)}$ or

$$2 - \frac{(x - 5k)(x - k)}{(x - 2k)(x - k)} = \frac{2(x - 2k)(x - k) - (x - 5k)(x - k)}{(x - 2k)(x - k)} \quad \left(= \frac{x^2 - k^2}{x^2 - 3kx + 2k^2} \right)$$

A1* Proceeds without any errors (including bracketing) to $= \frac{x + k}{(x - 2k)}$

(b)

M1 Applies $\frac{vu' - uv'}{v^2}$ to $y = \frac{x+k}{x-2k}$ with $u = x+k$ and $v = x-2k$.

If the rule it is stated it must be correct. It can be implied by $u = x+k$ and $v = x-2k$ with their u', v' and $\frac{vu' - uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $f'(x) = \frac{x-2k-x \pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y = (x+k)(x-2k)^{-1}$. If the rule it is stated it must be correct. It can be implied by $u = x+k$ and $v = (x-2k)^{-1}$ with their u', v' and $vu' + uv'$

If it is neither stated nor implied only accept expressions of the form

$$f'(x) = (x-2k)^{-1} \pm (x+k)(x-2k)^{-2}$$

Alternatively writes $y = \frac{x+k}{x-2k}$ as $y = 1 + \frac{3k}{x-2k}$ and differentiates to $\frac{dy}{dx} = \frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $f'(x)$.

$$f'(x) = \frac{(x-2k) \times 1 - (x+k) \times 1}{(x-2k)^2} \text{ by quotient rule}$$

$$f'(x) = (x-2k)^{-1} - (x+k)(x-2k)^{-2} \text{ by product rule}$$

$$\text{and } f'(x) = \frac{-3k}{(x-2k)^2} \text{ by the third method}$$

$$A1 \text{ cao } f'(x) = \frac{-3k}{(x-2k)^2}. \text{ Allow } f'(x) = \frac{-3k}{x^2 - 4kx + 4k^2}$$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $f'(x) = \frac{-Ck}{(x-2k)^2}$, look for $f(x)$ is an increasing function as $f'(x) / \text{gradient} > 0$

Accept a version that states as $k < 0 \Rightarrow -Ck > 0$ hence increasing

If in part (b) $f'(x) = \frac{(+Ck)}{(x-2k)^2}$, look for $f(x)$ is an decreasing function as $f'(x) / \text{gradient} < 0$

Similarly accept a version that states as $k < 0 \Rightarrow (+)Ck < 0$ hence decreasing

A1 Must have $f'(x) = \frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

Question 6

Question Number	Scheme	Marks
	$x^2 - 9 = (x+3)(x-3)$ $\frac{4x}{x^2 - 9} - \frac{2}{x+3} = \frac{4x - 2(x-3)}{(x+3)(x-3)}$ $= \frac{2x+6}{(x+3)(x-3)}$ $= \frac{2(x+3)}{(x+3)(x-3)}$ $= \frac{2}{x-3}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p>

B1 $x^2 - 9 = (x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\frac{4x}{x^2 - 9} - \frac{2}{x+3} = \frac{4x(x+3) - 2(x^2 - 9)}{(x+3)(x^2 - 9)}$

accept separately $\frac{4x}{(x+3)(x-3)} - \frac{2}{x+3} = \frac{4x}{(x+3)(x-3)} - \frac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\frac{4x}{x^2 - 9} - \frac{2}{x+3} = \frac{4x(x+3) - 2}{(x+3)(x^2 - 9)}$ as only one numerator has been adapted

A1 A correct intermediate form of $\frac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\frac{2x+6}{(x+3)(x-3)}$, $\frac{2x+6}{x^2 - 9}$, and even $\frac{(2x+6)(x+3)}{(x^2 - 9)(x+3)}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\frac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\frac{4x - 2(x-3)}{(x+3)(x-3)} = \frac{4x - 2x + 6}{\dots} = \frac{2x+6}{(x+3)(x-3)} = \frac{2}{x-3}$

This is not a "show that" question.

Note: Watch out for an answer of $\frac{2}{x+3}$ probably scored from $\frac{4x - 2(x-3)}{(x+3)(x-3)} = \frac{2x-6}{(x+3)(x-3)} = \frac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0