Areas under Parametric Curves - Edexcel Past Exam Questions MARK SCHEME

Question 1: Jan 08 Q7

Question Number	Scheme		Marks
(a)	$\left[x=\ln(t+2),\ y=\frac{1}{t+1}\right],\quad \Rightarrow \frac{dx}{dt}=\frac{1}{t+2}$	Must state $\frac{dx}{dt}=\frac{1}{t+2}$	B1
	$\text{Area}(R)=\int_{\ln 2}^{\ln 4}\frac{1}{t+1}dx;=\int_0^2\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	Area $=\int\frac{1}{t+1}dx$. Ignore limits.	M1;
		$\int\left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x=\ln 2\ \Rightarrow \ln 2=\ln(t+2)\ \Rightarrow\ 2=t+2\ \Rightarrow t=0$ $x=\ln 4\ \Rightarrow \ln 4=\ln(t+2)\ \Rightarrow\ 4=t+2\ \Rightarrow t=2$	changes limits $x\to t$ so that $\ln 2\to 0$ and $\ln 4\to 2$	B1
	Hence, $\text{Area}(R)=\int_0^2\frac{1}{(t+1)(t+2)}dt$		
			[4]
(b)	$\left(\frac{1}{(t+1)(t+2)}\right)=\frac{A}{(t+1)}+\frac{B}{(t+2)}$	$\frac{A}{(t+1)}+\frac{B}{(t+2)}$ with A and B found	M1
	$1=A(t+2)+B(t+1)$		
	Let $t=-1,\quad 1=A(1)\quad\Rightarrow \underline{A=1}$ Let $t=-2,\quad 1=B(-1)\quad\Rightarrow \underline{B=-1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\int_0^2\frac{1}{(t+1)(t+2)}dt=\int_0^2\frac{1}{(t+1)}-\frac{1}{(t+2)}dt$		
	$=\left[\ln(t+1)-\ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$	dM1
		Both ln terms correctly ft.	A1 ✓
	$=(\ln 3-\ln 4)-(\ln 1-\ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$=\ln 3-\ln 4+\ln 2=\ln 3-\ln 2=\ln\left(\frac{3}{2}\right)$	$\underline{\ln 3-\ln 4+\ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right)-\ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3-\ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw
			[6]

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x = \ln(t+2), \quad y = \frac{1}{t+1}$		
(c)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t = \ldots$ the subject giving $t = e^x - 2$	M1 A1
	$y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$	dM1 A1
			[4]
Aliter (c) **Way 2**	$t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$	Attempt to make $t = \ldots$ the subject Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$	M1 A1
	$x = \ln\left(\frac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\frac{1-y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\frac{1}{y} + 1\right)$		
	$e^x = \frac{1}{y} + 1$		
	$e^x - 1 = \frac{1}{y}$		
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1
			[4]
(d)	Domain : $\underline{x > 0}$	$\underline{x > 0}$ or just > 0	B1
			[1]
			15 marks

Question Number	Scheme		Marks
Aliter (c) **Way 3**	$e^x = t+2 \Rightarrow t+1 = e^x - 1$	Attempt to make $t+1 = \ldots$ the subject	M1
		giving $t+1 = e^x - 1$	A1
	$y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y	dM1
		giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]
Aliter (c) **Way 4**	$t+1 = \dfrac{1}{y} \Rightarrow t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	Attempt to make $t+2 = \ldots$ the subject	M1
		Either $t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y}+1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y}+1\right)$		
	$e^x = \dfrac{1}{y}+1 \Rightarrow e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]