Differentiating functions & Expressions 2 - Edexcel Past Exam Questions

1. Differentiate with respect to x, giving your answer in its simplest form,

(*a*) $x^2 \ln(3x)$, **(4)**

(*b*) $\dfrac{\sin 4x}{x^3}$. **(5)**

Jan 12 Q1

---

2. The point P is the point on the curve $x = 2\tan\left(y + \dfrac{\pi}{12}\right)$ with y-coordinate $\dfrac{\pi}{4}$.

Find an equation of the normal to the curve at P. **(7)**

Jan 12 Q4

3.

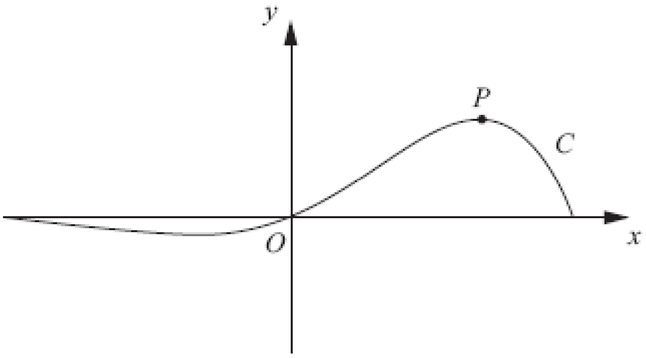

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(*a*) Find the x-coordinate of the turning point P on C, for which $x > 0$. Give your answer as a multiple of π. **(6)**

(*b*) Find an equation of the normal to C at the point where $x = 0$. **(3)**

June 12 Q3

---

4. (*a*) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form. **(6)**

(*b*) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x. **(5)**

June 12 Q7

---

5. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(*a*) the value of w, **(2)**

(*b*) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants. **(5)**

Jan 13 Q1

---

6. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$. **(6)**

Given that $x = \cot y$,

(ii) show that $\dfrac{dy}{dx} = \dfrac{-1}{1+x^2}$. **(5)**

Jan 13 Q5

---

7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0.$$

(*a*) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$. **(4)**

(*b*) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

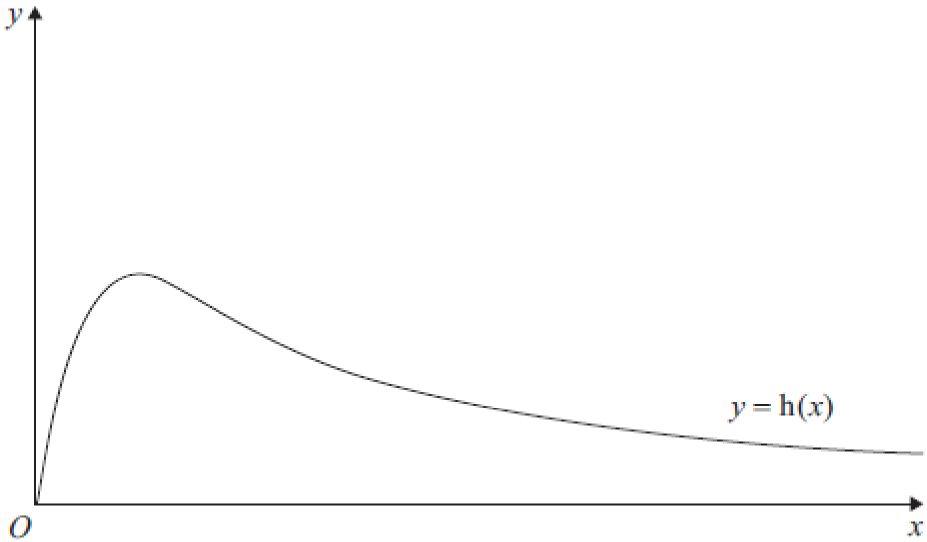

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(*c*) Calculate the range of $\mathrm{h}(x)$. **(5)**

Jan 13 Q7*(edited)*

---

8. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(*a*) find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **(2)**

(*b*) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$ **(4)**

(*c*) Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of x. Give your answer in its simplest form. **(4)**

June 13 Q5

9. (*a*) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to *x*. **(3)**

(*b*) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant. **(3)**

(*c*) Given $x = 2\sin\left(\frac{y}{3}\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x*, simplifying your answer. **(4)**

June 13(R) Q5

---

10. The curve *C* has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(*a*) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$ **(3)**

Given that *P* is a point on *C* such that $\mathrm{f}'(x) = -1$,

(*b*) find the coordinates of *P*. **(3)**

June 14 Q1

---

11. The curve *C* has equation $x = 8y \tan 2y$.

The point *P* has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(*a*) Verify that *P* lies on *C*. **(1)**

(*b*) Find the equation of the tangent to *C* at *P* in the form $ay = x + b$, where the constants *a* and *b* are to be found in terms of π. **(7)**

June 14 Q3

---

12. (i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$ **(4)**

(ii) Given that

$$y = \left(x^2 + x^3\right)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form. **(5)**

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where g(x) is an expression to be found. **(3)**

June 14(R) Q4

---

13. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p. **(1)**

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A. **(6)**

June 15 Q5

14.

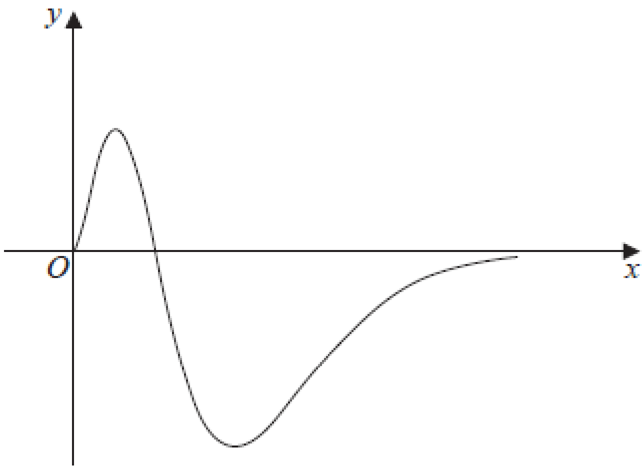

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \qquad x \geq 0.$$

(*a*) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found. **(3)**

(*b*) Hence find the range of g. **(6)**

(*c*) State a reason why the function $g^{-1}(x)$ does not exist. **(1)**

June 15 Q7

---

15. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

(*a*) show that $f(x) = \dfrac{x+k}{x-2k}$. **(3)**

(*b*) Hence find $f'(x)$, giving your answer in its simplest form. **(3)**

(*c*) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.
Justify your answer. **(2)**

June 15 Q9

16.

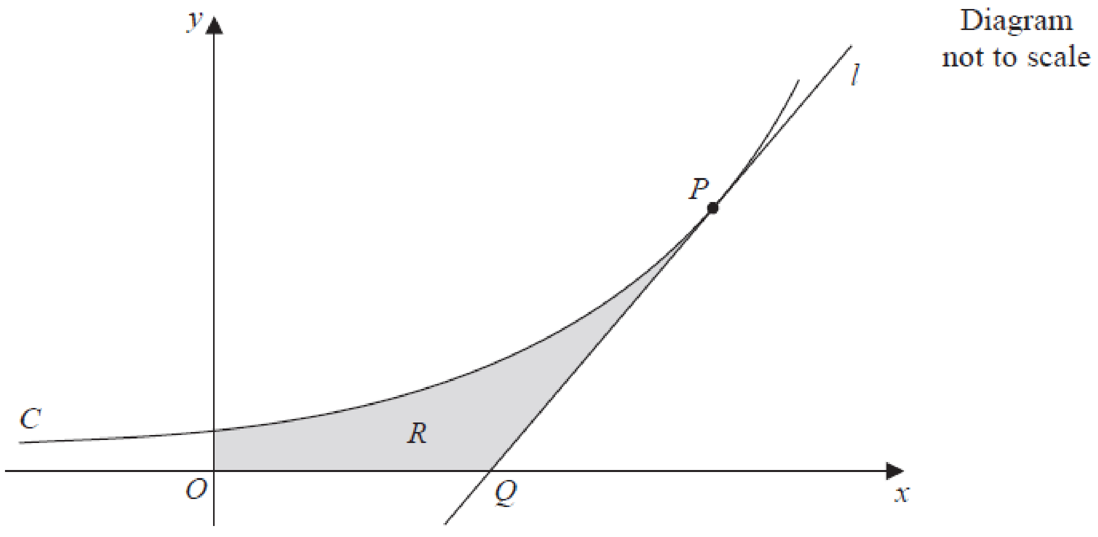

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = 3^x$.

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(*a*) Find the exact value of the x coordinate of Q. **(4)**

June 15 Q8 *(edited)*

17.

$$y = \frac{4x}{x^2 + 5}.$$

(*a*) Find $\frac{dy}{dx}$, writing your answer as a single fraction in its simplest form. **(4)**

(*b*) Hence find the set of values of x for which $\frac{dy}{dx} < 0$. **(3)**

June 16 Q2

18. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x}\cos 4x, \quad \frac{\pi}{4} \le x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places. **(5)**

(ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p\,\text{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined. **(5)**

June 16 Q5

19.
$$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, \qquad x \in \mathbb{R}.$$

(*a*) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x-2},$$

find the values of the constants A and B. **(4)**

(*b*) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$. **(5)**

June 16 Q6

---

20. (i) Given $y = 2x(x^2 - 1)^5$, show that

(*a*) $\frac{dy}{dx} = g(x)(x^2 - 1)^4$ where $g(x)$ is a function to be determined. **(4)**

(*b*) Hence find the set of values of x for which $\frac{dy}{dx} \geqslant 0$ **(2)**

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\frac{dy}{dx}$ as a function of x in its simplest form. **(4)**

June 17 Q7

21.

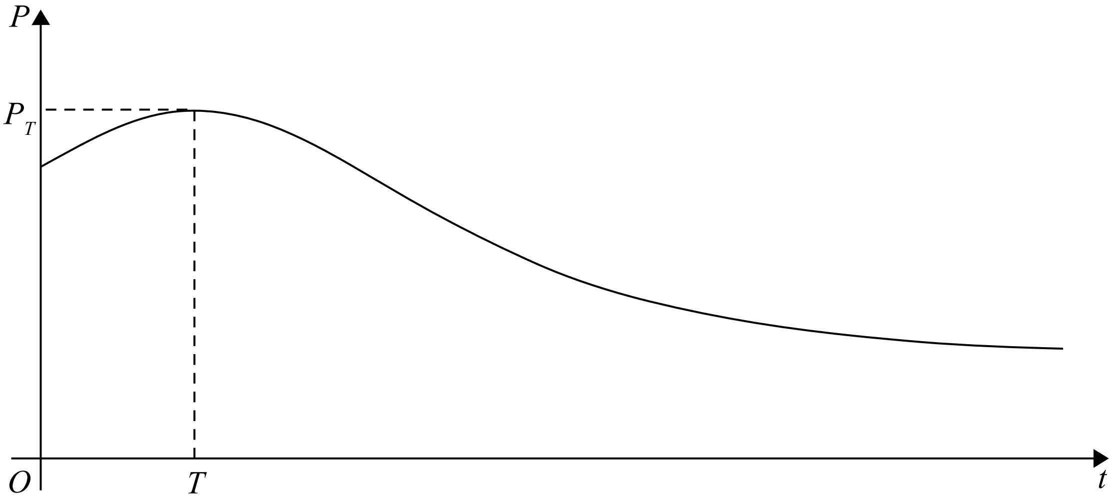

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1+3e^{-0.9t}} + 40, \qquad t \in \mathbb{R},\ t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

(*a*) Calculate the number of rabbits that were introduced onto the island. **(1)**

(*b*) Find $\frac{\mathrm{d}P}{\mathrm{d}t}$ **(3)**

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(*c*) Using your answer from part (*b*), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.) **(4)**

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(*d*) Use the model to state the maximum value of k. **(1)**

June 17 Q8