## Finding Areas using Integration 2 - Edexcel Past Exam Questions

1. 



Figure 3
Figure 3 shows a sketch of part of the curve with equation $y=x^{\frac{1}{2}} \ln 2 x$.
The finite region $R$, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and the lines $x=1$ and $x=4$.
(a) Find $\int x^{\frac{1}{2}} \ln 2 x d x$.
(b) Hence find the exact area of $R$, giving your answer in the form $a \ln 2+b$, where $a$ and $b$ are exact constants.
2.


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y=\frac{x}{1+\sqrt{ } x}$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis, the line with equation $x=1$ and the line with equation $x=4$.

Use the substitution $u=1+\sqrt{ } x$, to find, by integrating, the exact area of $R$.
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3.


Figure 1
Figure 1 shows part of the curve with equation $x=4 t e^{-\frac{1}{3} t}+3$. The finite region $R$ shown shaded in Figure 1 is bounded by the curve, the $x$-axis, the $t$-axis and the line $t=8$.

Use calculus to find the exact value for the area of $R$.
4.


Figure 1
Figure 1 shows a sketch of part of the curve with equation $y=4 x-x \mathrm{e}^{\frac{1}{2} x}, x \geq 0$.
The curve meets the $x$-axis at the origin $O$ and cuts the $x$-axis at the point $A$.
(a) Find, in terms of $\ln 2$, the $x$ coordinate of the point $A$.
(b) Find $\int x \mathrm{e}^{\frac{1}{2} x} \mathrm{~d} x$.

The finite region $R$, shown shaded in Figure 1, is bounded by the $x$-axis and the curve with equation $y=4 x-x \mathrm{e}^{\frac{1}{2} x}, \quad x \geq 0$.
(c) Find, by integration, the exact value for the area of $R$.

Give your answer in terms of $\ln 2$.
5.


Figure 2
Figure 2 shows a sketch of the curve with equation $y=\sqrt{(3-x)(x+1)}, 0 \leq x \leq 3$.

The finite region $R$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis, and the $y$-axis.
(a) Use the substitution $x=1+2 \sin \theta$ to show that

$$
\begin{equation*}
\int_{0}^{3} \sqrt{(3-x)(x+1)} \mathrm{d} x=k \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos ^{2} \theta \mathrm{~d} \theta \tag{5}
\end{equation*}
$$

where $k$ is a constant to be determined.
(b) Hence find, by integration, the exact area of $R$.
6.


Figure 1
Figure 1 shows a sketch of part of the curve with equation $y=x^{2} \ln x, x \geq 1$.
The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the line $x=2$.

Use integration to find the exact value for the area of $R$.
7.


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y=\frac{6}{\left(\mathrm{e}^{x}+2\right)}, x \in \mathbb{R}$
The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $y$-axis, the $x$-axis and the line with equation $x=1$
(c) Use the substitution $u=\mathrm{e}^{x}$ to show that the area of $R$ can be given by

$$
\begin{equation*}
\int_{a}^{b} \frac{6}{u(u+2)} \mathrm{d} u \tag{2}
\end{equation*}
$$

where $a$ and $b$ are constants to be determined.
(d) Hence use calculus to find the exact area of $R$.
[Solutions based entirely on graphical or numerical methods are not acceptable.]

