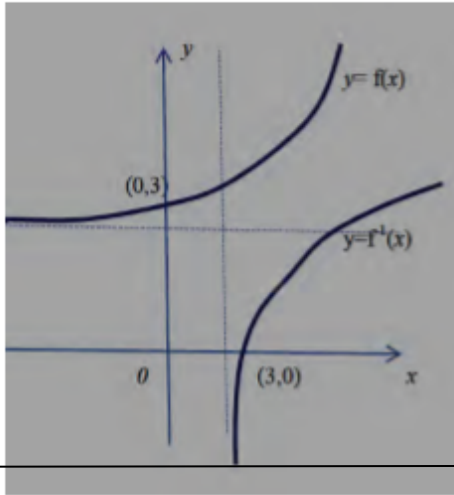## Functions 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question No	Scheme		Marks
	(a) $\quad 2x^2 + 7x - 4 = (2x - 1)(x + 4)$		B1
	$\dfrac{3(x + 1)}{(2x - 1)(x + 4)} - \dfrac{1}{(x + 4)} = \dfrac{3(x + 1) - (2x - 1)}{(2x - 1)(x + 4)}$		M1
	$\qquad\qquad = \dfrac{x+4}{(2x-1)(x+4)}$		M1
	$\qquad\qquad = \dfrac{1}{2x-1}$		A1*
			(4)
	(b) $\quad y = \dfrac{1}{2x-1} \Rightarrow y(2x - 1) = 1 \Rightarrow 2xy - y = 1$		
	$2xy = 1 + y \Rightarrow x = \dfrac{1 + y}{2y}$		M1M1
	$y \text{ OR } f^{-1}(x) = \dfrac{1 + x}{2x}$		A1
			(3)
	(c) $\qquad\qquad\qquad x > 0$		B1
			(1)
	(d) $\quad \dfrac{1}{2\ln(x+1) - 1} = \dfrac{1}{7}$		M1
	$\ln(x + 1) = 4$		A1
	$x = e^4 - 1$		M1A1
			(4)
			12 Marks

Question 2

Question Number	Scheme	Marks
	(a) f(x)>2	B1 (1)
	(b) $fg(x) = e^{\ln x} + 2, = x + 2$	M1,A1 (2)
	(c) $e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$ $\Rightarrow 2x + 3 = \ln 4$ $\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1A1 M1A1 (4)
	(d) Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$ $f^{-1}(x) = \ln(x-2), \quad x > 2.$	M1 A1 , **B1ft** (3)
	(e)	Shape for f(x) B1 (0, 3) B1 Shape for $f^{-1}(x)$ B1 (3, 0) B1 (4)

(a) B1   Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1   For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0

   A1   Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks

(c) M1   Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = ...$

   A1 $e^{2x+3} = 4$

   M1   Takes ln's both sides, $2x + 3 = \ln..$ and proceeds to x=....

   A1 $x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1   Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
    All ln work must be correct. The 2 must be dealt with first.
    Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x - 2)$ or $y = \ln(x - 2)$ or $y = \ln|x - 2|$ There must be some form of bracket

B1ft Either $x > 2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
    Do not accept y>2 or $f^{-1}(x)$>2.

(e) B1   Shape for $y = e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient
    that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1.
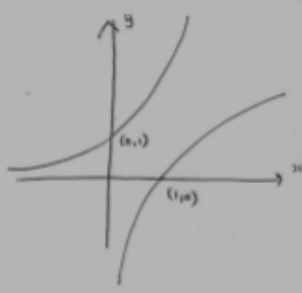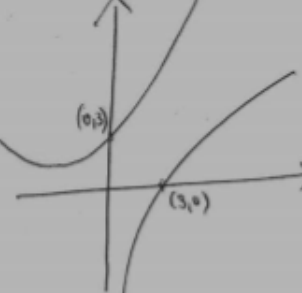   You should not see a minimum point on the graph.

  B1   (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

  B1   Shape for $y = \ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient
    that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves
    into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y = e^x$

  B1   (3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect
	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question 3

Question Number	Scheme	Marks
(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	$ff(0) = f(5). \ = 3$	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y - 4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4 \quad$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to (d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2 \quad or \quad 5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4 \quad$ oe	A1 (5)

Notes for Question

(a)

B1   Correct range. Allow $0 \leqslant f(x) \leqslant 10$, $0 \leqslant f \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0, 10]$

       Allow $f(x) \geqslant 0$ and $f(x) \leqslant 10$ but not $f(x) \geqslant 0$ or $f(x) \leqslant 10$

       Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends

(b)

B1   For correct one application of the function at $x = 0$

       Possible ways to score this mark are   f(0)=5,   f(5) $0 \to 5 \to \ldots$

B1:   3   ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1   For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1   Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one**   sign error for this mark

A1   Correct answer. No need to state the domain. Allow $g^{-1}(x) = \dfrac{5x - 4}{3 + x}$ $y = \dfrac{5x - 4}{3 + x}$

       Accept alternatives such as $y = \dfrac{4 - 5x}{-3 - x}$ and $y = \dfrac{5 - \frac{4}{x}}{1 + \frac{3}{x}}$

(d)

M1   Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\dfrac{4 + 3f(x)}{5 - f(x)} = 16 \Rightarrow f(x) = ..$

A1   Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$

B1 $x = 6$ and may be given if there is no working

M1   Full method to obtain other value from line $y = 5 - 2.5x$

       $5 - 2.5x = 4 \Rightarrow x = \ldots$.

       Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2 - x}{4}$ (oe) $\Rightarrow x = ..$

A1   0.4 or 2/5

Alt 1 to (d)

M1   Writes $gf(x) = 16$ with a linear $f(x)$. The order of $gf(x)$ must be correct

       Condone invisible brackets. Even accept if there is a modulus sign.

A1   Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$

B1 $x = 6$ and may be given if there is no working

M1   Attempt at solving $\dfrac{4 + 3(5 - 2.5x)}{5 - (5 - 2.5x)} = 16 \Rightarrow x = \ldots$. The bracketing must be correct and there must be

no more than one error in their calculation

A1 $x = 0.4, \dfrac{2}{5}$ or equivalent

Question 4

Question Number	Scheme	Marks
(a)	$f(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $fg(1) = 5$	A1
		(2)
(c)	$y = 3 - 4x \Rightarrow 4x = 3 - y \Rightarrow x = \dfrac{3-y}{4}$	M1
	$g^{-1}(x) = \dfrac{3-x}{4}$	A1
		(2)
(d)	$\left[g(x)\right]^2 = (3-4x)^2$	B1
	$gg(x) = 3 - 4(3-4x)$	M1
	$gg(x) + \left[g(x)\right]^2 = 0 \Rightarrow -9 + 16x + 9 - 24x + 16x^2 = 0$	
	$16x^2 - 8x = 0$	A1
	$8x(2x-1) = 0 \Rightarrow x = 0, 0.5 \quad$ oe	M1A1
		(5)
		(11 marks)

Notes for Question

(a)

M1   Attempt at calculating f at $x=0$. Sight of 3 is sufficient. Accept $f(x) > 3$ and $x > 3$ for M1,

A1 $f(x) \geqslant 3$. Accept $y \geqslant 3$, range $\geqslant 3$, $[3, \infty)$

Do not accept $f(x) > 3$, $x \geqslant 3$

The correct answer is sufficient for both marks.

(b)

M1   A full method of finding fg(1). The order of substituting into the expressions must be correct and $2|x| + 3$ must be used as opposed to $2x + 3$

Accept an attempt to calculate $2|x| + 3$ when $x = -1$.

Accept an attempt to put $x=1$ into $3 - 4x$ and then substituting their answer to $3 - 4x|_{x=1}$ into $2|x| + 3$

Do not accept the substitution of $x=1$ into $2|x| + 3$, followed by their result into '3-4x'

This is evidence of incorrect order.

A1   fg(1)=5.

Watch for $1 \xrightarrow{3-4x} 1 \xrightarrow{2|x|+3} 5$ which is M1A0

(c)

M1   Award for an attempt to make x or a swapped y the subject of the formula. It must be a full method and cannot finish $4x = ..$

You can condone at most one 'arithmetic' error for this method mark.

$y = 3 - 4x \Rightarrow 4x = 3 + y \Rightarrow x = \dfrac{3+y}{4}$ is fine for the M1 as there is only one error

$y = 3 - 4x \Rightarrow 4x = 3 - y \Rightarrow x = \dfrac{3}{4} - y$ is fine for the M1 as there is only one error

$y = 3 - 4x \Rightarrow 4x = 3 + y \Rightarrow x = \dfrac{3}{4} + y$ is M0 as there are two arithmetic errors

A1   Obtaining a correct expression $g^{-1}(x) = \dfrac{3-x}{4}$ oe such as $g^{-1}(x) = \dfrac{x-3}{-4}$, $g^{-1}(x) = \dfrac{3}{4} - \dfrac{x}{4}$

It must be in terms of x, but could be expressed 'y=' or $g^{-1}(x) \rightarrow$

(d)

B1   Sight of $[g(x)]^2 = (3 - 4x)^2$. If only the expanded version appears it must be correct

M1   A full attempt to find $gg(x) = 3 - 4(3 - 4x)$
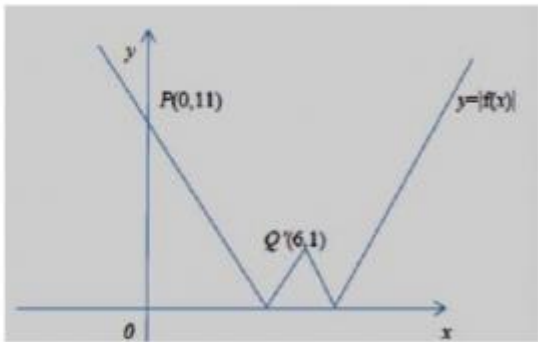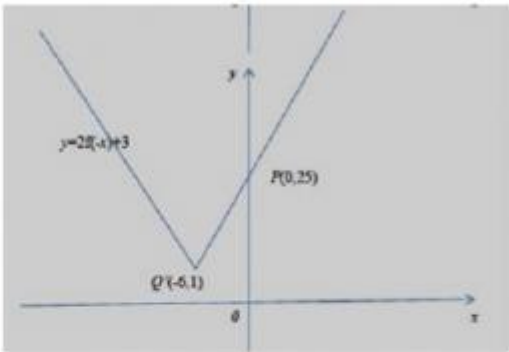
Condone invisible brackets. Note that it may appear in an equation

A1 $16x^2 - 8x = 0$ Accept other alternatives such as $2x^2 = x$

M1   For factorising their quadratic or cancelling their $Ax^2 = Bx$ by x to get $\geqslant 1$ value of x

If they have a 3TQ then usual methods are applicable.

A1   Both values correct $x = 0, 0.5$ oe

Question 5

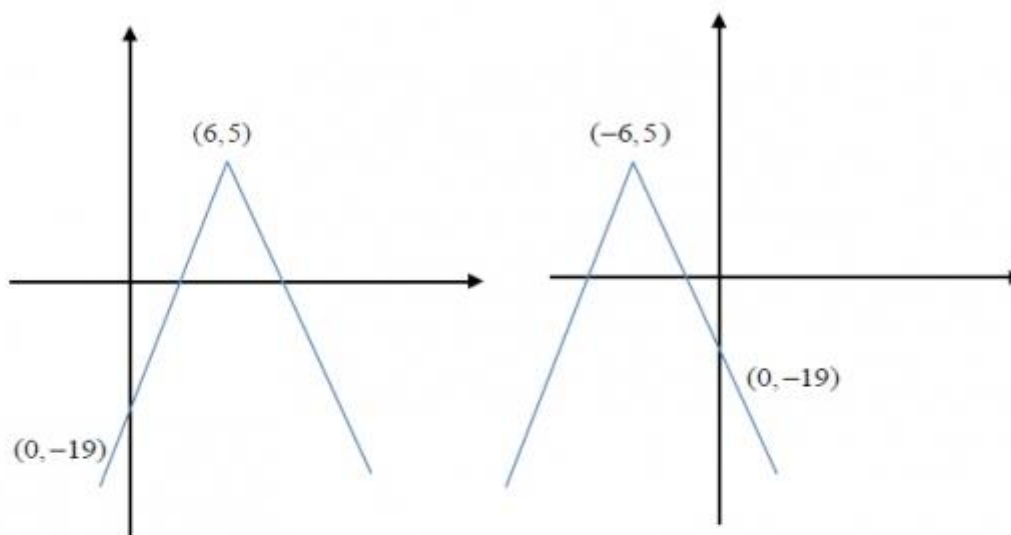| Question Number	Scheme		Marks
(a)		'W' Shape (0, 11) and (6, 1)	B1 B1 (2)
(b)		'V' shape (-6,1) (0,25)	B1 B1 B1 (3)
(c)	One of $a = 2$ or $b = 6$ $a = 2$ and $b = 6$		B1 B1 (2) **(7 marks)**

(a)

B1   A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.

A correct sketch of $y = f(|x|)$ would score this mark.

B1   A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

(b)

B1   Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.

Special case: A candidate who mistakenly sketches $y = -2f(x) + 3$ or $y = -2f(-x) + 3$ will arrive at one of the following. They can be awarded SC   B1B0B0

(c)

B1   Either states $a = 2$ **or** $b = 6$.

This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x - 6| - 1$ or $y = 2|x - ..| - 1$. If they are both stated and written, the stated answer takes precedence.

B1   States both $a = 2$ **and** $b = 6$

This can be implied by the candidate stating that $y = 2|x - 6| - 1$

If they are both stated and written, the stated answer takes precedence.

Question 6

Question Number	Scheme	Marks
(a)	$x^2 + x - 6 = (x+3)(x-2)$	B1
	$\dfrac{x}{x+3} + \dfrac{3(2x+1)}{(x+3)(x-2)} = \dfrac{x(x-2) + 3(2x+1)}{(x+3)(x-2)}$	M1
	$= \dfrac{x^2 + 4x + 3}{(x+3)(x-2)}$	A1
	$= \dfrac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$= \dfrac{(x+1)}{(x-2)}$ $\qquad$ cso	A1*
		(4)
(b)	One end either $(y) > 1, (y) \geqslant 1$ or $(y) < 4, (y) \leqslant 4$	B1
	$1 < y < 4$	B1
		(2)
(c)	Attempt to set	
	Either $g(x) = x$ or $g(x) = g^{-1}(x)$ or $g^{-1}(x) = x$ or $g^2(x) = x$	
	$\dfrac{(x+1)}{(x-2)} = x \qquad \dfrac{x+1}{x-2} = \dfrac{2x+1}{x-1} \qquad \dfrac{2x+1}{x-1} = x \qquad \dfrac{\frac{x+1}{x-2} + 1}{\frac{x+1}{x-2} - 2} = x$	M1
	$x^2 - 3x - 1 = 0 \Rightarrow x = \ldots$	A1, dM1
	$a = \dfrac{3 + \sqrt{13}}{2}$ oe $\left(1.5 + \sqrt{3.25}\right)$ $\qquad$ cso	A1
		(4)
		(10 marks)

(a)

B1 $x^2 + x - 6 = (x+3)(x-2)$ This can occur anywhere in the solution.

M1   For combining the two fractions with a common denominator. The denominator must be correct for their fractions and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

Accept $\dfrac{x}{x+3} + \dfrac{3(2x+1)}{x^2+x-6} = \dfrac{x(x^2+x-6)+3(2x+1)(x+3)}{(x+3)(x^2+x-6)}$

Condone $\dfrac{x}{x+3} + \dfrac{3(2x+1)}{(x+3)(x-2)} = \dfrac{x \times x - 2}{(x+3)(x-2)} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

A1   A correct intermediate form of $\dfrac{\text{simplified quadratic}}{\text{simplified quadratic}}$

Accept $\dfrac{x^2+4x+3}{(x+3)(x-2)}, \dfrac{x^2+4x+3}{x^2+x-6},$ OR $\dfrac{x^3+7x^2+15x+9}{(x+3)(x^2+x-6)} \rightarrow \dfrac{(x+1)(x+3)(x+3)}{(x+3)(x^2+x-6)}$

As in question one they can score this mark having 'invisible' brackets on line 1.

A1*   Further factorises and cancels (which may be implied) to complete the proof to reach the given

answer $= \dfrac{(x+1)}{(x-2)}$. All aspects including bracketing must be correct. If a cubic is formed then it needs

to be correct.

(b)

B1   States either end of the range. Accept either $y < 4,\ y \leqslant 4$ or $y > 1,\ y \geqslant 1$ with or without the y's.

B1   Correct range. Accept $1 < y < 4,\ 1 < g < 4,\ y > 1$ and $y < 4,\ (1, 4),\ 1 < \text{Range} < 4,$ even $1 < f < 4,$

Do not accept $1 < x < 4,\ 1 < y \leqslant 4,\ [1, 4)$ etc.

Special case, allow B1B0 for $1 < x < 4$

(c)

M1   Attempting to set $g(x) = x,\ g^{-1}(x) = x$ or $g(x) = g^{-1}(x)$ or $g^2(x) = x$.

If $g^{-1}(x)$ has been used then a full attempt must have been made to make x the subject of the formula. A full attempt would involve cross multiplying, collecting terms, factorising and ending with division.

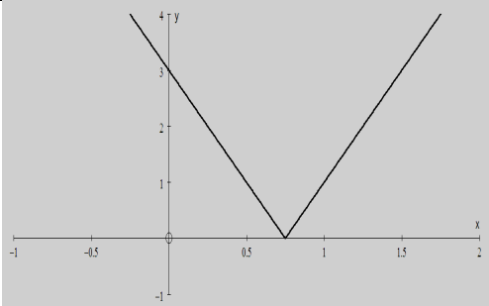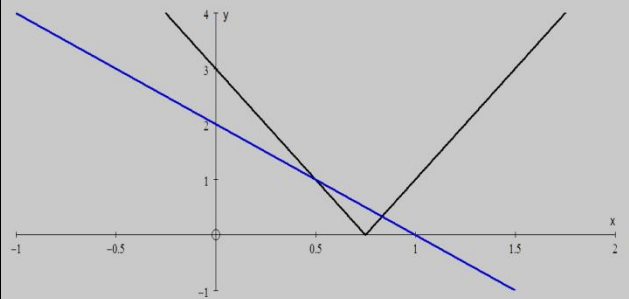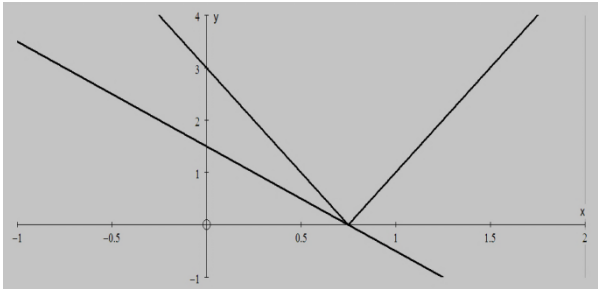As a result, it must be in the form $g^{-1}(x) = \dfrac{\pm 2x \pm 1}{\pm x \pm 1}$

Accept as evidence $\dfrac{(x+1)}{(x-2)} = x$ OR $\dfrac{x+1}{x-2} = \dfrac{\pm 2x \pm 1}{\pm x \pm 1}$ OR $\dfrac{\pm 2x \pm 1}{\pm x \pm 1} = x$ OR $\dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2} = x$

A1 $x^2 - 3x - 1 = 0$ or exact equivalent. The $=0$ may be implied by subsequent work.

dM1   For solving a 3TQ=0. It is dependent upon the first M being scored. Do not accept a method using factors unless it clearly factorises. Allow the answer written down awrt 3.30 (from a graphical calculator).

A1 a or $x = \dfrac{3 + \sqrt{13}}{2}$. Ignore any reference to $\dfrac{3 - \sqrt{13}}{2}$

Question 7

Question Number	Scheme	Marks
7. (a)	V shaped graph Touches x axis at ¾ and cuts y axis at 3	B1 B1 (2)
(b)	Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x Both $x=\dfrac{5}{6}$ and $x=\dfrac{1}{2}$ or $x>\dfrac{5}{6}$ or $x<\dfrac{1}{2}$ $x<\dfrac{1}{2}$ or $x>\dfrac{5}{6}$	M1 A1 dM1A1 (4)
(c)	Draws graph Or solves $\lvert 4x-3\rvert=1\frac{1}{2}-2x$ to give one soln $x=$ ¾ Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\mathbb{R},)$ $x\neq\frac{3}{4}$, or $x<\frac{3}{4}, x>\frac{3}{4}$	M1 A1 (2) (8 marks)

(a)

B1 A 'V' shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a 'dotted' extension of $y = 4x - 3$ appearing under the x axis.

B1 The graph **meets** the x axis at $x = \dfrac{3}{4}$ and **crosses** the y axis at $y = 3$. Do not allow multiple meets or crosses

If they have lost the previous B1 mark for an extra section of graph underneath the x axis allow for **crossing the** x axis at $x = \dfrac{3}{4}$ and **crosses the** y axis at $y = 3$.

Accept marked elsewhere on the page with A and B marked on the graph and $A = \left(\dfrac{3}{4}, 0\right)$ and $B = (0, 3)$

Condone $\left(0, \dfrac{3}{4}\right)$ and $(3, 0)$ marked on the correct axis

(b)

M1 Attempts to solve $|4x - 3| \ldots 2 - 2x$ finding at least one solution. You may see … replaced by either $=$ or $>$
Accept as evidence $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$
Accept as evidence $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..,$ or $x < ..$

A1 Both critical values $x = \dfrac{5}{6}$ and $x = \dfrac{1}{2}$, or one inequality, accept $x > \dfrac{5}{6}$ or $x < \dfrac{1}{2}$

Accept $x = 0.83$ and $x = 0.5$ for the critical values
Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.
Eg if M1 has been scored for $4x - 3 = 2 - 2x \Rightarrow x = 0.83$ and $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$
A correct application of M1 would be $x < -2.5, x > 0.83$

A1 Correct answer only $x < \dfrac{1}{2}$ or $x > \dfrac{5}{6}$.

Accept $x < 0.5, x > 0.8\dot{3}$

(c)

M1 Either sketch both lines showing a single intersection at the point $x = \dfrac{3}{4}$

Or solves $|4x - 3| = 1\tfrac{1}{2} - 2x$ using both $4x - 3 = 1\tfrac{1}{2} - 2x$ and $-4x + 3 = 1\tfrac{1}{2} - 2x$ **giving one solution** $x = \dfrac{3}{4}$

Accept $|4x - 3| > 1\tfrac{1}{2} - 2x$ using both $4x - 3 > 1\tfrac{1}{2} - 2x$ and $-4x + 3 > 1\tfrac{1}{2} - 2x$ **giving one solution** $x \ldots \dfrac{3}{4}$

If two values are obtained using either method it is M0A0

A1 States that the solution set is all values apart from $x = \dfrac{3}{4}$. Do not isw in this question. Score their final statement. Accept versions of all values of x except $x = \tfrac{3}{4}$ or $x \in \mathbb{R}$, $x \neq \tfrac{3}{4}$, or $x < \tfrac{3}{4}, x > \tfrac{3}{4}$

Question 8

Question Number	Scheme	Marks
8.(a)	$f(x) > k^2$	B1
		(1)
(b)	$y = e^{2x} + k^2 \Rightarrow e^{2x} = y - k^2$	M1
	$\Rightarrow x = \dfrac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow f^{-1}(x) = \dfrac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = e^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\dfrac{e}{\sqrt[6]{8}}\right) = \dfrac{e}{\sqrt{2}}$ (Ignore any reference to $-\dfrac{e}{\sqrt{2}}$)	A1
		(4)
(d)	$fg(x) = e^{2 \times \ln(2x)} + k^2$	M1
	$\Rightarrow fg(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$fg(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \dfrac{k}{2}$ **only**	A1
		(2)
		12 marks

Question 9

Question	Scheme	Marks
(a)	$\text{fg}(x) = \dfrac{28}{x-2} - 1$ $\left(= \dfrac{30-x}{x-2}\right)$	M1
	Sets $\text{fg}(x) = x \Rightarrow \dfrac{28}{x-2} - 1 = x$	
	$\Rightarrow 28 = (x+1)(x-2)$	
	$\Rightarrow x^2 - x - 30 = 0$	M1
	$\Rightarrow (x-6)(x+5) = 0$	
	$\Rightarrow x = 6, \ x = -5$	dM1 A1
		(4)
(b)	$a = 6$	B1 ft
		(1)
		5 marks
Alt 1(a)	$\text{fg}(x) = x \Rightarrow g(x) = f^{-1}(x)$	
	$\dfrac{4}{x-2} = \dfrac{x+1}{7}$	M1
	$\Rightarrow x^2 - x - 30 = 0$	M1
	$\Rightarrow (x-6)(x+5) = 0$	
	$\Rightarrow x = 6, \ x = -5$	dM1 A1
		4 marks
S. Case	Uses $gf(x)$ instead $\text{fg}(x)$ $\dfrac{4}{7x-1-2} = x$ $\Rightarrow 7x^2 - 3x - 4 = 0$ $\Rightarrow (7x+4)(x-1) = 0$ $\Rightarrow x = -\dfrac{4}{7}, \ x = 1$ Makes an error on $\text{fg}(x)$ Sets $\text{fg}(x) = x \Rightarrow \dfrac{7\times4}{7\times(x-2)} - 1 = x$ $\Rightarrow x^2 - x - 6 = 0$ $\Rightarrow (x+2)(x-3) = 0$ $\Rightarrow x = -2, \ x = 3$	M0 M1 dM1 A0 2 out of 4 marks

(a)

M1   Sets or implies that $\text{fg}(x) = \dfrac{28}{x-2} - 1$ Eg accept $\text{fg}(x) = 7\left(\dfrac{4}{x-2}\right) - 1$ followed by $\text{fg}(x) = \dfrac{7\times4}{x-2} - 1$

     Alternatively sets $g(x) = f^{-1}(x)$ where $f^{-1}(x) = \dfrac{x\pm1}{7}$

     Note that $\text{fg}(x) = 7\left(\dfrac{4}{x-2}\right) - 1 = \dfrac{28}{7(x-2)} - 1$ is M0

M1   Sets up a 3TQ $(= 0)$ from an attempt at $\text{fg}(x) = x$ or $g(x) = f^{-1}(x)$

dM1   Method of solving 3TQ $(= 0)$ to find at least one value for x. See "General Priciples for Core
     Mathematics" on page 3 for the award of the mark for solving quadratic equations
     This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1   Both $x = 6$ and $x = -5$

(b)

B1ft   For $a = 6$ but you may follow through on the largest solution from part (a) provided more than one
     answer was found in (a). Accept 6, $a = 6$ and even $x = 6$
     Do not award marks for part (a) for work in part (b).

Question 10

Question Number	Scheme	Marks
(a)	$y \geq 3$	B1
		(1)
(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x..3$	A1
		(3)
(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7 + \sqrt{21}}{2} \text{ only}$	M1, A1
		(4)
(d)	$a = \dfrac{7 + \sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x) = x \Rightarrow (x-3)^2 - 2 = x$	
	$\Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7 + \sqrt{21}}{2} \text{ only}$	dM1, A1
		(4)

(a)

B1   States the correct range for g Accept $g(x)..33g..3$, Range $..3, [3, \infty)$ Range is greater than or equal to 3

Condone f $..3$ Do not accept $g(x) > 3, x..3, (3, \infty)$

(b)

M1   Attempts to make x or a swapped y the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2} = y \pm 3 \Rightarrow x + 2 = y^2 \pm 9$

A1   Achieves $x = (y-3)^2 - 2$ or if swapped $y = (x-3)^2 - 2$ or equivalent such as $x = y^2 - 6y + 7$

A1   Requires a correct function in x + correct domain or a correct function in x with a correct follow through on the range in (a) but do not follow through on $x \in \mathbb{R}$

Accept for example $g^{-1}(x)=(x-3)^2-2, \ x..3$ Condone $f^{-1}(x)=(x-3)^2-2, \ x..3$

or variations such as $y=(x-3)^2-2, \ x>3$ if (a) was $y>3$

Accept expanded versions such as $g^{-1}(x)=x^2-6x+7, \ x..3$ but remember to isw after a correct answer

(Condone $f^{-1}(x)=x^2-6x+7, \ x..3$)

(c)

M1   Sets $3+\sqrt{x+2}=x$, moves the 3 over and then attempts to square both sides.

Can be scored for $\sqrt{x+2}=x-3 \Rightarrow x+2=x^2 \pm 9$

A1 $x^2-7x+7=0$. The $=0$ may be implied by subsequent working

M1   Correct method of solving their 3TQ by the formula/ completing the square. The equation must have real roots.

It is dependent upon them having attempted to set $3+\sqrt{x+2}=x$ and proceeding to a quadratic.

You may just see both roots written down which is fine.

Allow for this mark decimal answers Eg 5.79 and 1.21 for $x^2-7x+7=0$ You may need to check with a calc.

A1 $(x)=\dfrac{7+\sqrt{21}}{2}$ or exact equivalent **only**.

This answer following the correct quadratic would imply the previous M

Allow $x=\dfrac{7}{2}+\sqrt{\dfrac{21}{4}}$ but **DO NOT** allow $x=\dfrac{7\pm\sqrt{21}}{2}$

(c) can of course be attempted by solving

$$3+\sqrt{x+2}="(x-3)^2-2" \Rightarrow x^4-12x^3+44x^2-49x+14=0$$
$$\vdots \qquad\qquad \Rightarrow (x^2-7x+7)(x^2-5x+2)=0$$

The scheme can be applied to this

(d)

B1ft $(a)=\dfrac{7+\sqrt{21}}{2}$ oe . You may condone $x=\dfrac{7+\sqrt{21}}{2}$. You may allow this following a re - start.

You may allow the correct decimal answer, awrt 5.79, following exact/decimal work in part (c) or a restart.

Follow through on their root, including decimals, coming from the **positive** root with the **positive** sign in (c).
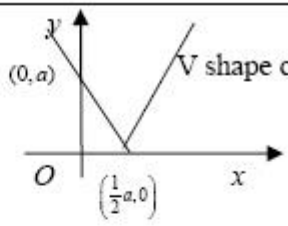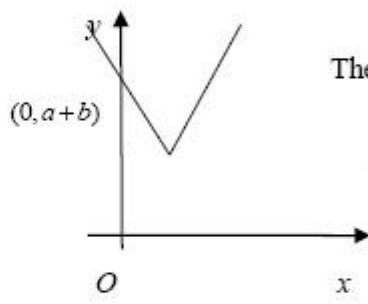
Eg In (c) . $x^2-7x+11=0 \Rightarrow x=\dfrac{7\pm\sqrt{5}}{2}$ So the correct follow through would be $x=\dfrac{7+\sqrt{5}}{2}$

If they only had one root in (c) then follow through on this as long as it is positive.

SC. If they give the correct roots in parts (c) and (d) without considering the correct answer then award B1 in (d)

following the A0 in (c). So $(x)=\dfrac{7\pm\sqrt{21}}{2}$ as their answer in part (c), allow $(x/a)=\dfrac{7\pm\sqrt{21}}{2}$ for B1 in (d).

Question 11

Question Number	Scheme	Marks
(a)(i)	V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ Correct shape, position and coordinates	B1 B1
(ii)	Their "V" shape translated up or $(0,a+b)$ Correct shape, position and $(0,a+b)$	B1ft B1 **(4)**
(b)	States or uses $a+b=8$ Attempts to solve $\|2x-a\|+b=\frac{3}{2}x+8$ in either x or with $x=c$ $2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$ Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	B1 M1 dM1 A1 **(4)** **(8 marks)**

(a)(i)

B1   V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ lying on the curve.

Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1   V shape sitting on the positive x-axis at $\left(\frac{1}{2}a,0\right)$, cutting the y-axis at $(0,a)$ and lying in both quadrants 1 and 2

Accept $\frac{1}{2}a$ and a marked on the correct axis. Condone say $(a,0)$ for $(0,a)$ as long as it is on the correct axis.

Condone a dotted line appearing on the diagram as many reflect $y=2x-a$ to sketch $y=|2x-a|$

If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft   Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.

Alternatively score for the $(0,a+b)$ lying on the curve

B1   V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0,a+b)$

Ignore any coordinates given for the vertex.

(b)

B1   States or uses $a+b=8$ or exact equivalent. Condone use of capital letters throughout

It is not scored for just $|0-a|+b=8$

M1   This M is for an understanding of the modulus.

It is scored for an attempt at solving $(2x-a)+b=\frac{3}{2}x+8$ or $-(2x-a)+b=\frac{3}{2}x+8$ in either x or with x

replaced by c. The signs of the $2x$ and the a must be different. $|2x-a|\neq 2x+a$

You may see $(2x-a)+b=\frac{3}{2}x+8\Rightarrow kx=\mathrm{f}(a,b)$

You may see $-2x+a+b=\frac{3}{2}x+8\Rightarrow kx=\mathrm{f}(a,b)$

You may see $(2x-a)+b=\frac{3}{2}x+8\Rightarrow kx=\mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

You may see $-2c+a+b=\frac{3}{2}c+8\Rightarrow kc=\mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

dM1   This dM mark is scored for combining $b=8-a$ with $(2x-a)+b=\frac{3}{2}x+8$ (or their $kx=\mathrm{f}(a,b)$ resulting from

that equation) resulting in a link between x and a **Both equations must have been correct initially.**

Alternatively for combining $b=8-a$ with their $2c-a+b=\frac{3}{2}c+8$ (or their $kc=\mathrm{f}(a,b)$ resulting from that

equation) resulting in a link between c and a

You may condone sign slips in finding the link between x (or c) and a

If you see an approach that involves making $|2x-a|$ the subject followed by squaring, and you feel that it

deserves credit, please send to review. The solution proceeds as follows

Look for $|2x-a|=\frac{3}{2}x+8-b\Rightarrow|2x-a|=\frac{3}{2}x+a\Rightarrow(2x-a)^2=\left(\frac{3}{2}x+a\right)^2\Rightarrow 7x\left(\frac{1}{4}x-a\right)=0$

A1 $c=4a$ ONLY

....

Special Case where they have the roots linked with the incorrect branch of the curve.

They have $x=0$ as the solution to $2x-a+b=\frac{3}{2}x+8\Rightarrow -a+b=8$...................(1)

They have $x=c$ as the solution to $-2x+a+b=\frac{3}{2}x+8\Rightarrow\frac{7}{2}x=a+b-8$...........(2)

Solve (1) and (2) $\Rightarrow x=\frac{4}{7}a$

Hence $\Rightarrow c=\frac{4}{7}a$

This would score B0 M1 dM0 A0 anyway but should be awarded SC B0, M1 dM1, A0 for above

work leading to either $x=\frac{4}{7}a$ or $c=\frac{4}{7}a$