Numerical Methods: Iteration 2 - Edexcel Past Exam Questions

1. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \qquad 0 \le x \le \pi.$$

(*a*) Show that the equation f(*x*) = 0 has a solution in the interval $0.8 < x < 0.9$. **(2)**

The curve with equation $y = f(x)$ has a minimum point *P*.

(*b*) Show that the *x*-coordinate of *P* is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}.$$ **(4)**

(*c*) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \qquad x_0 = 2,$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places. **(3)**

(*d*) By choosing a suitable interval, show that the *x*-coordinate of *P* is 1.9078 correct to 4 decimal places. **(3)**

Jan 12 Q6

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2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation f(*x*) = 0 can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \ne -3.$$ **(3)**

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \qquad n \ge 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3. **(3)**

The root of f(*x*) = 0 is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places. **(3)**

June 12 Q2

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3. $$\mathrm{g}(x) = \mathrm{e}^{x-1} + x - 6$$

(a) Show that the equation $\mathrm{g}(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of $\mathrm{g}(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places. **(3)**

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places. **(3)**

Jan 13 Q2

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4. $$\mathrm{f}(x) = 25x^2\mathrm{e}^{2x} - 16, \qquad x \in \mathbb{R}.$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = \mathrm{f}(x)$.

(5)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written as $x = \pm\frac{4}{5}\mathrm{e}^{-x}$.

(1)

The equation $\mathrm{f}(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}\mathrm{e}^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

June 13 Q4

5.

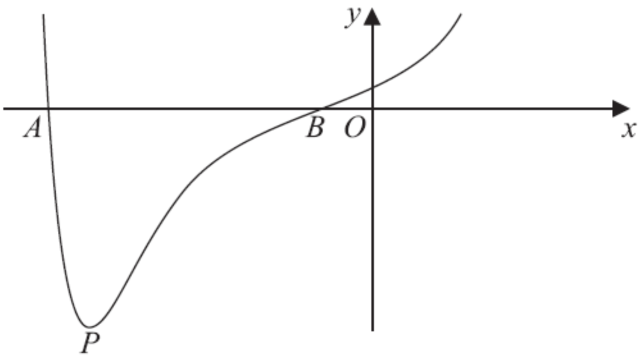

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x-coordinate of A and the x-coordinate of B, giving your answers to 3 decimal places. **(2)**

(b) Find $f'(x)$. **(3)**

The curve has a minimum turning point P as shown in Figure 2.

(c) Show that the x-coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$ **(3)**

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places. **(3)**

The x-coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places. **(2)**

June 13(R) Q7

6.

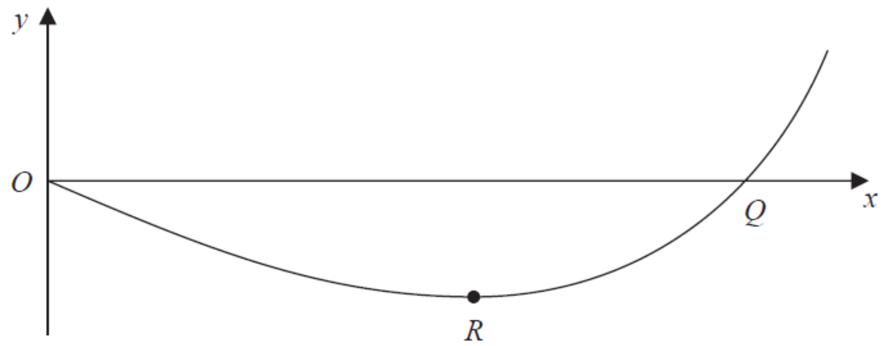

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(*a*) Show that the x coordinate of Q lies between 2.1 and 2.2. **(2)**

(*b*) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$ **(4)**

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n{}^2\right)}, \qquad x_0 = 1.3$$

(*c*) find the values of x_1 and x_2 to 3 decimal places. **(2)**

June 14 Q6

7. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$.

(a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$ **(3)**

(b) On a pair of axes, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes. **(4)**

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root. **(1)**

The iteration formula

$$x_{n+1} = \left(-2 - e^{4x_n}\right)^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

(d) Calculate the values of x_1 and x_2, giving your answers to 5 decimal places. **(2)**

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C. **(2)**

June 14(R) Q6

8.

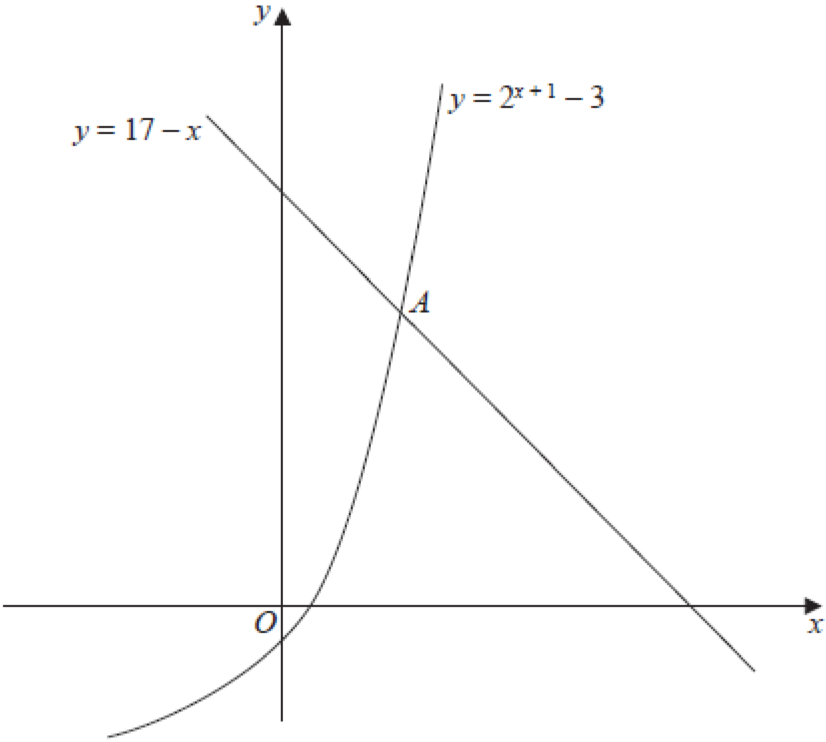

Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(a) Show that the x-coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1.$$ **(3)**

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \qquad x_0 = 3,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places. **(3)**

(c) Use your answer to part (b) to deduce the coordinates of the point A, giving your answers to one decimal place. **(2)**

June 15 Q6

9.

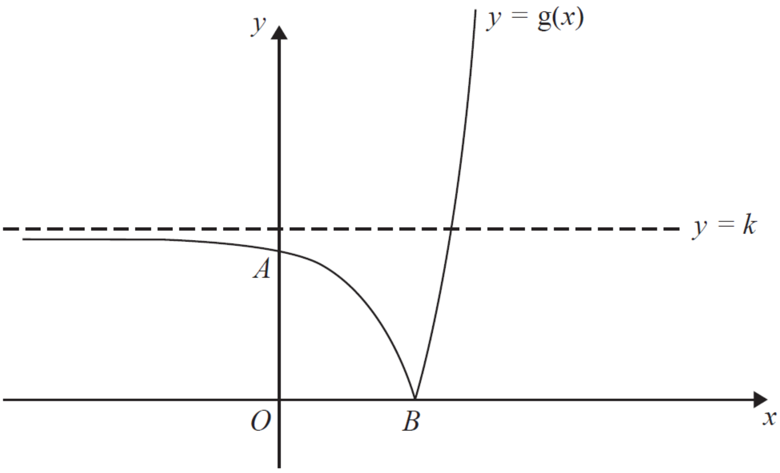

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}.$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

(*a*) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k. **(5)**

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$.

(*b*) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$. **(2)**

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α.

(*c*) Taking $x_0 = 1.4$, find the values of x_1 and x_2. Give each answer to 4 decimal places. **(2)**

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places. **(2)**

June 16 Q4

10.

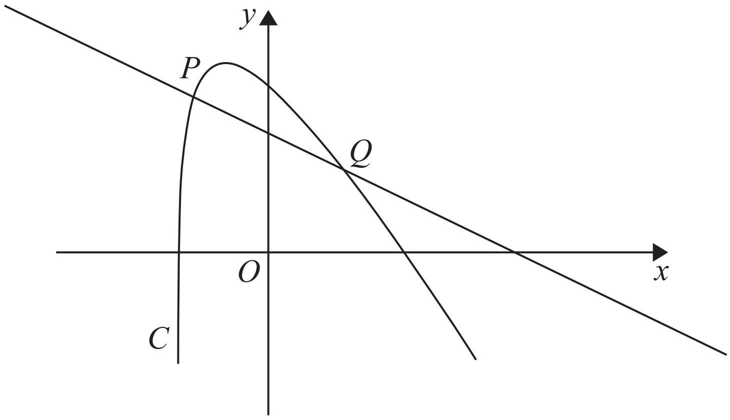

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers. **(5)**

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$ **(3)**

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places. **(2)**

June 17 Q5