Numerical Methods: Iterations - Edexcel Past Exam Questions

1. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$. **(3)**

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$. **(2)**

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places. **(2)**

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places. **(2)**

June 05 Q4

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2. $$f(x) = 2x^3 - x - 4.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$ **(3)**

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3. **(3)**

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places. **(3)**

Jan 06 Q5

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3. **Figure 2**

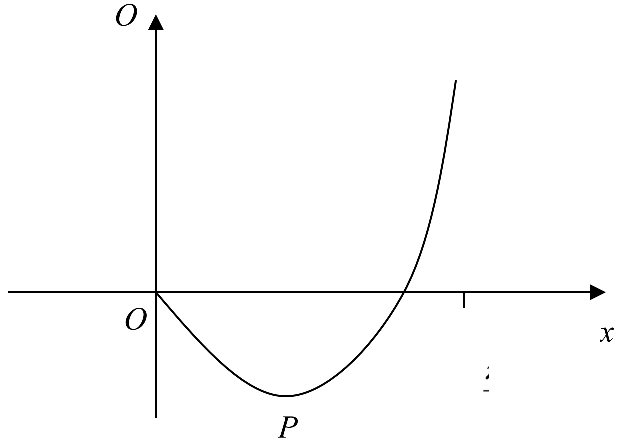

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \tfrac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimals places. **(3)**

(c) Show that $k = 0.277$, correct to 3 significant figures. **(2)**

June 06 Q5

4. The function f is defined by

$$f : x \mapsto \ln (4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \tfrac{1}{2} e^{x}$$

and write down the domain of f^{-1}. **(4)**

(*b*) Write down the range of f^{-1}. **(1)**

(*c*) Sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes. **(4)**

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\tfrac{1}{2} e^{x_n}, \quad x_0 = -0.3,$$

is used to find an approximate value for k.

(*d*) Calculate the values of x_1 and x_2, giving your answer to 4 decimal places. **(2)**

(*e*) Find the values of k to 3 decimal places. **(2)**

Jan 07 Q6

5. $\mathrm{f}(x) = -x^3 + 3x^2 - 1.$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(2)

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(c) Show that $x = 0.653$ is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places. **(3)**

June 07 Q4

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6. $\mathrm{f}(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$

(a) Show that there is a root of $\mathrm{f}(x) = 0$ in the interval $2 < x < 3$. **(2)**

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places. **(3)**

(c) Show that $x = 2.505$ is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places. **(2)**

Jan 08 Q3

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7. $\mathrm{f}(x) = 3x^3 - 2x - 6.$

(a) Show that f (x) = 0 has a root, α, between $x = 1.4$ and $x = 1.45$.
(2)

(b) Show that the equation f (x) = 0 can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places. **(3)**

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places. **(3)**

June 08 Q7

8. $$f(x) = 3xe^x - 1.$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P. **(5)**

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}.$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3. **(3)**

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places. **(3)**

Jan 09 Q7

9.

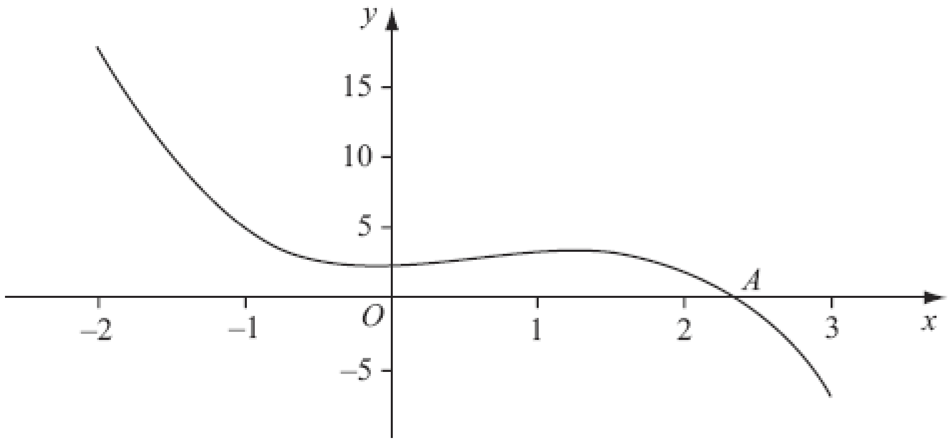

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate. **(3)**

(b) Show that $\alpha = 2.359$ correct to 3 decimal places. **(3)**

June 09 Q1

10. $f(x) = 4 \operatorname{cosec} x - 4x + 1$, where x is in radians.

(*a*) Show that there is a root α of $f(x) = 0$ in the interval [1.2, 1.3]. **(2)**

(*b*) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places. **(3)**

(*d*) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places. **(2)**

June 10 Q3

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11. $f(x) = x^3 + 2x^2 - 3x - 11$

(*a*) Show that $f(x) = 0$ can be rearranged as **(2)**

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

The equation $f(x) = 0$ has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$ is used to find an approximation to α.

(*b*) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4. **(3)**

(*c*) Show that $\alpha = 2.057$ correct to 3 decimal places. **(3)**

Jan 10 Q2

12.

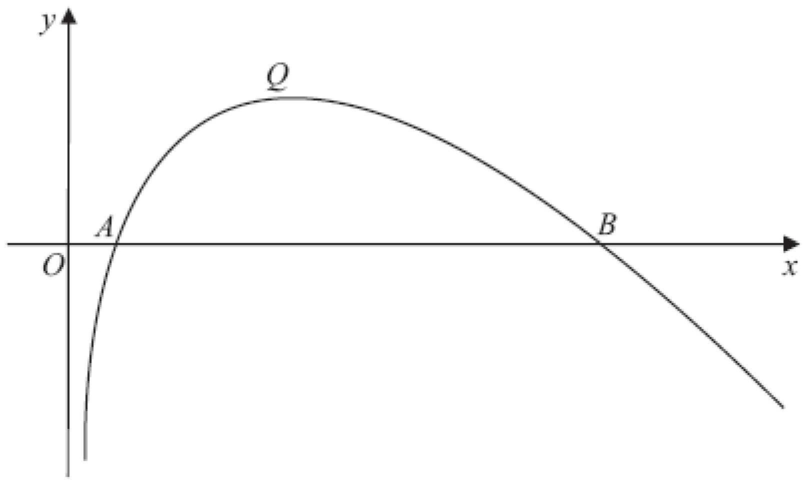

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0.$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B. **(2)**

(b) Find $\mathrm{f}'(x)$ **(3)**

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6 **(2)**

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}.$$ **(3)**

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places. **(3)**

Jan 11 Q5

13. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \le x < 2\pi.$$

(*a*) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$. **(2)**

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(*b*) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places. **(3)**

(*c*) Show that $\alpha = 0.80157$ is correct to 5 decimal places. **(3)**

June 11 Q2