## Recurrence Relations 2 - Edexcel Past Exam Questions

1. A sequence $x_{1}, x_{2}, x_{3}, \ldots$ is defined by

$$
\begin{aligned}
& x_{1}=1, \\
& x_{n+1}=a x_{n}+5, \quad n \geq 1,
\end{aligned}
$$

where $a$ is a constant.
(a) Write down an expression for $x_{2}$ in terms of $a$.
(b) Show that $x_{3}=a^{2}+5 a+5$.

Given that $x_{3}=41$
(c) find the possible values of $a$.
2. A sequence of numbers $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{aligned}
& a_{1}=3 \\
& a_{n+1}=2 a_{n}-c, \quad(n \geq 1),
\end{aligned}
$$

where $c$ is a constant.
(a) Write down an expression, in terms of $c$, for $a_{2}$.
(b) Show that $a_{3}=12-3 c$.

Given that $\sum_{i=1}^{4} a_{i} \geq 23$,
(c) find the range of values of $c$.
3. A sequence $u_{1}, u_{2}, u_{3}, \ldots$, satisfies

$$
u_{n+1}=2 u_{n}-1, \quad n \geq 1 .
$$

Given that $u_{2}=9$,
(a) find the value of $u_{3}$ and the value of $u_{4}$,
(b) evaluate $\sum_{r=1}^{4} u_{r}$.
4. A sequence $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{aligned}
& a_{1}=4, \\
& a_{n+1}=k\left(a_{n}+2\right), \quad \text { for } n \geq 1
\end{aligned}
$$

where $k$ is a constant.
(a) Find an expression for $a_{2}$ in terms of $k$.

Given that $\sum_{i=1}^{3} a_{i}=2$,
(b) find the two possible values of $k$.
5. A sequence $x_{1}, x_{2}, x_{3}, \ldots$ is defined by

$$
\begin{aligned}
& x_{1}=1, \\
& x_{n+1}=\left(x_{n}\right)^{2}-k x_{n}, \quad n \geq 1,
\end{aligned}
$$

where $k$ is a constant.
(a) Find an expression for $x_{2}$ in terms of $k$.
(b) Show that $x_{3}=1-3 k+2 k^{2}$.

Given also that $x_{3}=1$,
(c) calculate the value of $k$.
(d) Hence find the value of $\sum_{n=1}^{100} x_{n}$.

June 13(R) Q6
6. A sequence of numbers $a_{1}, a_{2}, a_{3} \ldots$ is defined by

$$
a_{n+1}=5 a_{n}-3, \quad n \geq 1 .
$$

Given that $a_{2}=7$,
(a) find the value of $a_{1}$.
(b) Find the value of $\sum_{r=1}^{4} a_{r}$.
7. A sequence $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{array}{ll}
a_{n+1}=4 a_{n}-3, & n \geq 1 \\
a_{1}=k, & \text { where } k \text { is a positive integer. }
\end{array}
$$

(a) Write down an expression for $a_{2}$ in terms of $k$.

Given that $\sum_{r=1}^{3} a_{r}=66$
(b) find the value of $k$.
8. (i) A sequence $U_{1}, U_{2}, U_{3}, \ldots$ is defined by

$$
\begin{gathered}
U_{n+2}=2 U_{n+1}-U_{n}, \quad n \geq 1, \\
U_{1}=4 \text { and } U_{2}=4 .
\end{gathered}
$$

Find the value of
(a) $U_{3}$,
(b) $\sum_{n=1}^{20} U_{n}$.
(2)
(ii) Another sequence $V_{1}, V_{2}, V_{3}, \ldots$ is defined by

$$
V_{n+2}=2 V_{n+1}-V_{n}, \quad n \geq 1,
$$

$V_{1}=k$ and $V_{2}=2 k$, where $k$ is a constant.
(a) Find $V_{3}$ and $V_{4}$ in terms of $k$.

Given that $\sum_{n=1}^{5} V_{n}=165$,
(b) find the value of $k$.
9. A sequence $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{aligned}
a_{1} & =4 \\
a_{n+1} & =5-k a_{n}, \quad n \geq 1,
\end{aligned}
$$

where $k$ is a constant.
(a) Write down expressions for $a_{2}$ and $a_{3}$ in terms of $k$.

Find
(b) $\sum_{r=1}^{3}\left(1+a_{r}\right)$ in terms of $k$, giving your answer in its simplest form,
(c) $\sum_{r=1}^{100}\left(a_{r+1}+k a_{r}\right)$.
10. A sequence $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{aligned}
a_{1} & =1 \\
a_{n+1} & =\frac{k\left(a_{n}+1\right)}{a_{n}}, \quad n \geqslant 1
\end{aligned}
$$

where $k$ is a positive constant.
(a) Write down expressions for $a_{2}$ and $a_{3}$ in terms of $k$, giving your answers in their simplest form.

Given that $\sum_{r=1}^{3} a_{r}=10$
(b) find an exact value for $k$.

