Recurrence Relations 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question	Scheme	Marks
(a)	$(x_2 =)\ a + 5$	B1 (1)
(b)	$(x_3) = a"(a+5)"+5$	M1
	$= a^2 + 5a + 5$ (*)	A1cso (2)
(c)	$41 = a^2 + 5a + 5 \Rightarrow a^2 + 5a - 36 (=0)$ or $36 = a^2 + 5a$	M1
	$(a+9)(a-4) = 0$	M1
	$a = 4$ or -9	A1 (3)
		6 marks
	Notes	
(a)	B1 accept $a1 + 5$ or $1 \times a + 5$ (etc)	
(b)	M1 must see $a($ their $x_2) + 5$	
	A1cso must have seen $a(a[1] + 5) + 5$ (etc or better) Must have both brackets (....) and no incorrect working seen	
(c)	1st M1 for forming a suitable equation using x_3 and 41 and an attempt to collect like terms and reduce to 3TQ (o.e). Allow one error in sign. Accept for example $a^2 + 5a + 46 (=0)$	
	If completing the square should get to $\left(a \pm \frac{5}{2}\right)^2 = 36 + \frac{25}{4}$	
	2nd M1 Attempting to solve their relevant 3TQ (see **General Principles**)	
	A1 for both 4 and -9 seen. If $a = 4$ and -9 is followed by $-9 < a < 4$ apply ISW. No working or trial and improvement leading to both answers scores 3/3 but no marks for only one answer.	
	Allow use of other letters instead of a	

Question 2

Question Number	Scheme	Marks
	$a_1 = 3,\ a_{n+1} = 2a_n - c,\ \ n \geq 1,\ c$ is a constant	
(a)	$\{a_2 =\}\ 2\times 3 - c$ or $2(3) - c$ or $6 - c$	B1
		[1]
(b)	$\{a_3 =\}\ 2\times(\text{"}6-c\text{"}) - c$	M1
	$= 12 - 3c$ (*)	A1 cso
		[2]
(c)	$a_4 = 2\times(\text{"}12-3c\text{"}) - c \qquad \{= 24 - 7c\}$	M1
	$\left\{\sum_{i=1}^{4} a_i =\right\}\ 3 + (6-c) + (12-3c) + (24-7c)$	M1
	$\text{"}45 - 11c\text{"} \geq 23$ or $\text{"}45 - 11c\text{"} = 23$	M1
	$c \leq 2$ or $2 \geq c$	A1 cso
		[4]
		7

	Notes
(a)	The answer to part (a) cannot be recovered from candidate's working in part (b) or part (c). Once the candidate has achieved the correct result you can ignore subsequent working in this part.
(b)	**M1:** For a correct substitution of *their* a_2 *which must include term(s) in c* into $2a_2 - c$ giving a result for a_3 in terms of only c. Candidates must use correct bracketing for this mark. **A1:** for correct solution only. No incorrect working/statements seen. (**Note:** the answer is given!)
(c)	**1st M1:** For a correct substitution of a_3 *which must include term(s) in c* into $2a_3 - c$ giving a result for a_4 in terms of only c. Candidates must use correct bracketing (can be implied) for this mark. **2nd M1:** for an attempt to sum their a_1, a_2, a_3 and a_4 only. **3rd M1:** for their sum (of 3 or 4 or 5 consecutive terms) $=$ or $\geq$ or >23 to form a linear inequality or equation in c. **A1:** for $c \leq 2$ or $2 \geq c$ from a correct solution only. **Beware:** $-11c \geq -22 \Rightarrow c \geq 2$ is A0. **Note:** $45 - 11c \geq 23 \Rightarrow -11c \leq -22 \Rightarrow c \leq 2$ would be A0 cso. **Note:** Applying either $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ is 2nd M0, 3rd M0. **Note:** If a candidate gives a numerical answer in part (a); they will then get M0A0 in part (b); but if they use the printed result of $a_3 = 12 - 3c$ they could potentially get M0M1M1A0 in part (c) **Note:** If a candidate only adds numerical values (not in terms of c) in part (c) then they could potentially get only M0M0M1A0. **Note:** For the 3rd M1 candidates will usually sum a_1, a_2, a_3 and a_4 or a_2, a_3 and a_4 or a_2, a_3, a_4 and a_5 or a_1, a_2, a_3, a_4 and a_5

Question 3

Question Number	Scheme		Marks
	For this question, mark (a) and (b) together and ignore labelling.		
(a)	$(a_2 =)\ k(4+2)\ \ (= 6k)$	Any correct (possibly un-simplified) expression	B1
			(1)
(b)	$a_3 = k(\text{their } a_2 + 2)\ \ (=6k^2 + 2k)$	An attempt at a_3. Can follow through their answer to (a) but a_2 must be an expression in k.	M1
	$a_1 + a_2 + a_3 = 4 + (6k) + (6k^2 + 2k)$	An attempt to find their $a_1 + a_2 + a_3$	M1
	$4 + (6k) + (6k^2 + 2k) = 2$	A **correct** equation in any form.	A1
	Solves $6k^2 + 8k + 2 = 0$ to obtain $k =$ $(6k^2 + 8k + 2 = 2(3k+1)(k+1))$	Solves their 3TQ as far as $k =$ .... according to the general principles. (An independent mark for solving their three term quadratic)	M1
	$k = -1/3$	Any equivalent fraction	A1
	$k = -1$	Must be from a correct equation. (Do not accept un-simplified)	B1
	Note that it is quite common to think the sequence is an AP. Unless they find a_3, this is likely only to score the M1 for solving their quadratic.		
			(6)
			[7]

Question 4

Question Number	Scheme	Notes	Marks
(a)	$x_2 = 1 - k$	Accept un-simplified e.g. 1^2 - $1k$	B1
			(1)
(b)	$x_3 = (1-k)^2 - k(1-k)$	Attempt to substitute their x_2 into $x_3 = (x_2)^2 - kx_2$ with their x_2 in terms of k.	M1
	$= 1 - 3k + 2k^2$ *	Answer given	A1*
			(2)
(c)	$1 - 3k + 2k^2 = 1$	Setting $1 - 3k + 2k^2 = 1$	M1
	$\left(2k^2 - 3k = 0\right)$		
	$k(2k-3) = 0 \Rightarrow k = ..$	Solving their quadratic to obtain a value for k. **Dependent on the previous M1.**	dM1
	$k = \dfrac{3}{2}$	Cao and cso (ignore any reference to $k = 0$)	A1
			(3)
(d)	$\sum_{n=1}^{100} x_n = 1 + \left(-\dfrac{1}{2}\right) + 1 + \ldots\ldots$ Or $= 1 + \left(1 - 'k'\right) + 1 + \ldots\ldots$		M1
	Writing out at least 3 terms with the third term equal to the first term. Allow in terms of k as well as numerical values. Evidence that the sequence is oscillating between 1 and $1-k$. This may be implied by a correct sum.		
	$50 \times \dfrac{1}{2}$ or $50 \times 1 - 50 \times \dfrac{1}{2}$ or $\dfrac{1}{2} \times 50 \times (1 - \dfrac{1}{2})$	An attempt to combine the terms correctly. Can be in terms of k here e.g $100 - 50k$	M1
	$= 25$	Allow an equivalent fraction, e.g. 50/2 or 100/4	A1
	Note that the use of $\dfrac{1}{2}n(a+l)$ is acceptable here but $\dfrac{1}{2}n(2a+(n-1)d)$ is not.		
			(3)
	Allow correct answer only		
			[9]

Question 5

Question Number	Scheme	Marks
	(a) $7 = 5a_1 - 3 \quad \Rightarrow a_1 = ..$	M1
	$a_1 = 2$	A1
		(2)
	(b) $a_3 = "32"$ and $a_4 = "157"$	M1
	$\sum_{r=1}^{r=4} a_r = a_1 + a_2 + a_3 + a_4$	
	$=$ "2"+ "7"+ "32"+ "157"	dM1
	$= 198$	A1
		(3)
		(5 marks)

Notes

(a) M1 Writes $7 = 5a_1 - 3$ and attempts to solve leading to an answer for a_1. If they rearrange wrongly before any substitution this is M0

A1 Cao $a_1 = 2$

Special case: Substitutes $n = 1$ into $5n - 3$ and obtains answer 2. This is fortuitous and gets M0A0 but full marks are available on (b).

(b) M1 Attempts to find either their a_3 or their a_4 using $a_{n+1} = 5a_n - 3, \; a_2 = 7$
Needs clear attempt to use formula or is implied by correct answers or correct follow through of their a_3

dM1 (Depends on previous M mark) Sum of their four adjacent terms from the given sequence.
n.b May be given for 9 + a_3 + a_4 as they may add 2 + 7 to give 9
(dM0 for sum of an Arithmetic series)

A1 cao 198

Special case
(a) $a_1 = 32$ is M0 A0
(b) Adds for example 7+32+157 + 782 = or 32+157 + 782 + 3907 is M1 M1 A0
Total mark possible is 2 / 5
(This is not treated as a misread – as it changes the question)

Question 6

Question Number	Scheme	Marks
(a)	$(a_2 =)\quad 4k-3$	B1 (1)
(b)	$a_3 = 4(4k-3)-3$	M1
	$\sum_{r=1}^{3} a_r = k + 4k - 3 + 4(4k-3) - 3 = ..k \pm ...$	M1
	$21k - 18 = 66 \Rightarrow k = ...$	dM1
	$k = 4$	A1 (4) (5 marks)

(a) B1 $4k-3$ cao

(b) M1 An attempt to find a_3 from iterative formula $a_3 = 4a_2 - 3$. Condone bracketing errors for the M mark

M1 Attempt to sum their a_1, a_2 and a_3 to get a linear expression in k (Sum of Arithmetic series is M0)

dM1 Sets their linear expression to 66 and solves to find a value for k. It is dependent upon the previous M mark

A1 cao $k = 4$

Question 7

Question Number	Scheme		Marks
(i).(a)	$U_3 = 4$	cao	B1
			(1)
(b)	$\sum_{n=1}^{n=20} U_n = 4+4+4.........+4$ or 20×4	For realising that all 20 terms are 4 and that the sum is required. Possible ways are 4+4+4…..+4 or 20×4 or $\frac{1}{2}\times20(2\times4+19\times0)$ or $\frac{1}{2}\times20(4+4)$ (Use of a correct sum formula with $n = 20$, $a = 4$ and $d = 0$ or $n = 20$, $a = 4$ and $l = 4$)	M1
	$= 80$	cao	A1
	Correct answer with no working scores M1A1		
			(2)
(ii)(a)	$V_3 = 3k,\quad V_4 = 4k$	May score in (b) if clearly identified as V_3 and V_4	B1, B1
			(2)
(b)	$\sum_{n=1}^{n=5} V_n = k+2k+3k+4k+5k = 165$ or $\frac{1}{2}\times5(2\times k+4\times k)=165$ or $\frac{1}{2}\times5(k+5k)=165$	Attempts V_5, adds their V_1,V_2,V_3,V_4,V_5 AND sets equal to 165 or Use of a correct sum formula with $a = k$, $d = k$ and $n = 5$ or $a = k$, $l = 5k$ and $n = 5$ AND sets equal to 165	M1
	$15k = 165 \Rightarrow k = ..$	Attempts to solve their linear equation in k **having set the sum of their first 5 terms equal to 165**. Solving $V_5 = 165$ scores no marks.	M1
	$k = 11$	cao and cso	A1
			(3)
			(8 marks)

Question 8

Question Number	Scheme	Notes	Marks
	$a_1 = 4,\ a_{n+1} = 5 - ka_n\ ,\ n \ldots 1$		
(a)	$a_2 = 5 - ka_1 \ = 5 - 4k$ $a_3 = 5 - ka_2 \ = 5 - k(5 - 4k)$	M1: Uses the recurrence relation correctly at least once. This may be implied by $a_2 = 5 - 4k$ or by the use of $a_3 = 5 - k\left(\text{their } a_2\right)$	M1A1
		A1: Two correct expressions – need not be simplified but must be seen in (a). Allow $a_2 = 5 - k4$ and $a_3 = 5 - 5k + k^2 4$ Isw if necessary for a_3.	
			[2]
(b)	$\sum_{r=1}^{3}(1) = 1+1+1$	Finds 1+1+1 or 3 somewhere in their solution (may be implied by e.g. $5 + 6 - 4k + 6 - 5k + 4k^2$). Note that $5 + 6 - 4k + 6 - 5k + 4k^2$ would score B1 and the M1 below.	B1
	$\sum_{r=1}^{3} a_r = 4 + "5 - 4k" + "5 - 5k + 4k^2"$	Adds 4 to their a_2 and their a_3 where a_2 and a_3 are functions of k. The statement as shown is sufficient.	M1
	$\sum_{r=1}^{3}(1 + a_r) = 17 - 9k + 4k^2$	Cao but condone '= 0' after the expression	A1
	Allow full marks in (b) for correct answer only		
			[3]
(c)	500	cao	B1
			[1]
			6 marks

Question 9

Question Number	Scheme		Marks
(a)	$(a_2 =)2k$	$2k$ only	B1
	$(a_3 =)\dfrac{k(\text{"}2k\text{"}+1)}{\text{"}2k\text{"}}$	For substituting their a_2 into $a_3 = \dfrac{k(a_2+1)}{a_2}$ to find a_3 in terms of just k	M1
	$(a_3 =)\dfrac{2k+1}{2}$	$(a_3 =)\dfrac{2k+1}{2}$ or exact simplified equivalent such as $(a_3 =)k+\dfrac{1}{2}$ or $\dfrac{1}{2}(2k+1)$ but not $k+\dfrac{k}{2k}$ **Must be seen in (a)** but isw once a correct simplified answer is seen.	A1
			(3)
	Note that there are no marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).		
(b)	$\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+\text{"}2k\text{"}+\text{"}\dfrac{2k+1}{2}\text{"}=10$	Writes 1 + their a_2 + their a_3 = 10. E.g. $1+2k+\dfrac{2k^2+k}{2k}=10$. Must be a correct follow through equation in terms of k only.	M1
	$\Rightarrow 2+4k+2k+1=20 \Rightarrow k=\ldots$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=\ldots$	Solves their equation in k which has come from the sum of 3 terms = 10, and reaches $k=\ldots$ Condone poor algebra but if a quadratic is obtained then the usual rules apply for solving – see General Principles. (Note that it does not need to be a 3-term quadratic in this case)	M1
	$(k=)\dfrac{17}{6}$	$k=\dfrac{17}{6}$ or exact equivalent e.g. $2\dfrac{5}{6}$ Do **not** allow $k=\dfrac{8.5}{3}$ or $k=\dfrac{17/2}{3}$ Ignore any reference to $k=0$. Allow 2.83 recurring as long as the recurring is clearly indicated e.g. a dot over the 3.	A1
			(3)
			(6 marks)

Question 10

Question Number	Scheme		Marks
	$u_2 = 9,\ u_{n+1} = 2u_n - 1,\ n \geqslant 1$		
(a)	$u_3 = 2u_2 - 1 = 2(9) - 1 \quad (=17)$	$u_3 = 2(9) - 1.$ Can be implied by $u_3 = 17$	M1
	$u_4 = 2u_3 - 1 = 2(17) - 1 = 33$	Both $u_3 = 17$ and $u_4 = 33$	A1
			[2]
(b)	$\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$		
	$(u_1) = 5$	$(u_1) = 5$	B1
	$\sum_{r=1}^{4} u_r = "5" + 9 + "17" + "33" = 64$	Adds their first four terms obtained legitimately (see notes below)	M1
		64	A1
			[3]
			5 marks
	Notes		
(a)	**M1**: Substitutes 9 into RHS of iteration formula **A1**: Needs **both** 17 and 33 (but **allow if either or both seen in part (b)**)		
(b)	**B1: for** u_1=5 (however obtained – may appear in (a)) May be called *a=5* **M1:** Uses their u_1 found from $u_2 = 2u_1 - 1$ stated explicitly, or uses $u_1 = 4$ *or* $5\frac{1}{2}$, and adds it to u_2, their u_3 and their u_4 only. (See special cases below). There should be no fifth term included. Use of sum of AP is irrelevant and scores M0 **A1**: 64		