Solving Differential Equations 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme		Marks
(a)	$1 \equiv A(5-P)+BP$	Can be implied.	M1
	$A=\frac{1}{5},\ B=\frac{1}{5}$	Either one.	A1
	giving $\dfrac{\frac{1}{5}}{P}+\dfrac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\displaystyle\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t\ \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\ \ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \qquad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4+e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \dfrac{5e^{\frac{1}{3}t}}{(4+e^{\frac{1}{3}t})}\qquad \left\{\begin{matrix}(\times e^{-\frac{1}{3}t})\\ (\div e^{\frac{1}{3}t})\end{matrix}\right\}$	Make P the subject.	dM1*
	$P = \dfrac{5}{(1+4e^{-\frac{1}{3}t})}$ or $P = \dfrac{25}{(5+20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1+4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **12**

	<u>*Alternative method for part (b)*</u>
	B1M1*A1: as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t\ \ (+c)$
	Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$
	Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$
	Award 2nd M1 for $t=0,\ P=1 \Rightarrow \frac{1}{5-1} = Ae^{0}\ \ \left\{\Rightarrow\ A=\frac{1}{4}\right\}$
	$\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$
	then award the final M1A1 in the same way.

Question 2

Question Number	Scheme		Marks	
	$\int y\,dy = \int \frac{3}{\cos^2 x}\,dx$ $= \int 3\sec^2 x\,dx$	Can be implied. Ignore integral signs	B1	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	(5)
				[5]

Question 3

Question Number	Scheme		Marks
	$\left\{\frac{d\theta}{dt} = \frac{(3-\theta)}{125}\right\} \Rightarrow \int \frac{1}{3-\theta}\, d\theta = \int \frac{1}{125}\, dt$ or $\int \frac{125}{3-\theta}\, d\theta = \int dt$		B1
	$-\ln(\theta - 3) = \frac{1}{125}t \ \{+c\}$ or $-\ln(3-\theta) = \frac{1}{125}t \ \{+c\}$	See notes.	M1 A1
	$\ln(\theta - 3) = -\frac{1}{125}t + c$		
	$\theta - 3 = e^{-\frac{1}{125}t + c}$ or $e^{-\frac{1}{125}t}e^{c}$	***Correct*** completion to $\theta = Ae^{-0.008t} + 3$.	
	$\theta = Ae^{-0.008t} + 3$ *		A1
			[4]
(b)	$\{t = 0,\ \theta = 16 \Rightarrow\}$ $16 = Ae^{-0.008(0)} + 3$; $\Rightarrow \underline{A = 13}$	***See notes.***	M1; A1
	$10 = 13e^{-0.008t} + 3$	Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. ***See notes.***	M1
	$e^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right)$	Correct algebra to $-0.008t = \ln k$, where k is a positive value. **See *notes.***	M1
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799\ldots = 77$ (nearest minute)	awrt 77	A1
			[5]
			9

(b)

M1: (B1 on epen) Substitutes $\theta = 16$, $t = 0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark.

A1: (M1 on epen) $A = 13$. **Note:** $\theta = 13e^{-0.008t} + 3$ without any working implies the first two marks, M1A1.

M1: Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1.

M1: Uses correct algebra to rearrange **their equation** into the form $-0.008t = \ln k$, where k is ***a positive numerical value***.

A1: awrt 77 or awrt 1 hour 17 minutes.

Alternative Method 1 for part (b)

$$\int \frac{1}{3-\theta}\, d\theta = \int \frac{1}{125}\, dt \;\Rightarrow\; -\ln(\theta - 3) = \frac{1}{125}t + c$$

$$\{t = 0,\ \theta = 16 \Rightarrow\}\quad -\ln(16-3) = \frac{1}{125}(0) + c$$
$$\Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(\theta - 3) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln(\theta - 3) = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln(\theta - 3) = -\frac{1}{125}t + \ln 13$$

$$-\ln(10-3) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(\theta - 3) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.

$$t = 77.3799\ldots = 77 \text{ (nearest minute)}$$

A1: awrt 77.

Alternative Method 2 for part (b)

$$\int \frac{1}{3-\theta}\, d\theta = \int \frac{1}{125}\, dt \;\Rightarrow\; -\ln|3 - \theta| = \frac{1}{125}t + c$$

$$\{t = 0,\ \theta = 16 \Rightarrow\}\quad -\ln|3-16| = \frac{1}{125}(0) + c$$
$$\Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(3 - \theta) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln|3 - \theta| = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln|3 - \theta| = -\frac{1}{125}t + \ln 13$$

$$-\ln(3-10) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3 - \theta) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.

$$t = 77.3799\ldots = 77 \text{ (nearest minute)}$$

A1: awrt 77.

(b)	***Alternative Method 3 for part (b)***	
	$\int_{16}^{10} \frac{1}{3-\theta}\,\mathrm{d}\theta = \int_{0}^{t} \frac{1}{125}\,\mathrm{d}t$	
	$= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_{0}^{t}$	**M1A1:** $\ln 13$
	$-\ln 7 - -\ln 13 = \frac{1}{125}t$	**M1:** Substitutes limit of $\theta = 10$ correctly. **M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.
	$t = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77.
	Alternative Method 4 for part (b)	
	$\{\theta = 16 \Rightarrow\}\quad 16 = Ae^{-0.008t} + 3$ $\{\theta = 10 \Rightarrow\}\quad 10 = Ae^{-0.008t} + 3$	**M1*:** Writes down a pair of equations in A and t, for $\theta = 16$ and $\theta = 10$ with either A unknown or A being a positive or negative value. **A1:** Two equations with an unknown A.
	$-0.008t = \ln\left(\frac{13}{A}\right)$ or $-0.008t = \ln\left(\frac{7}{A}\right)$	**M1:** Uses ***correct algebra*** to solve both of **their equations** leading to answers of the form $-0.008t = \ln k$, where k is ***a positive numerical value***.
	$t_{(1)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008}$ and $t_{(2)} = \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	
	$t = t_{(1)} - t_{(2)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008} - \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	**M1:** Finds difference between the two times. (either way round).
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77. Correct solution only.

Question 4

Question Number	Scheme		Marks
	$\frac{d\theta}{dt} = \lambda(120-\theta), \quad \theta \leqslant 100$		
(a)	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$	or $\int \frac{1}{\lambda(120-\theta)}\,d\theta = \int dt$	B1
	$-\ln(120-\theta); = \lambda t + c$	or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$ See notes	M1 A1; M1 A1
	$\{t=0, \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either...	*or...*	
	$-\lambda t = \ln(120-\theta) - \ln 100$	$\lambda t = \ln 100 - \ln(120-\theta)$	
	$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$	$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$	
	$e^{-\lambda t} = \frac{120-\theta}{100}$	$e^{\lambda t} = \frac{100}{120-\theta}$	dddM1
	$100e^{-\lambda t} = 120-\theta$	$(120-\theta)e^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100e^{-\lambda t}$	
	leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01, \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$	
	$t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$		dM1
	$\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379\ldots = 161$ (s) (nearest second)	awrt 161	A1
			[3] 11

<table>
<tr><th colspan="3">Notes for Question</th></tr>
<tr><td>(a)</td><td colspan="2">

B1: Separates variables as shown. $d\theta$ and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

Either

M1: $\displaystyle\int \frac{1}{120-\theta}\,d\theta \to \pm A\ln(120-\theta)$

A1: $\displaystyle\int \frac{1}{120-\theta}\,d\theta \to -\ln(120-\theta)$

M1: $\int \lambda\,dt \to \lambda t$

A1: $\int \lambda\,dt \to \lambda t + c$

or

$\displaystyle\int \frac{1}{\lambda(120-\theta)}\,d\theta \to \pm A\ln(120-\theta)$, A is a constant.

$\displaystyle\int \frac{1}{\lambda(120-\theta)}\,d\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$,

$\int 1\,dt \to t$

or $\int 1\,dt \to t + c$ The $+c$ can appear on either side of the equation.

IMPORTANT: $+c$ can be on either side of their equation for the 2nd A1 mark.

M1: Substitutes $t=0$ AND $\theta=20$ in an integrated or changed equation containing c (or A or $\ln A$).

Note that this mark can be implied by the correct value of c. { Note that $-\ln 100 = -4.60517...$ }.

dddM1: Uses their value of c which must be a ln term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded.

A1*: This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either:

(1): $e^{-\lambda t} = \dfrac{120-\theta}{100} \Rightarrow 100e^{-\lambda t} = 120-\theta \Rightarrow \theta = 120-100e^{-\lambda t}$

or (2): $e^{\lambda t} = \dfrac{100}{120-\theta} \Rightarrow (120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t} \Rightarrow \theta = 120-100e^{-\lambda t}$

is required for A1.

Note: $\displaystyle\int \frac{1}{(120\lambda-\lambda\theta)}\,d\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).

</td></tr>
<tr><td>(b)</td><td colspan="2">

M1: Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working.

dM1: Candidate uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to $t = ...$

Note: that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b).

A1: awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).

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<tr><td>*Aliter* (a) Way 2</td><td>

$\displaystyle\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$

$-\ln(120-\theta) = \lambda t + c$ See notes

$-\ln(120-\theta) = \lambda t + c$

$\ln(120-\theta) = -\lambda t + c$

$120-\theta = Ae^{-\lambda t}$

$\theta = 120 - Ae^{-\lambda t}$

$\{t=0,\ \theta=20 \Rightarrow\}\ 20 = 120 - Ae^{0}$

$A = 120 - 20 = 100$

So, $\theta = 120 - 100e^{-\lambda t}$

</td><td>

B1

M1 A1;
M1 A1

M1

dddM1 A1 *

[8]

</td></tr>
</table>

<table>
<tr><th colspan="3">Notes for Question Continued</th></tr>
<tr><td>(a)</td><td colspan="2">B1M1A1M1A1: Mark as in the original scheme.
M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Note that this mark can be implied by the correct value of c or A.
dddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.
Note: This mark is dependent on all three previous method marks being awarded.
Note: $\ln(120-\theta) = -\lambda t + c$ leading to $120-\theta = \mathrm{e}^{-\lambda t} + \mathrm{e}^{c}$ or $120-\theta = \mathrm{e}^{-\lambda t} + A$, would be dddM0.
A1*: Same as the original scheme.
Note: The jump from $\ln(120-\theta) = -\lambda t + c$ to $120-\theta = A\mathrm{e}^{-\lambda t}$ with no incorrect working is condoned in part (a).</td></tr>
<tr><td>Aliter (a) Way 3</td><td>$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t\right\}$

$-\ln|\theta-120| = \lambda t + c$ Modulus required for 1st A1.

$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln|20-120| = \lambda(0) + c$ Modulus not required here!

$\Rightarrow c = -\ln 100 \Rightarrow -\ln|\theta-120| = \lambda t - \ln 100$

then either...
$-\lambda t = \ln|\theta-120| - \ln 100$
$-\lambda t = \ln\left|\frac{\theta-120}{100}\right|$
As $\theta \leqslant 100$
$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$
$\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100}$
$100\mathrm{e}^{-\lambda t} = 120-\theta$

or...
$\lambda t = \ln 100 - \ln|\theta-120|$
$\lambda t = \ln\left|\frac{100}{\theta-120}\right|$
$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$
$\mathrm{e}^{\lambda t} = \frac{100}{120-\theta}$
$(120-\theta)\mathrm{e}^{\lambda t} = 100$
$\Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t}$

Understanding of modulus is required here!

leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$</td><td>B1

M1 A1
M1 A1
M1

dddM1

A1 *

[8]</td></tr>
<tr><td></td><td colspan="2">B1: Mark as in the original scheme.
M1: Mark as in the original scheme ignoring the modulus.
A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln|\theta-120|$. (The modulus is required here).
M1A1: Mark as in the original scheme.
M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus.
dddM1: Mark as in the original scheme AND the candidate must demonstrate that they have converted $\ln|\theta-120|$ to $\ln(120-\theta)$ in their working. Note: This mark is dependent on all three previous method marks being awarded.
A1: Mark as in the original scheme.</td></tr>
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Notes for Question Continued		
Aliter (a) Way 4	*Use of an integrating factor*	
	$\frac{d\theta}{dt} = \lambda(120-\theta) \Rightarrow \frac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\frac{d}{dt}\left(e^{\lambda t}\theta\right) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t = 0, \theta = 20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1

Question 5

Question Number	Scheme		Marks
(i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\ \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\,\{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\,\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\,\{+c\}\}$	*{Ignore subsequent working}*.	[2]
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y \sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec} 2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^x\,\{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ or $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ or $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,dy = \int e^x\,dx$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^x\,\{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^x \to e^x$ *as part of solving their DE.*	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^0 + c$ or $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
			[7]
			12

		Question Notes
(i)	M1	Integration by parts is applied in the form $\pm \alpha x e^{4x} - \int \beta e^{4x}\{dx\}$, where $\alpha \neq 0,\ \beta > 0$. (must be in this form).
	A1	$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\{dx\}$ or equivalent.
	A1	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$ with/without $+c$. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{dv}{dx} = e^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	M1	$\pm\lambda(2x-1)^{-2},\ \lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without $+c$. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	B1	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x$ or $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y\cos y$ or $\sin 2y \to 2\sin y\cos y$ or $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y,\ \mu \neq 0$ or $\pm\alpha\sin 3y \pm \beta\sin y,\ \alpha \neq 0,\ \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) or integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x *as part of solving their DE.*
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ or $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

<u>**Alternative Method 2 (Using integration by parts twice)**</u>

$\int \sin 2y \sin y\, dy = \int e^x\, dx$		B1 oe
	Applies integration by parts twice to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	M2
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x\ \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ *as part of solving their DE.*	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]

Question 6

<table>
<tr><th>Question Number</th><th colspan="3">Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td colspan="3">$\dfrac{dN}{dt} = \dfrac{(kt-1)(5000-N)}{t}, \quad t>0,\ 0<N<5000$
$\displaystyle\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or } = \int\left(k - \frac{1}{t}\right)dt\right\}$ See notes
$-\ln(5000-N) = kt - \ln t; +c$ See notes</td><td>B1

M1 A1; A1</td></tr>
<tr><td></td><td>then eg either...
$-kt + c = \ln(5000-N) - \ln t$
$-kt + c = \ln\left(\dfrac{5000-N}{t}\right)$
$e^{-kt+c} = \dfrac{5000-N}{t}$</td><td>or...
$kt + c = \ln t - \ln(5000-N)$
$kt + c = \ln\left(\dfrac{t}{5000-N}\right)$
$e^{kt+c} = \dfrac{t}{5000-N}$</td><td>or...
$\ln(5000-N) = -kt + \ln t + c$
$5000 - N = e^{-kt + \ln t + c}$
$5000 - N = t\,e^{-kt+c}$</td><td></td></tr>
<tr><td></td><td colspan="3">leading to $N = 5000 - Ate^{-kt}$ with no incorrect working/statements. See notes</td><td>A1 * cso
[5]</td></tr>
<tr><td>(b)</td><td colspan="3">$\{t=1,\ N=1200 \Rightarrow\}\quad 1200 = 5000 - Ae^{-k}$
$\{t=2,\ N=1800 \Rightarrow\}\quad 1800 = 5000 - 2Ae^{-2k}$ At least one correct statement written down using the boundary conditions
So $Ae^{-k} = 3800$
and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$
Eg. $\dfrac{e^{-k}}{2e^{-2k}} = \dfrac{3800}{3200}$ or $\dfrac{2e^{-2k}}{e^{-k}} = \dfrac{3200}{3800}$ An attempt to eliminate A by producing an equation in only k.
So $\dfrac{1}{2}e^{k} = \dfrac{3800}{3200}$ or $2e^{-k} = \dfrac{3200}{3800}$
$k = \ln\left(\dfrac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\dfrac{19}{8}\right)\right\}$ At least one of $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent
$\left\{A = 3800(e^{k}) = 3800\left(\dfrac{19}{8}\right) \Rightarrow\right\}\ A = 9025$ Both $A = 9025$ cao or $k = \ln\left(\dfrac{7600}{3200}\right)$ or exact equivalent</td><td>B1

M1

A1

A1
[4]</td></tr>
<tr><td></td><td colspan="3"><u>Alternative Method for the M1 mark in (b)</u>
$e^{-k} = \dfrac{3800}{A}$
$2A\left(\dfrac{3800}{A}\right)^2 = 3200$ An attempt to eliminate k by producing an equation in only A</td><td>M1</td></tr>
<tr><td>(c)</td><td colspan="3">$\left\{t = 5,\ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$
$N = 4402.828401... = 4400$ (fish) (nearest 100) anything that rounds to 4400</td><td>B1
[1]
10</td></tr>
</table>

		Question Notes
(a)		
	B1	Separates variables as shown. dN and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	M1	**Either** $\pm\lambda\ln(5000-N)$ or $\pm\lambda\ln(N-5000)$ **or** $kt-\ln t$ where $\lambda \neq 0$ is a constant.
	A1	For $-\ln(5000-N)=kt-\ln t$ or $\ln(5000-N)=-kt+\ln t$ or $-\frac{1}{k}\ln(5000-N)=t-\frac{1}{k}\ln t$ oe
	A1	**which is dependent on the 1st M1 mark being awarded.** For applying a constant of integration, eg. $+c$ or $+\ln e^c$ or $+\ln c$ or A to their integrated equation
	Note	$+c$ can be on either side of their equation for the 2nd A1 mark.
	A1	Uses a constant of integration eg. "c" or "$\ln e^c$" "$\ln c$" or and applies a fully correct method to prove the result $N=5000-Ate^{-kt}$ with no incorrect working seen. (Correct solution only.)
	NOTE	**IMPORTANT** There needs to be an intermediate stage of justifying the A **and** the e^{-kt} in Ate^{-kt} by for example • **either** $5000-N=e^{\ln t-kt+c}$ • **or** $5000-N=te^{-kt+c}$ • **or** $5000-N=te^{-kt}e^{c}$ or equivalent needs to be stated before achieving $N=5000-Ate^{-kt}$
(b)	B1	At least one of **either** $1200=5000-Ae^{-k}$ (or equivalent) **or** $1800=5000-2Ae^{-2k}$ (or equivalent)
	M1	• **Either** an attempt to eliminate A by producing an equation in only k. • or an attempt to eliminate k by producing an equation in only A
	A1	At least one of $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	A1	Both $A=9025$ cao or $k=\ln\left(\frac{7600}{3200}\right)$ or equivalent
	Note	Alternative correct values for k are $k=\ln\left(\frac{19}{8}\right)$ or $k=-\ln\left(\frac{8}{19}\right)$ or $k=\ln 7600-\ln 3200$ or $k=-\ln\left(\frac{3800}{9025}\right)$ or equivalent.
	Note	$k=0.8649...$ without a correct exact equivalent is A0.
(c)	B1	anything that rounds to 4400

Question 7

Question Number	Scheme		Marks
(a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{dP}{dt} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,dP = \int \cos 2t\ dt$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P,$ $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c\quad \{\Rightarrow\ c = -\ln 3\ \text{or}\ \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,$ $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t}$ gives $3P - Pe^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k,$ $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467\ldots$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] 13

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
(b)	$\ln(P-2)-\ln P=\frac{1}{2}\sin 2t\ \ (+c)$	As before for…	B1M1A1
	$\ln\left(\frac{(P-2)}{P}\right)=\frac{1}{2}\sin 2t+c$		
	$\frac{(P-2)}{P}=e^{\frac{1}{2}\sin 2t+c}$ or $\frac{(P-2)}{P}=Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta)\pm\mu\ln P=\pm K\sin\delta t+c$, $\lambda,\mu,\beta,K,\delta\neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	3rd M1
	$(P-2)=APe^{\frac{1}{2}\sin 2t}\Rightarrow P-APe^{\frac{1}{2}\sin 2t}=2$ $\Rightarrow P(1-Ae^{\frac{1}{2}\sin 2t})=2\Rightarrow P=\frac{2}{(1-Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	4th dM1
	$\{t=0,P=3\Rightarrow\}\quad 3=\frac{2}{(1-Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	2nd M1
	$\left\{\Rightarrow 3=\frac{2}{(1-A)}\Rightarrow A=\frac{1}{3}\right\}$		
	$\Rightarrow P=\frac{2}{\left(1-\frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)}\Rightarrow P=\frac{6}{(3-e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso

	Question Notes	
(a)	M1	Forming a correct identity. For example, $2\equiv A(P-2)+BP$ from $\frac{2}{P(P-2)}=\frac{A}{P}+\frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A=-1$ or $B=1$.
	A1	$\frac{1}{(P-2)}-\frac{1}{P}$ or any equivalent form. This answer *cannot* be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A=-1$ and $B=1$ and $\frac{A}{P}+\frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)}-\frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2\equiv A(P-2)+BP$ gives $A+B=2,\ -2A=2\Rightarrow A=-1,\ B=1$

(b)	B1	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	Eg: $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ o.e. are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP;\ M, N$ can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. c, A, $\ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$. Note: This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c} \rightarrow \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is final M1M0A0
		<u>4th M1 for making P the subject</u> **Note there are three type of manipulations here which are considered acceptable for making P the subject.** (1) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t} \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ (2) M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - e^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ (3) M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3e^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3e^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2}\sin 2t}$ leading to $P = ..$
(c)	M1	States $P = 4$ or applies $P = 4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. *(Do not apply isw here)*
	Note	*Do not apply ignore subsequent working for A1.* (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P = 4000$</u>: Without the mention of $P = 4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t =$ awrt 27.1 will usually imply M1M1A0

Question 8

Question Number	Scheme	Notes	Marks
	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) Way 1	$\int \frac{1}{x}\,dx = \int -\frac{5}{2}\,dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) Way 2	$\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,dx$	**Either** $\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x}\,dx$	B1
	$t = -\frac{2}{5}\ln x + c$	Integrates both sides to give **either** $t = ...$ **or** $\pm\alpha\ln px;\ \alpha \neq 0,\ p > 0$	M1
		$t = -\frac{2}{5}\ln x + c$, including "$+c$"	A1
	$\{t=0, x=60 \Rightarrow\}\ c = \frac{2}{5}\ln 60 \Rightarrow t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) Way 3	$\int_{60}^{x} \frac{1}{x}\,dx = \int_{0}^{t} -\frac{5}{2}\,dt$	Ignore limits	B1
	$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$	Integrates both sides to give **either** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$ including the correct limits	A1
	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

(b)	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of **either** $x = \pm\lambda e^{\pm\mu t} \pm \beta$ **or** $x = \pm\lambda e^{\pm\mu t \pm \alpha\ln\delta x}$ **or** $\pm\alpha\ln\delta x = \pm\mu t \pm \beta$ **or** $t = \pm\lambda\ln\delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449...\text{ (days)}\}$ **Note: t must be greater than 0**	**dependent on the previous M mark** Uses correct algebra to achieve an equation of the form of either $t = A\ln\left(\frac{60}{20}\right)$ or $A\ln\left(\frac{20}{60}\right)$ or $A\ln 3$ or $A\ln\left(\frac{1}{3}\right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. $(A \in \square, t > 0)$	dM1
	$\Rightarrow t = 632.8006... = 633$(to the nearest minute)	awrt 633 **or** 10 hours and awrt 33 minutes	A1 cso
	Note: dM1 can be implied by $t =$ awrt 0.44 from no incorrect working.		
			7

Question Number	Scheme	Notes	Marks
	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geqslant 0$		
(a) **Way 4**	$\int \frac{2}{5x}\,dx = -\int dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\frac{2}{5}\ln(5x) = -t + c$	Integrates both sides to give **either** $\pm\alpha\ln(px)$ **or** $\pm k \to \pm kt$ (with respect to t); $k, \alpha \neq 0$; $p > 0$	M1
		$\frac{2}{5}\ln(5x) = -t + c$, including "$+c$"	A1
	$\{t = 0, x = 60 \Rightarrow\}$ $\frac{2}{5}\ln 300 = c$ $\frac{2}{5}\ln(5x) = -t + \frac{2}{5}\ln 300 \Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ **with no incorrect working seen**	A1 cso
			[4]
(a) **Way 5**	$\left\{\frac{dt}{dx} = -\frac{2}{5x} \Rightarrow\right\}$ $t = \int_{60}^{x} -\frac{2}{5x}\,dx$	Ignore limits	B1
	$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$	Integrates both sides to give **either** $\pm k \to \pm kt$ (with respect to t) **or** $\pm\frac{\alpha}{x} \to \pm\alpha\ln x$; $k, \alpha \neq 0$	M1
		$t = \left[-\frac{2}{5}\ln x\right]_{60}^{x}$ including the correct limits	A1
	$t = -\frac{2}{5}\ln x + \frac{2}{5}\ln 60 \Rightarrow -\frac{5}{2}t = \ln x - \ln 60$ $\Rightarrow \underline{x = 60e^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{e^{\frac{5}{2}t}}}$	Correct algebra leading to a correct result	A1 cso
			[4]

		Question Notes
(a)	**B1**	For the correct separation of variables. E.g. $\int \frac{1}{5x}\,\mathrm{d}x = \int -\frac{1}{2}\,\mathrm{d}t$
	Note	B1 can be implied by seeing **either** $\ln x = -\frac{5}{2}t + c$ **or** $t = -\frac{2}{5}\ln x + c$ with or without $+c$
	Note	B1 can also be implied by seeing $[\ln x]_{60}^{x} = \left[-\frac{5}{2}t\right]_{0}^{t}$
	Note	Allow A1 for $x = 60\sqrt{\mathrm{e}^{-5t}}$ or $x = \frac{60}{\sqrt{\mathrm{e}^{5t}}}$ with no incorrect working seen
	Note	Give final A0 for $x = \mathrm{e}^{-\frac{5}{2}t} + 60 \rightarrow x = 60\mathrm{e}^{-\frac{5}{2}t}$
	Note	Give final A0 for writing $x = \mathrm{e}^{-\frac{5}{2}t + \ln 60}$ as their final answer (without seeing $x = 60\mathrm{e}^{-\frac{5}{2}t}$)
	Note	Way 1 to Way 5 do not exhaust all the different methods that candidates can give.
	Note	Give B0M0A0A0 for writing down $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}$ with no evidence of working or integration seen.
(b)	**A1**	You can apply cso for the work only seen in part (b).
	Note	Give dM1(Implied) A1 for $\frac{5}{2}t = \ln 3$ followed by $t =$ awrt 633 from no incorrect working.
	Note	Substitutes $x = 40$ into their equation from part (a) is M0dM0A0

Question 9

Question Number	Scheme	Notes	Marks
	$\frac{dh}{dt} = k\sqrt{(h-9)},\ \ 9 < h \le 200;\quad h = 130,\ \frac{dh}{dt} = -1.1$		
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = ...$	Substitutes $h = 130$ and either $\frac{dh}{dt} = -1.1$ or $\frac{dh}{dt} = 1.1$ into the printed equation and rearranges to give $k = ...$	M1
	so, $k = -\frac{1}{10}$ or -0.1	$k = -\frac{1}{10}$ or -0.1	A1
			[2]
(b) **Way 1**	$\int \frac{dh}{\sqrt{(h-9)}} = \int k\,dt$	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs.	B1
	$\int (h-9)^{-\frac{1}{2}}\,dh = \int k\,dt$		
	$\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt\ (+c)$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} =$ (their k)t, with/without $+c$, or equivalent, which can be un-simplified or simplified.	A1
	$\{t = 0, h = 200 \Rightarrow\}\ 2\sqrt{(200-9)} = k(0) + c$	Some evidence of applying both $t = 0$ and $h = 200$ to changed equation containing a constant of integration, e.g. c or A	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = ...$	**dependent on the previous M mark** Applies $h = 50$ and their value of c to their changed equation and rearranges to find the value of $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ isw or awrt 148	A1 cso
			[6]

(b) **Way 2**	$\int_{200}^{50} \frac{dh}{\sqrt{(h-9)}} = \int_0^T k\ dt$	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary.	B1
	$\int_{200}^{50} (h-9)^{-\frac{1}{2}} dh = \int_0^T k\ dt$		
	$\left[\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{200}^{50} = [kt]_0^T$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} =$ (their k)t, with/without limits, or equivalent, which can be un-simplified or simplified.	A1
	$2\sqrt{41} - 2\sqrt{191} = kt$ or kT	Attempts to apply limits of $h = 200, h = 50$ and (can be implied) $t = 0$ to their changed equation	M1
	$t = \frac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$	**dependent on the previous M mark** Then rearranges to find the value of $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148 **or** 2 hours and awrt 28 minutes	A1 cso
			[6]
			8

	Question Notes	
(b)	**Note**	Allow first B1 for writing $\frac{dt}{dh} = \frac{1}{k\sqrt{(h-9)}}$ or $\frac{dt}{dh} = \frac{1}{(\text{their } k)\sqrt{(h-9)}}$ or equivalent
	Note	$\frac{dt}{dh} = \frac{1}{k\sqrt{(h-9)}}$ leading to $t = \frac{2}{k}\sqrt{(h-9)}\ (+c)$ with/without $+c$ is B1M1A1
	Note	After finding $k = 0.1$ in part (a), it is only possible to gain full marks in part (b) by **initially writing** $\frac{dh}{dt} = -k\sqrt{(h-9)}$ **or** $\int \frac{dh}{\sqrt{(h-9)}} = \int -k\,dt$ **or** $\frac{dh}{dt} = -0.1\sqrt{(h-9)}$ **or** $\int \frac{dh}{\sqrt{(h-9)}} = \int -0.1\,dt$ Otherwise, those candidates who find $k = 0.1$ in part (a), should lose at least the final A1 mark in part (b).