Integration : Solving Differential Equations 2 - Edexcel Past Exam Questions

1. (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions. **(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers. **(8)**

(*c*) Hence show that the population cannot exceed 5000. **(1)**

Jan 12 Q8

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2. Given that $y = 2$ at $x = \dfrac{\pi}{4}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{y\cos^2 x}.$$

(5)

June 12 Q4

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3. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation

$$\frac{d\theta}{dt} = \frac{(3-\theta)}{125}.$$

(*a*) By solving the differential equation, show that

$$\theta = Ae^{-0.008t} + 3,$$

where A is a constant. **(4)**

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(*b*) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute. **(5)**

Jan 13 Q8

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4. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ°C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120-\theta), \qquad \theta \leq 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(*a*) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$ **(8)**

When the temperature of the water reaches 100°C, the kettle switches off.

(*b*) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

June 13 Q6

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5. (i) Find

$$\int xe^{4x}\,dx \qquad \textbf{(3)}$$

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \qquad x > \frac{1}{2} \qquad \textbf{(2)}$$

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad \textbf{(7)}$$

June 14 Q6

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6. The rate of increase of the number, N, of fish in a lake is modelled by the differential equation

$$\frac{dN}{dt} = \frac{(kt-1)(5000-N)}{t}, \qquad t > 0,\ 0 < N < 5000$$

In the given equation, the time t is measured in years from the start of January 2000 and k is a positive constant.

(*a*) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where A is a positive constant. **(5)**

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

(*b*) Find the exact value of the constant A and the exact value of the constant k. **(4)**

(*c*) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish. **(1)**

June 14(R) Q7

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7. (*a*) Express $\dfrac{2}{P(P-2)}$ in partial fractions. **(3)**

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{2}P(P-2)\cos 2t, \qquad t \geq 0,$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(*b*) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(*c*) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures. **(3)**

June 15 Q7

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8. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad t \geq 0,$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(*a*) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form. **(4)**

(*b*) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.
Give your answer to the nearest minute. **(3)**

June 16 Q4

9.

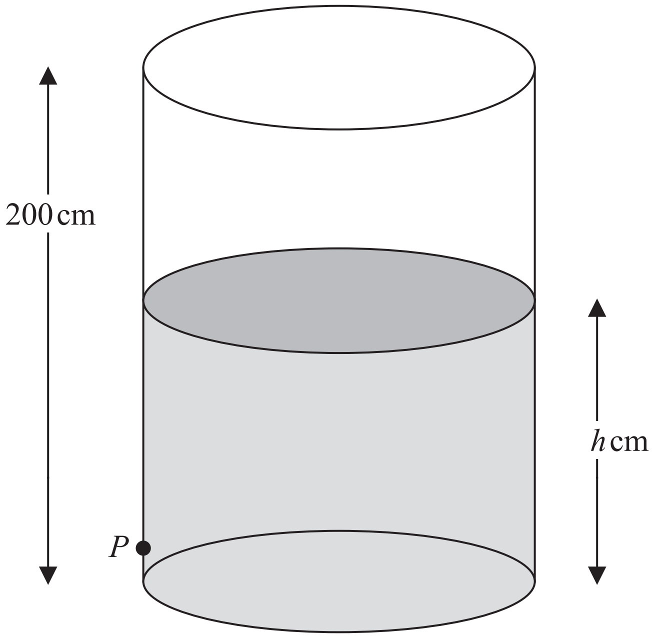

Diagram not drawn to scale

Figure 3

Figure 3 shows a vertical cylindrical tank of height 200 cm containing water.
Water is leaking from a hole P on the side of the tank.

At time t minutes after the leaking starts, the height of water in the tank is h cm.

The height h cm of the water in the tank satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k(h-9)^{\frac{1}{2}}, \qquad 9 < h \leqslant 200$$

where k is a constant.

Given that, when $h = 130$, the height of the water is falling at a rate of 1.1 cm per minute,

(*a*) find the value of k. **(2)**

Given that the tank was full of water when the leaking started,

(*b*) solve the differential equation with your value of k, to find the value of t when $h = 50$ **(6)**

June 17 Q7