Solving Trigonometric Equations 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question number	Scheme	Marks
(i)	$\sin(3x-15)=\frac{1}{2}$ so $3x-15=30$ (α) and $x=15$	M1 A1
	Need $3x-15=180-\alpha$ or $3x-15=540-\alpha$	M1
	Need $3x-15=180-\alpha$ and $3x-15=360+\alpha$ and $3x-15=540-\alpha$	M1
	$x=55$ or 175	A1
	$x=55$, 135, 175	A1
		(6)
(ii)	At least one of $(\frac{a\pi}{10}-b)=0$ (or $n\pi$)	M1
	$(\frac{a3\pi}{5}-b)=\pi$ {or $(n+1)\pi$} or in degrees	
	or $(\frac{a11\pi}{10}-b)=2\pi$ {or $(n+2)\pi$}	
	If two of **above equations** used eliminates a or b to find one or both of these or uses period property of curve to find a	M1
	or uses other valid method to find either a or b (May see $\dfrac{5\pi}{10}a=\pi$ so $a=$)	
	Obtains $a=2$	A1
	Obtains $b=\frac{\pi}{5}$ (must be in radians)	A1
		(4)

Question 2

Question Number	Scheme		Marks
	(i) $9\sin(\theta + 60^{\circ}) = 4$; $0 \leq \theta < 360^{\circ}$		
	(ii) $2\tan x - 3\sin x = 0$; $-\pi \leq x < \pi$		
(i)	$\sin(\theta + 60^{\circ}) = \dfrac{4}{9}$, so $(\theta + 60^{\circ}) = 26.3877...$ $(\alpha = 26.3877...)$	Sight of $\sin^{-1}\left(\dfrac{4}{9}\right)$ or awrt 26.4° or 0.461^{c} Can also be implied for $\theta = $ awrt -33.6 (i.e. $26.4 - 60$)	M1
	So, $\theta + 60^{\circ} = \{153.6122..., 386.3877...\}$	$\theta + 60^{\circ} = $ either "$180 - $ their α" or "$360^{\circ} + $ their α" and not for $\theta = $ either "$180 - $ their α" or "$360^{\circ} + $ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta = \{93.6122..., 326.3877...\}$	A1: At least one of awrt 93.6° or awrt 326.4° A1: Both awrt 93.6° and awrt 326.4°	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range.		
	In an otherwise fully correct solution deduct the final A1 for any extra solutions in range		
			[4]
(ii)	$2\left(\dfrac{\sin x}{\cos x}\right) - 3\sin x = 0$	Applies $\tan x = \dfrac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x = \dfrac{\sin x}{\cos x}$ can be implied by $2\tan x - 3\sin x = 0 \Rightarrow \tan x(2 - 3\cos x)$		
	$2\sin x - 3\sin x \cos x = 0$		
	$\sin x(2 - 3\cos x) = 0$		
	$\cos x = \dfrac{2}{3}$	$\cos x = \dfrac{2}{3}$	A1
	$x = $ awrt $\{0.84, -0.84\}$	A1: One of either awrt 0.84 or awrt -0.84 A1ft: You can apply ft for $x = \pm \alpha$, where $\alpha = \cos^{-1}k$ and $-1 \leq k \leq 1$	A1 A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x = 0 \Rightarrow\} \; x = 0$ and $-\pi$	Both $x = 0$ and $-\pi$ or awrt -3.14 from $\sin x = 0$ **In this part of the solution, ignore extra solutions in range.**	B1
	Note solutions are: $x = \{-3.1415..., -0.8410..., 0, 0.8410...\}$		
	Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Question 3

Question Number	Scheme	Marks
	$1 - 2\cos\left(\theta - \dfrac{\pi}{5}\right) = 0$; $-\pi < \theta \leqslant \pi$	
(i)	$\cos\left(\theta - \dfrac{\pi}{5}\right) = \dfrac{1}{2}$ Rearranges to give $\cos\left(\theta - \dfrac{\pi}{5}\right) = \dfrac{1}{2}$ or $-\dfrac{1}{2}$	M1
	$\theta = \left\{-\dfrac{2\pi}{15}, \dfrac{8\pi}{15}\right\}$ At least one of $-\dfrac{2\pi}{15}$ or $\dfrac{8\pi}{15}$ or $-24°$ or $96°$ or awrt 1.68 or awrt -0.419	A1
	Both $-\dfrac{2\pi}{15}$ and $\dfrac{8\pi}{15}$	A1
		[3]
NB Misread	Misreading $\dfrac{\pi}{5}$ as $\dfrac{\pi}{6}$ or $\dfrac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark	
(ii)	$4\cos^2 x + 7\sin x - 2 = 0$, $0 \leqslant x < 360°$	
	$4(1 - \sin^2 x) + 7\sin x - 2 = 0$ Applies $\cos^2 x = 1 - \sin^2 x$	M1
	$4 - 4\sin^2 x + 7\sin x - 2 = 0$	
	$4\sin^2 x - 7\sin x - 2 = 0$ Correct 3 term, $4\sin^2 x - 7\sin x - 2 \{= 0\}$	A1 oe
	$(4\sin x + 1)(\sin x - 2) \{= 0\}$, $\sin x = ...$ Valid attempt at solving and $\sin x = ...$	M1
	$\sin x = -\dfrac{1}{4}$, $\{\sin x = 2\}$ $\sin x = -\dfrac{1}{4}$ (See notes.)	A1 cso
	$x = \text{awrt}\{194.5, 345.5\}$ At least one of awrt 194.5 or awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
	awrt 194.5 and awrt 345.5	A1
		[6]
		9
NB Misread	Writing equation as $4\cos^2 x - 7\sin x - 2 = 0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6	
	$4(1 - \sin^2 x) - 7\sin x - 2 = 0$	M1
	$4\sin^2 x + 7\sin x - 2 = 0$	A0
	$(4\sin x - 1)(\sin x + 2) \{= 0\}$, $\sin x = ...$ Valid attempt at solving and $\sin x = ...$	M1
	$\sin x = +\dfrac{1}{4}$, $\{\sin x = -2\}$ $\sin x = \dfrac{1}{4}$ (See notes.)	A0
	$x = \text{awrt} 165.5$	A1ft
	Incorrect answers	A0

		Question Notes
(i)	M1	Rearranges to give $\cos\left(\theta - \dfrac{\pi}{5}\right) = \pm\dfrac{1}{2}$
	Note	M1 can be implied by seeing either $\dfrac{\pi}{3}$ or $60°$ as a result of taking $\cos^{-1}(...)$.
	A1	Answers **may be in degrees or radians** for this mark and may have just one correct answer. Ignore mixed units in working if correct answers follow (recovery)
	A1	Both answers correct and in radians as multiples of π $-\dfrac{2\pi}{15}$ and $\dfrac{8\pi}{15}$ Ignore EXTRA solutions outside the range $-\pi < \theta \le \pi$ but lose this mark for extra solutions in this range.
(ii)	1st M1	Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.]
	1st A1	Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2\{=0\}$ or $-4\sin^2 x + 7\sin x + 2\{=0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc.
	2nd M1	For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, s, y, x or $\sin x$, and an attempt to find at least one of the solutions for $\sin x$. This solution may be outside the range for $\sin x$
	2nd A1	$\sin x = -\dfrac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$.
	Note	$\sin x = -\dfrac{1}{4}$ can be implied by later correct working if no errors are seen.
	3rd A1ft	At least one of awrt 194.5 or awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through. Only follow through on the error $\sin x = \dfrac{1}{4}$ and allow for 165.5 special case (as this is equivalent work) This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip.
	4th A1	awrt 194.5 and awrt 345.5
	Note	If there are any EXTRA solutions inside the range $0 \leqslant x < 360°$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark. Ignore EXTRA solutions outside the range $0 \leqslant x < 360°$.
	Special Cases	Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error) Answers in radians:– lose **final** mark so either or both of 3.4, 6.0 gets A1ftA0 It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in $\sin x = -1/4$ then correct work follows.