Transformations of Graphs 2 - Edexcel Past Exam Questions MARK SCHEME
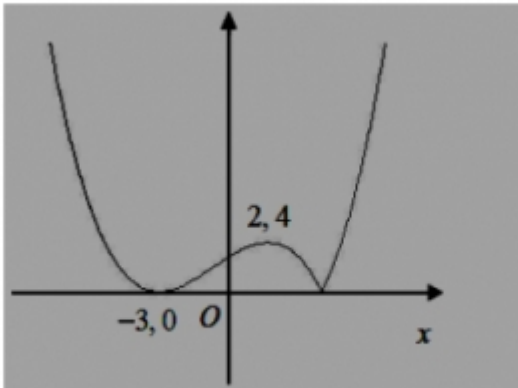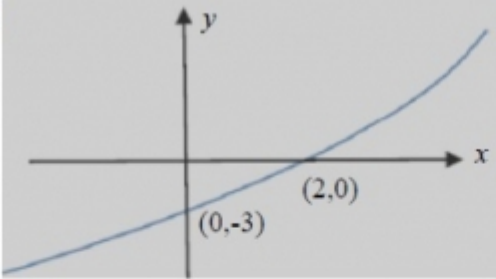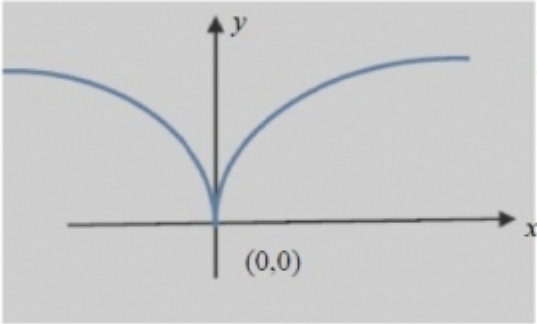
Question 1
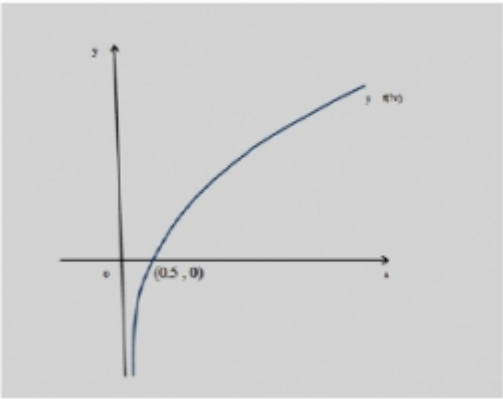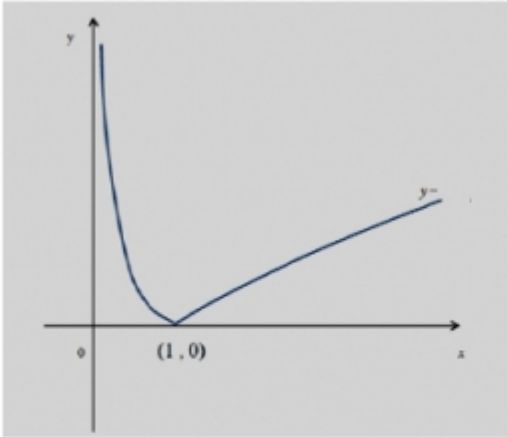
Question No	Scheme	Marks
	(a)	Shape B1 x coordinates correct B1 y coordinates correct B1 (3)
	(b)	Shape B1 Max at (2,4) B1 Min at (-3,0) B1 (3)
		6 marks

Question 2

Question Number	Scheme	Marks
	(a) ff(-3)= f(0),=2	M1,A1 (2)
	(b) $y = f^{-1}(x)$	Shape B1 (0,-3) and (2,0) B1 (2)
	(c) $y=f(|x|)-2$	Shape B1 (0,0) B1 (2)
	(d)	Shape B1 (-6,0) or (0,4) B1 (-6,0) and (0,4) B1 (3)
		(9 marks)

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.

Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes

through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.

A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)
B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum

B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P' and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)
B1 Award for a correct shape 'roughly' symmetrical about the y- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.
B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations
B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question 3

Question Number	Scheme	Marks
(a)		Shape B1 (0.5, 0) B1 (2)
(b)		Shape B1 (1,0) B1 Cusp at (1,0) B1 (3) (5 marks)

Notes for Question

(a)

B1   Award for the correct shape. Look for an increasing function with decreasing gradient. Condone linear looking functions in the first quadrant. It needs to look asymptotic at the y axis and have no obvious maximum point. It must be wholly contained in quadrants 1 and 4
See practice and qualification items for clarification.

B1   Crosses x axis at $\left(\dfrac{1}{2}, 0\right)$. Accept $\dfrac{1}{2}$, 0.5 or even $\left(0, \dfrac{1}{2}\right)$ marked on the correct axis.
There must be a graph for this mark to be scored.

(b)

B1   Correct shape wholly contained in quadrant 1.
The shape to the rhs of the cusp must not have an obvious maximum.
Accept linear, or positive with decreasing gradient. The gradient of the curve to the lhs of the cusp/minimum should always be negative. The curve in this section should not 'bend' back past (1, 0) forming a 'C' shape or have incorrect curvature.
See practice and qualification for clarification.

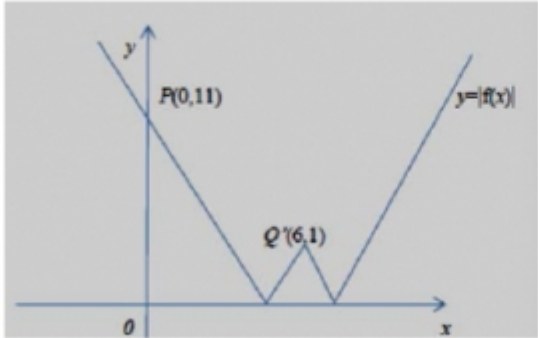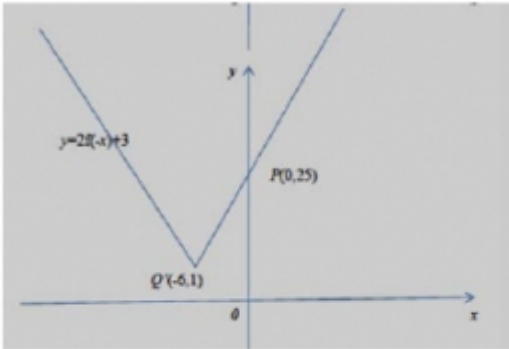**B1**   The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1'

on the x axis. Condone the point marked on the correct axis as (0, 1)

B1   Award for a cusp, not a minimum at (1,0)

Note that $f\left(|x|\right)$ scores B0 B1 B0 under the scheme.

If the graphs are not labelled (a) and (b), then they are to be marked in the order they are presented

Question 4

| Question Number	Scheme	Marks
(a)		'W' Shape **B1** (0, 11) and (6, 1) **B1** (2)
(b)		'V' shape **B1** (-6,1) **B1** (0,25) **B1** (3)
(c)	One of $a = 2$ or $b = 6$ $a = 2$ and $b = 6$	**B1** **B1** (2) **(7 marks)**

(a)

B1   A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = f(|x|)$ would score this mark.

B1   A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

(b)

B1   Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.
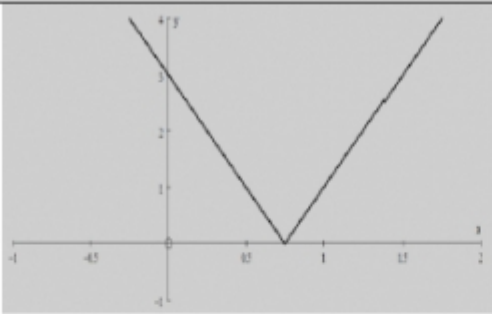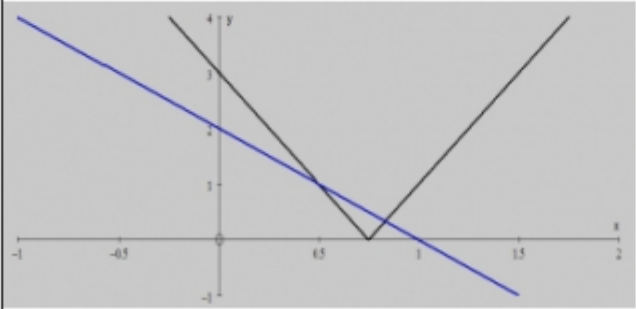
Special case: A candidate who mistakenly sketches $y = -2f(x) + 3$ or $y = -2f(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0
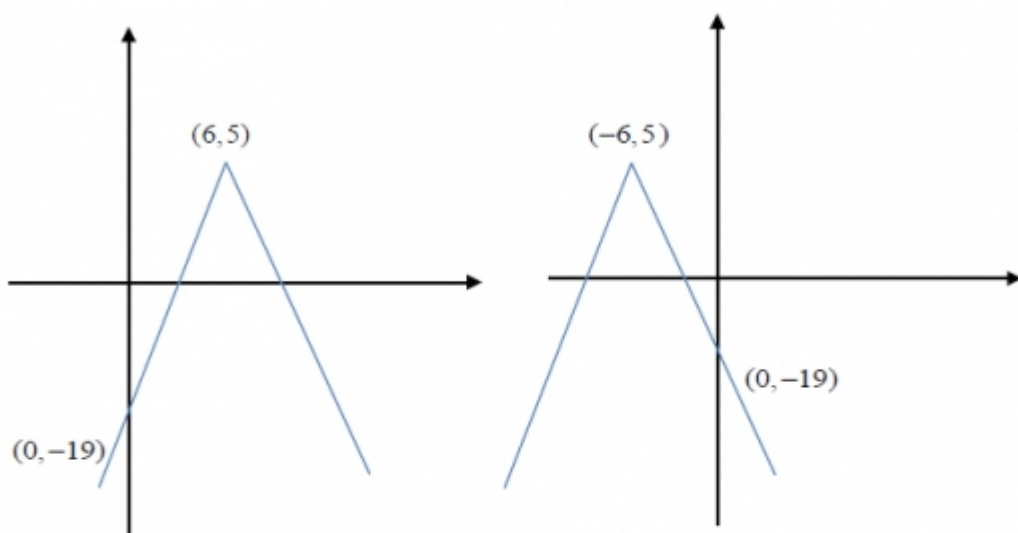
(c)

B1   Either states $a = 2$ or $b = 6$.
This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x - 6| - 1$ or $y = 2|x - ..| - 1$. If they are both stated and written, the stated answer takes precedence.

B1   States both $a = 2$ **and** $b = 6$
This can be implied by the candidate stating that $y = 2|x - 6| - 1$
If they are both stated and written, the stated answer takes precedence.

Question 5

Question Number	Scheme	Marks
(a)		V shaped graph B1 Touches x axis at ¾ and cuts y axis at 3 B1 (2)
(b)		Solves $4x - 3 = 2 - 2x$ or $3 - 4x = 2 - 2x$ to give either value of x M1 Both $x = \dfrac{5}{6}$ and $x = \dfrac{1}{2}$ A1 or $x > \dfrac{5}{6}$ or $x < \dfrac{1}{2}$

(a)

B1 A 'V' shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a 'dotted' extension of $y = 4x - 3$ appearing under the x axis.

B1 The graph **meets** the x axis at $x = \dfrac{3}{4}$ and **crosses** the y axis at $y = 3$. Do not allow multiple meets or crosses

If they have lost the previous B1 mark for an extra section of graph underneath the x axis allow for **crossing the** x axis at $x = \dfrac{3}{4}$ and **crosses** the y axis at $y = 3$.

Accept marked elsewhere on the page with A and B marked on the graph and $A = \left(\dfrac{3}{4}, 0\right)$ and $B = (0, 3)$

Condone $\left(0, \dfrac{3}{4}\right)$ and $(3, 0)$ marked on the correct axis

(b)

M1 Attempts to solve $|4x - 3| \dots 2 - 2x$ finding at least one solution. You may see $\dots$ replaced by either $=$ or $>$

Accept as evidence $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$

Accept as evidence $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..,$ or $x < ..$

A1 Both critical values $x = \dfrac{5}{6}$ and $x = \dfrac{1}{2}$, or one inequality, accept $x > \dfrac{5}{6}$ or $x < \dfrac{1}{2}$

Accept $x = 0.83$ and $x = 0.5$ for the critical values

Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.

Eg if M1 has been scored for $4x - 3 = 2 - 2x \Rightarrow x = 0.83$ and $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$

A correct application of M1 would be $x < -2.5, x > 0.83$

A1 Correct answer only $x < \dfrac{1}{2}$ or $x > \dfrac{5}{6}$.

Accept $x < 0.5, x > 0.8\dot{3}$

(c)

M1 **Either** sketch both lines showing a single intersection at the point $x = \dfrac{3}{4}$

Or solves $|4x - 3| = 1\frac{1}{2} - 2x$ using both $4x - 3 = 1\frac{1}{2} - 2x$ and $-4x + 3 = 1\frac{1}{2} - 2x$ **giving one solution** $x = \dfrac{3}{4}$
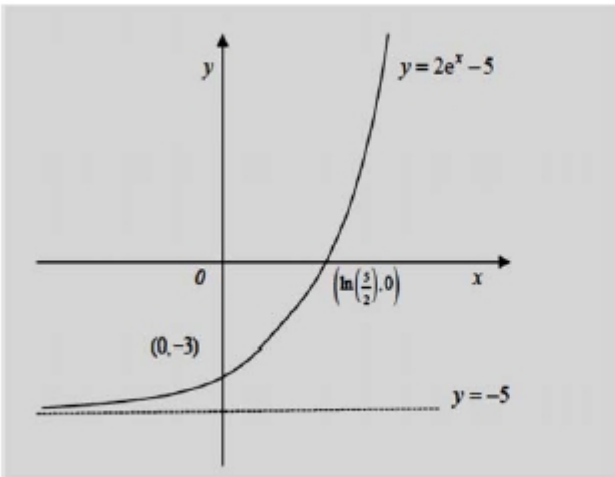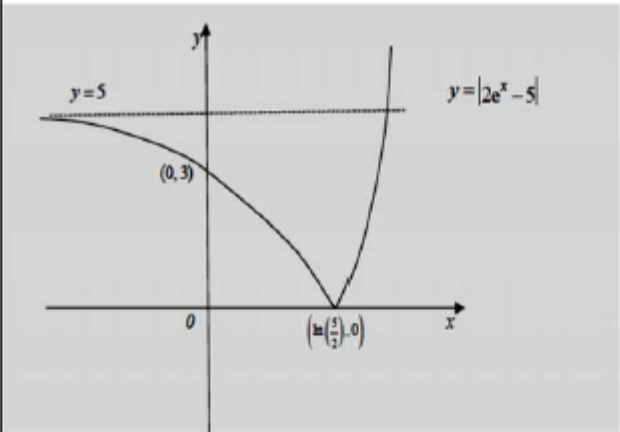
Accept $|4x - 3| > 1\frac{1}{2} - 2x$ using both $4x - 3 > 1\frac{1}{2} - 2x$ and $-4x + 3 > 1\frac{1}{2} - 2x$ **giving one solution** $x \dots \dfrac{3}{4}$

If two values are obtained using either method it is M0A0

A1 States that the solution set is all values apart from $x = \dfrac{3}{4}$. Do not isw in this question. Score their final statement. Accept versions of all values of x except $x = \frac{3}{4}$ or $x \in \mathbb{R}$, $x \neq \frac{3}{4}$, or $x < \frac{3}{4}, x > \frac{3}{4}$

Question 6

Question Number	Scheme		Marks
(ai)		Shape	B1
		$\left(\ln\left(\dfrac{5}{2}\right),0\right)$ and $(0,-3)$	B1
		$y=-5$	B1
			(3)
(aii)		Shape inc cusp	B1ft
		$\left(\ln\left(\dfrac{5}{2}\right),0\right)$ and $(0,3)$	B1ft
		$y=5$	B1ft
			(3)
(b)	$x \geqslant \ln\left(\dfrac{5}{2}\right)$		B1 ft
			(1)
(c)	$2e^x-5=-2 \Rightarrow (x)=\ln\left(\dfrac{3}{2}\right)$		M1A1
	$(x)=\ln\left(\dfrac{7}{2}\right)$		B1
			(3)
			(10 marks)

(a)(i)

B1   For an exponential (growth) shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1   Intersections with the axes at $\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$.

Allow $\ln\left(\frac{5}{2}\right)$ and -3 being marked on the correct axes.

Condone $\left(0,\ln\left(\frac{5}{2}\right)\right)$ and $(-3,0)$ being marked on the x and y axes respectively.

Do not allow $\left(\ln\left(\frac{5}{2}\right),0\right)$ appearing as awrt $(0.92,0)$ for this mark unless seen elsewhere. Allow if seen in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then **the ones on the curve take precedence.**

B1   Equation of the asymptote given as $y=-5$. Note that the curve must appear to have an asymptote at $y=-5$, not necessarily drawn. It is not enough to have -5 marked on the axis or indeed $x=-5$. An extra asymptote **with an equation** gets B0

(a)(ii)

B1ft   For **either** the correct shape **or** a reflection of their curve from (a)(i) in the x- axis. For this to be scored it must have appeared both above and below the x - axis. The shape must be correct including the cusp. The curve to the lhs of the cusp must appear to have the correct curvature

B1ft   Score for both intersections or follow through on both the intersections given in part (a)(i), including decimals, as long as the curve appeared both above and below the x - axis. See part (a) for acceptable forms

B1ft   Score for an asymptote of $y=5$ or follow through on an asymptote of $y=-C$ from part (a)(i). Note that the curve must appear to have an asymptote at $y=C$ but do not penalise if the first mark in (a)(ii) has been withheld for incorrect curvature on the lhs.

(b)

B1ft   Score for $x \geqslant \ln\left(\frac{5}{2}\right)$, $x \geqslant$ awrt 0.92 or follow through on the x intersection in part (a)

(c)

M1   Accept $2e^x - 5 = -2$ or $-2e^x + 5 = 2 \Rightarrow x = ..\ln(..)$

Allow squaring so $\left(2e^x - 5\right)^2 = 4 \Rightarrow e^x = ..$ and $.. \Rightarrow x = \ln(..), \ln(..)$

A1 $x = \ln\left(\frac{3}{2}\right)$ or exact equivalents such as $x = \ln 1.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 0.405

B1 $x = \ln\left(\frac{7}{2}\right)$ or exact equivalents such as $x = \ln 3.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 1.25

If both answers are given in decimals and there is no working $x =$ awrt 1.25, 0.405 award SC 100

Question 7

Question Number	Scheme	Marks
(a)(i)	V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$ Correct shape, position and coordinates	B1 B1
(ii)	Their "V" shape translated up or $(0,a+b)$ Correct shape, position and $(0,a+b)$	B1ft B1 (4)
(b)	States or uses $a+b=8$ Attempts to solve $\left\|2x-a\right\|+b=\frac{3}{2}x+8$ in either x or with $x=c$ $2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$ Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \Rightarrow c=4a$	B1 M1 dM1 A1 (4) **(8 marks)**

(a)(i)

B1   V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ lying on the curve.

         Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1   V shape sitting on the positive x-axis at $\left(\frac{1}{2}a,0\right)$, cutting the y-axis at $(0,a)$ and lying in both quadrants 1 and 2

         Accept $\frac{1}{2}a$ and a marked on the correct axis. Condone say $(a,0)$ for $(0,a)$ as long as it is on the correct axis.

         Condone a dotted line appearing on the diagram as many reflect $y=2x-a$ to sketch $y=\left|2x-a\right|$

         If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft   Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.

         Alternatively score for the $(0,a+b)$ lying on the curve

B1   V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0,a+b)$

         Ignore any coordinates given for the vertex.

(b)

B1   States or uses $a+b=8$ or exact equivalent. Condone use of capital letters throughout

It is not scored for just $|0-a|+b=8$

M1   This M is for an understanding of the modulus.

It is scored for an attempt at solving $(2x-a)+b=\dfrac{3}{2}x+8$ or $-(2x-a)+b=\dfrac{3}{2}x+8$ in either x or with x

replaced by c. The signs of the $2x$ and the a must be different. $|2x-a| \neq 2x+a$

You may see $(2x-a)+b=\dfrac{3}{2}x+8 \Rightarrow kx = \mathrm{f}(a,b)$

You may see $-2x+a+b=\dfrac{3}{2}x+8 \Rightarrow kx = \mathrm{f}(a,b)$

You may see $(2x-a)+b=\dfrac{3}{2}x+8 \Rightarrow kx = \mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

You may see $-2c+a+b=\dfrac{3}{2}c+8 \Rightarrow kc = \mathrm{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

dM1 This dM mark is scored for combining $b=8-a$ with $(2x-a)+b=\dfrac{3}{2}x+8$ (or their $kx=\mathrm{f}(a,b)$ resulting from

that equation) resulting in a link between x and a **Both equations must have been correct initially.**

Alternatively for combining $b=8-a$ with their $2c-a+b=\dfrac{3}{2}c+8$ (or their $kc=\mathrm{f}(a,b)$ resulting from that

equation) resulting in a link between c and a

You may condone sign slips in finding the link between x (or c) and a

If you see an approach that involves making $|2x-a|$ the subject followed by squaring, and you feel that it

deserves credit, please send to review. The solution proceeds as follows

Look for $|2x-a|=\dfrac{3}{2}x+8-b \Rightarrow |2x-a|=\dfrac{3}{2}x+a \Rightarrow (2x-a)^2=\left(\dfrac{3}{2}x+a\right)^2 \Rightarrow 7x\left(\dfrac{1}{4}x-a\right)=0$

A1 $c=4a$ ONLY

....

Special Case where they have the roots linked with the incorrect branch of the curve.

They have $x=0$ as the solution to $2x-a+b=\dfrac{3}{2}x+8 \Rightarrow -a+b=8$.....................(1)

They have $x=c$ as the solution to $-2x+a+b=\dfrac{3}{2}x+8 \Rightarrow \dfrac{7}{2}x=a+b-8$.............(2)
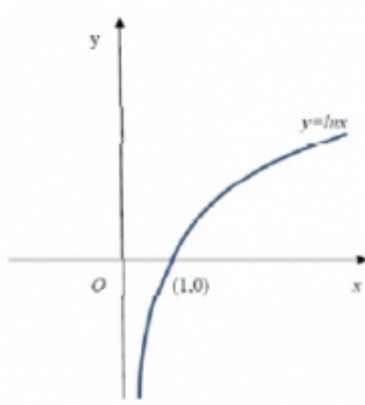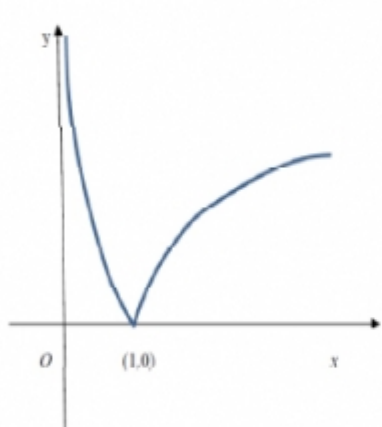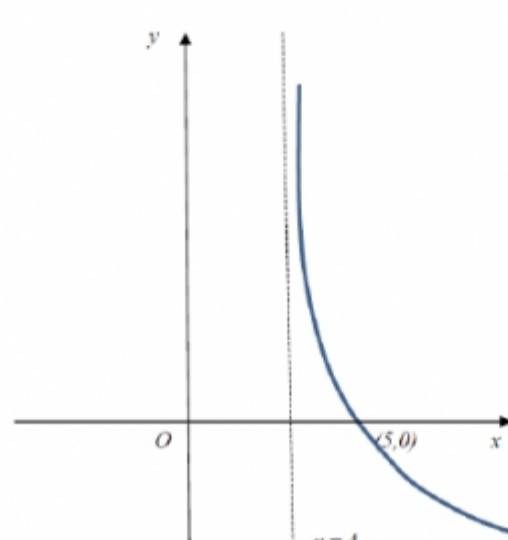
Solve (1) and (2) $\Rightarrow x=\dfrac{4}{7}a$

Hence $\Rightarrow c=\dfrac{4}{7}a$

This would score B0 M1 dM0 A0 anyway but should be awarded SC B0, M1 dM1, A0 for above

work leading to either $x=\dfrac{4}{7}a$ or $c=\dfrac{4}{7}a$

Question 8

Question Number	Scheme	Marks
Alt 1 **By Multiplication**	$* \; 3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$	
	Compares the x^4 terms $a = 3$	B1
	Compares coefficients to obtain a numerical value of one **further** constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$	M1
	Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
	All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
		(4 marks)

Notes for Question

B1   Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$

M1   Multiply out expression given to get *. Condone slips only on signs of either expression.

Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either b or c
The method may be implied by a correct additional constant to a.

A1   Achieving two of $b = -2 \; c = 7 \; d = -8 \; e = 24$

A1   Achieving all of $b = -2 \; c = 7 \; d = -8$ and $e = 24$

Question Number	Scheme	Marks
(i)		*ln* graph crossing *x* axis at (1,0) and asymptote at *x*=0 — B1
(ii)		Shape including cusp — B1ft Touches or crosses the *x* axis at (1,0) — B1ft Asymptote given as *x*=0 — B1
(iii)		Shape — B1 Crosses at (5, 0) — B1ft Asymptote given as *x*=4 — B1 (7 marks)

Notes for Question

(i) B1   Correct shape, correct position and passing through (1, 0).

Graph must 'start' to the rhs of the *y* - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (*x*=0) the asymptote is incorrect or not given.**

(ii) B1ft   Correct shape **including the cusp** wholly contained in quadrant 1.

The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)

Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.

Follow through on an incorrect sketch in part (i) as long as it was above and below the *x* axis.

     B1ft   The curve touches or crosses the *x* axis at (1, 0). Allow for the curve passing through a point marked '1' on the *x* axis. Condone the point marked on the correct axis as (0, 1)

         Follow through on an incorrect intersection in part (i).

     B1   Award for the asymptote to the curve given/ marked as $x = 0$. Do not allow for it given/ marked as 'the *y* axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at $x = 0$. Accept if *x*=0 is drawn separately to the y axis.

(iii)

     B1   Correct shape.

The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape. The position is not important for this mark. See practice and qualification for clarification.

     B1ft   The graph crosses (or touches) the *x* axis at (5, 0). Allow for the curve passing through a point marked '5' on the *x* axis. Condone the point marked on the correct axis as (0, 5)

         Follow through on an incorrect intersection in part (i). Allow for $((i) + 4, 0)$

     B1   The asymptote is given/ marked as $x = 4$. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the *y* axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.
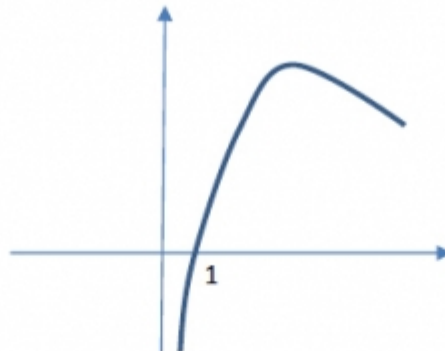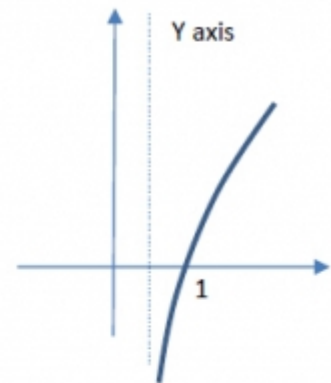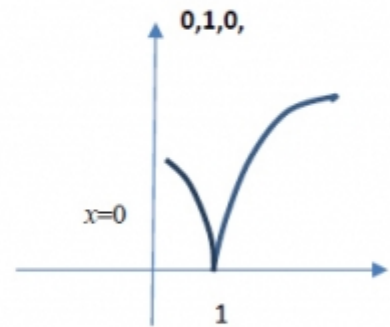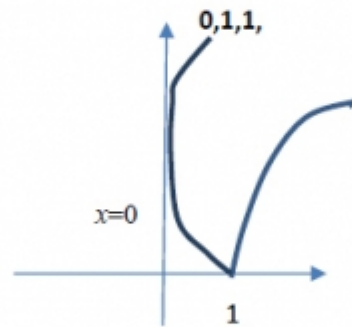
Examples of graphs

Part (i)

Condoned

Not condoned

Condone

(0,1)

1

Y axis

1

Part (ii)

1,1,1

$x=0$

1

0,1,1,

$x=0$

1

0,1,0,

$x=0$

1
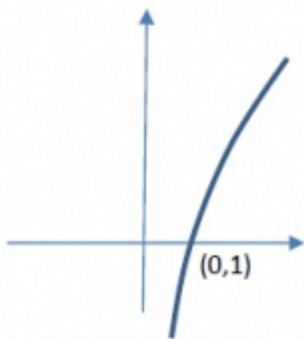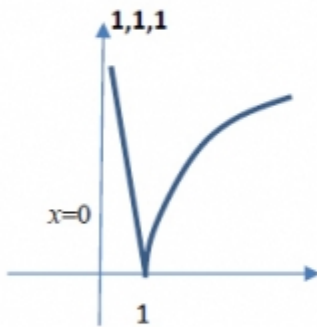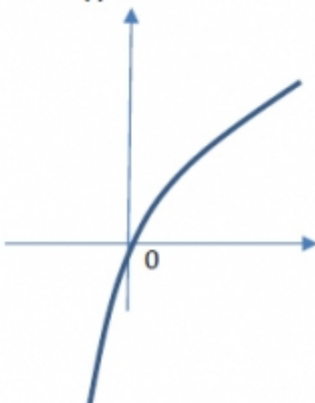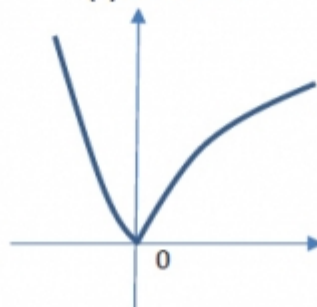
Example of follow through in part (ii) and (iii)

(i) B0

0

(ii) B1ftB1ftB0

0

(iii)   B0B1ftB0

4