Integration: Trapezium Rule 2 - Edexcel Past Exam Questions

1.

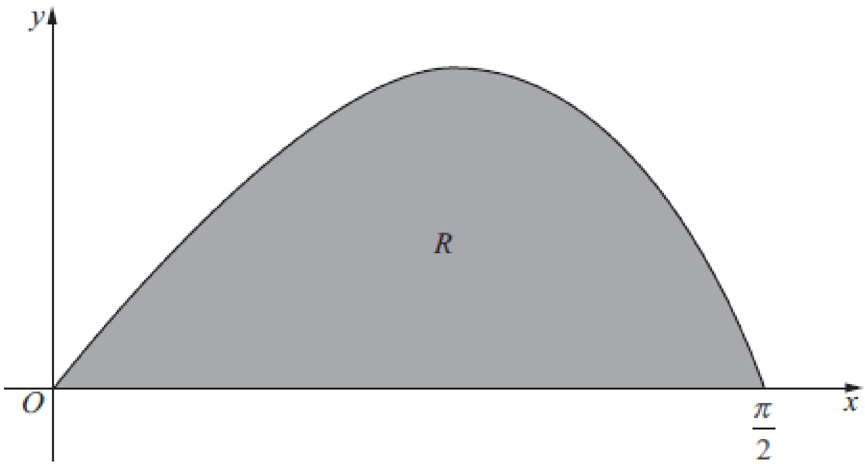

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \le x \le \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places. **(1)**

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant. **(5)**

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures. **(3)**

Jan 12 Q6

2.

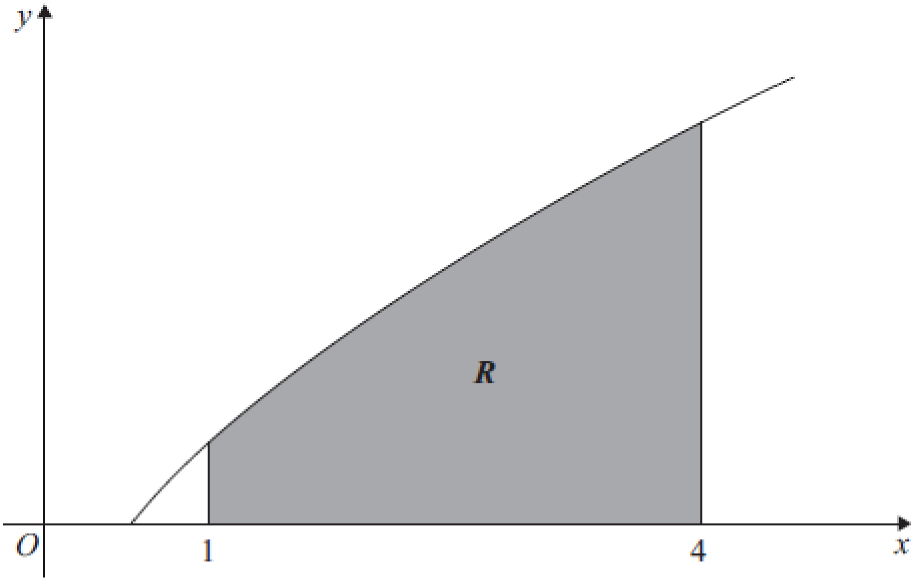

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(*a*) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places. **(4)**

(*b*) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$. **(4)**

(*c*) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants. **(3)**

June 12 Q7

3.

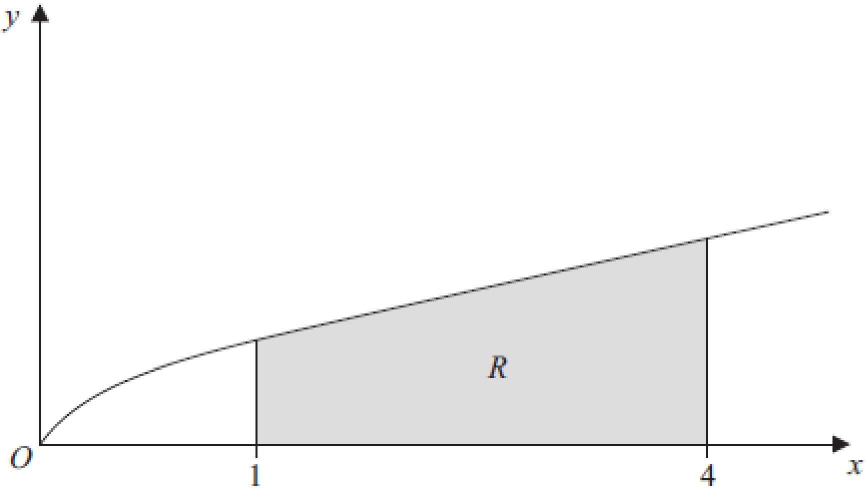

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(*a*) Copy and complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places. **(1)**

x	1	2	3	4
y	0.5	0.8284		1.3333

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places. **(3)**

(*c*) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R. **(8)**

Jan 13 Q4

4.

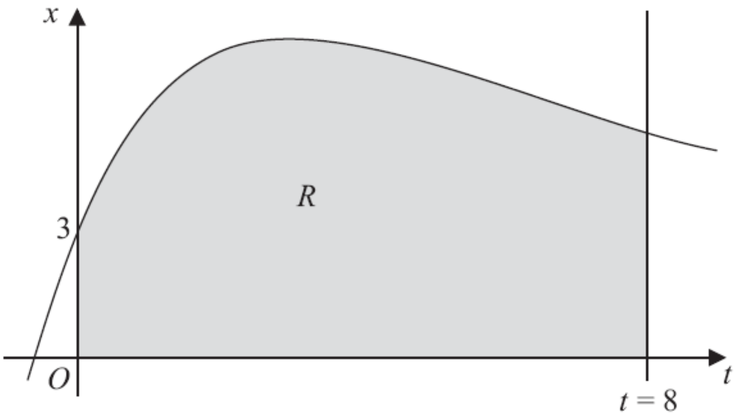

Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(*a*) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

(*b*) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places. **(3)**

(*c*) Use calculus to find the exact value for the area of R. **(6)**

(*d*) Find the difference between the values obtained in part (*b*) and part (*c*), giving your answer to 2 decimal places. **(1)**

June 13(R) Q5

5.

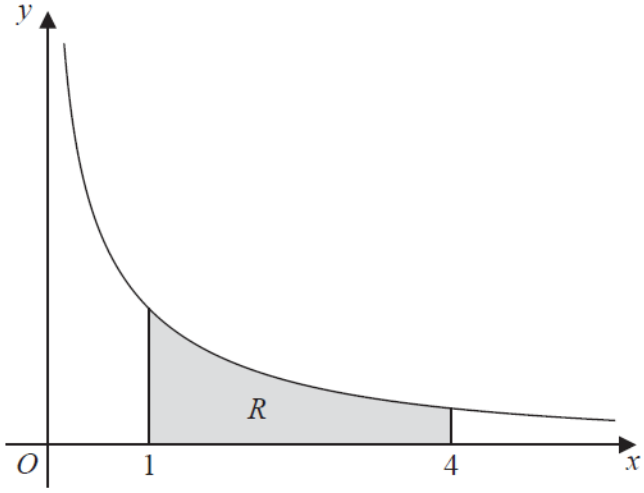

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(a) Complete the table above by giving the missing value of y to 5 decimal places. **(1)**

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R. **(1)**

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$ **(6)**

June 14 Q3

6.

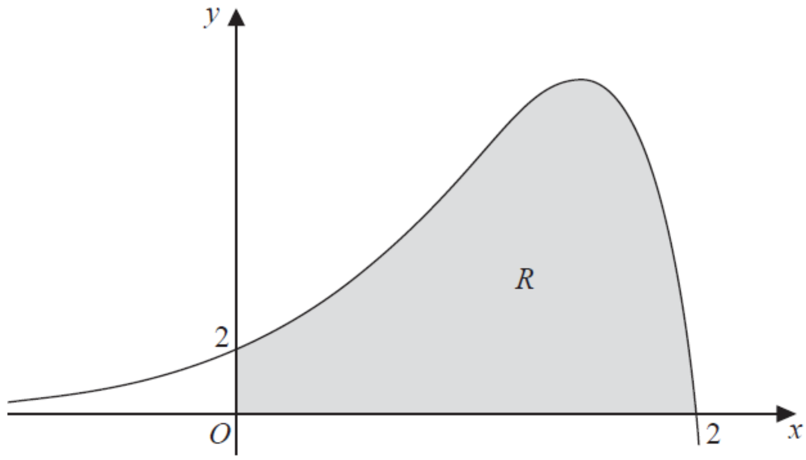

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \sim$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$.

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(*a*) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places. **(3)**

(*b*) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R. **(1)**

(*c*) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form. **(5)**

June 14(R) Q2

7.

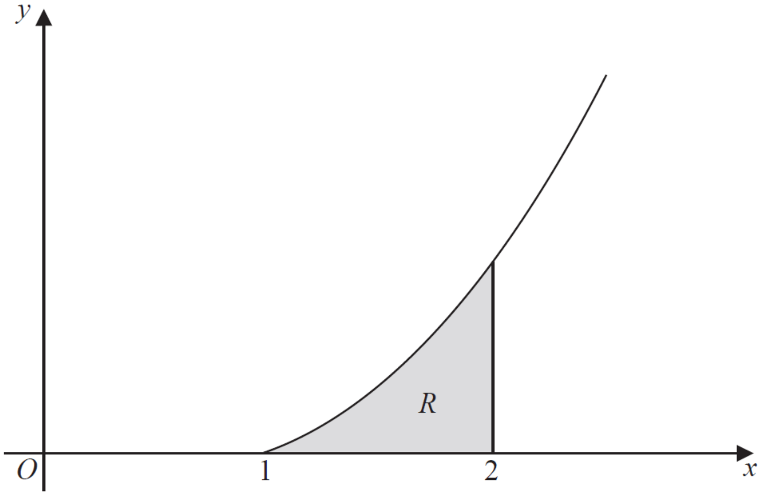

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geq 1$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The table below shows corresponding values of x and y for $y = x^2 \ln x$.

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places. **(1)**

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places. **(3)**

(c) Use integration to find the exact value for the area of R. **(5)**

June 16 Q2

8.

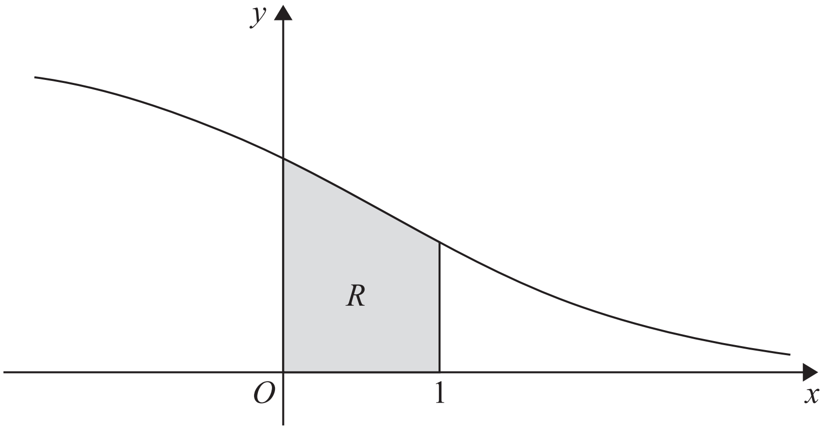

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \frac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(*a*) Complete the table above by giving the missing value of y to 5 decimal places. **(1)**

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(*c*) Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,du$$

where a and b are constants to be determined. **(2)**

(*d*) Hence use calculus to find the exact area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*] **(6)**

June 17 Q3