Integration: Trapezium Rule - Edexcel Past Exam Questions

1. **Figure 1**

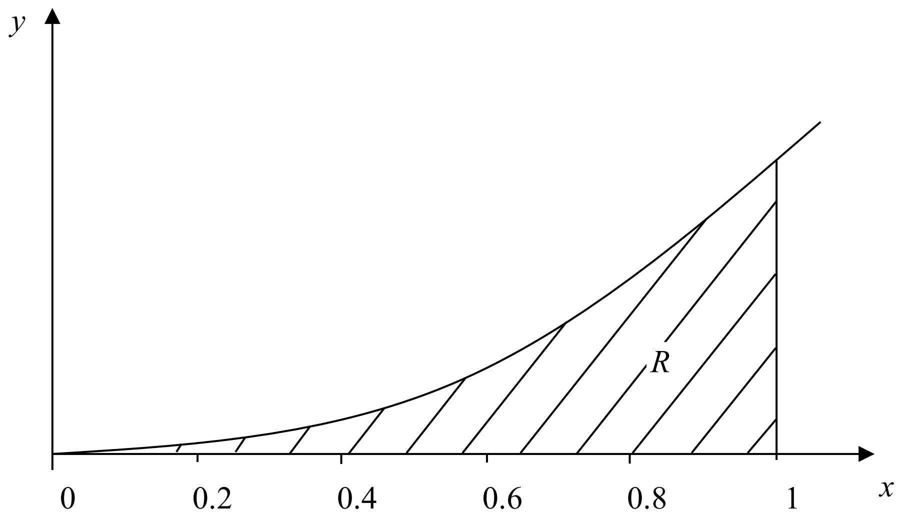

Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value of the area for R. **(5)**

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures. **(4)**

June 05 Q5

2. (*a*) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(*b*) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, dx$. Show all the steps of your working and give your answer to 4 decimal places. **(3)**

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, dx$ is $\ln(1 + \sqrt{2})$.

(*c*) Calculate the % error in using the estimate you obtained in part (*b*). **(2)**

Jan 06 Q2

3.
$$I = \int_0^5 e^{\sqrt{(3x+1)}} \, dx.$$

(*a*) Given that $y = e^{\sqrt{(3x+1)}}$, copy and complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures. **(3)**

(*c*) Use the substitution $t = \sqrt{(3x + 1)}$ to show that I may be expressed as $\int_a^b kte^t \, dt$, giving the values of a, b and k. **(5)**

(*d*) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)

Jan 07 Q8

4. **Figure 3**

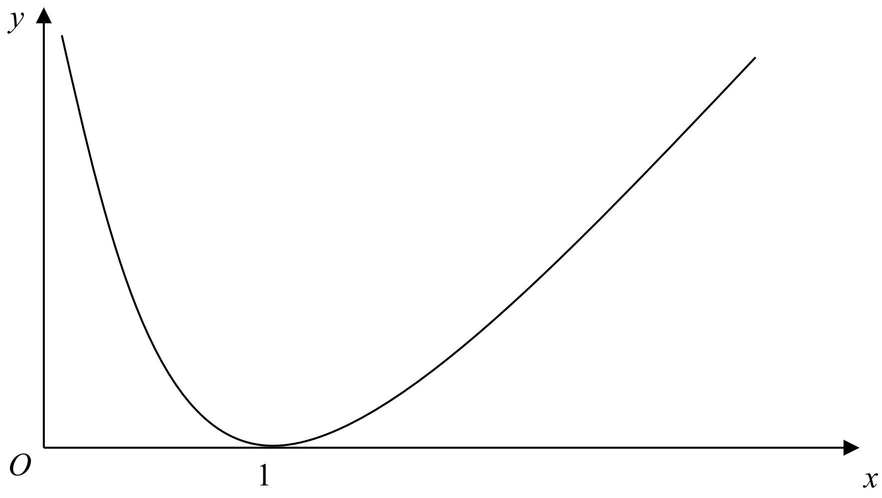

Figure 3 shows a sketch of the curve with equation $y = (x - 1) \ln x$, $x > 0$.

(a) Copy and complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		ln 2		2 ln 3

(1)

Given that $I = \int_1^3 (x-1) \ln x \ \mathrm{d}x$,

(b) use the trapezium rule

(i) with values at y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values at y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures. **(5)**

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation. **(1)**

(d) Show, by integration, that the exact value of $\int_1^3 (x-1) \ln x \ \mathrm{d}x$ is $\frac{3}{2} \ln 3$. **(6)**

June 06 Q6

5.

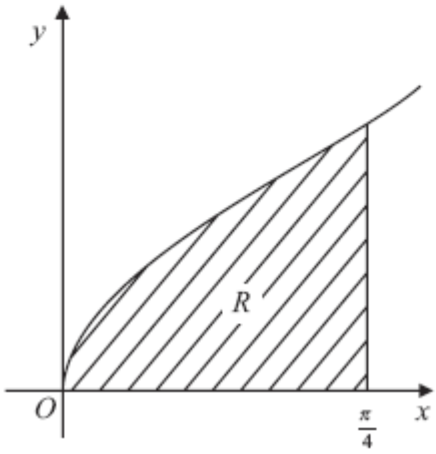

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(*a*) Given that $y = \sqrt{(\tan x)}$, copy and complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places. **(4)**

June 07 Q7*(edited)*

6.

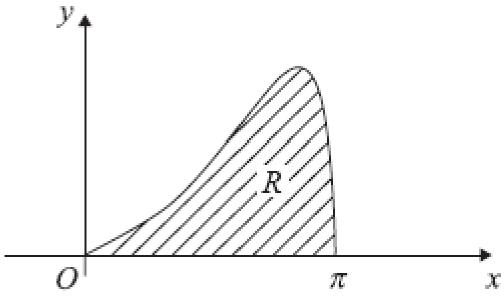

Figure 1

The curve shown in Figure 1 has equation $e^x\sqrt{(\sin x)}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(*a*) Copy and complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(*b*) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places. **(4)**

Jan 08 Q1

7.

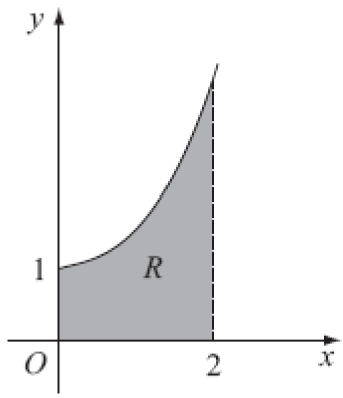

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures. **(3)**

June 08 Q1

8.

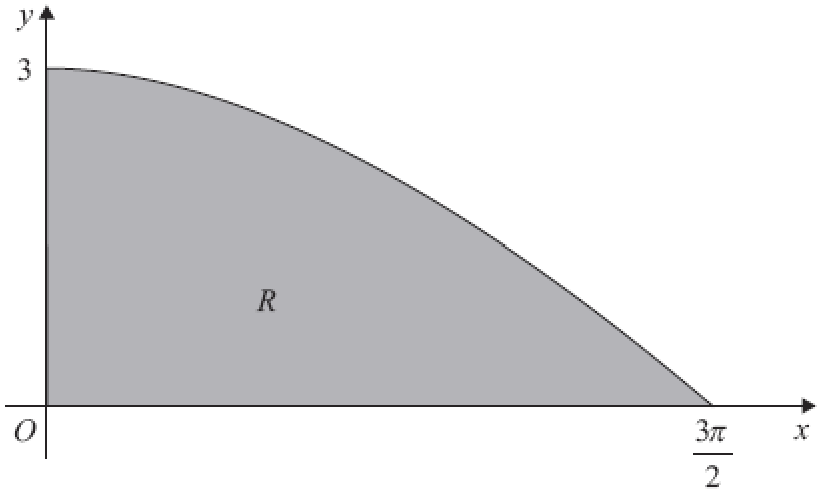

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(*a*) Copy and complete the table above giving the missing value of y to 5 decimal places. **(1)**

(*b*) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places. **(4)**

(*c*) Use integration to find the exact area of R. **(3)**

June 09 Q2

9.

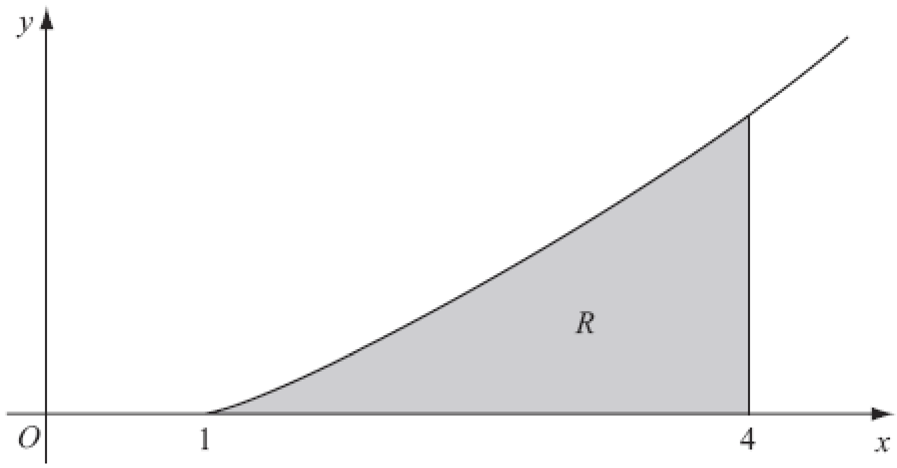

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. **(2)**

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places. **(4)**

(*c*) (i) Use integration by parts to find $\int x \ln x \; dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. **(7)**

Jan10 Q2

10.

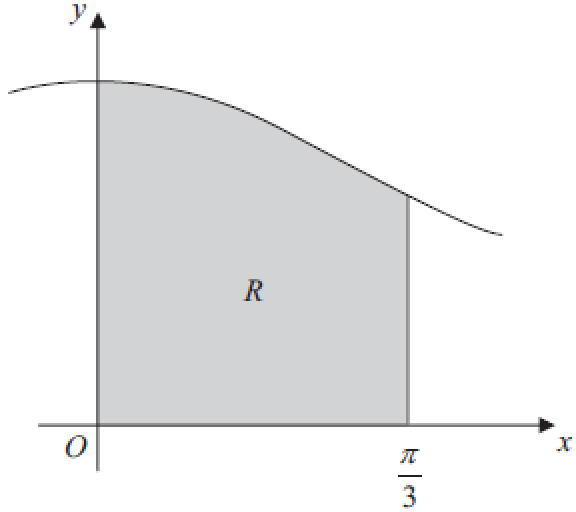

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(*a*) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(*b*) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R. Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places. **(6)**

June 10 Q1

11. $$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx.$$

(*a*) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(*b*) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places. **(4)**

(*c*) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I. **(8)**

Jan 11 Q7

12.

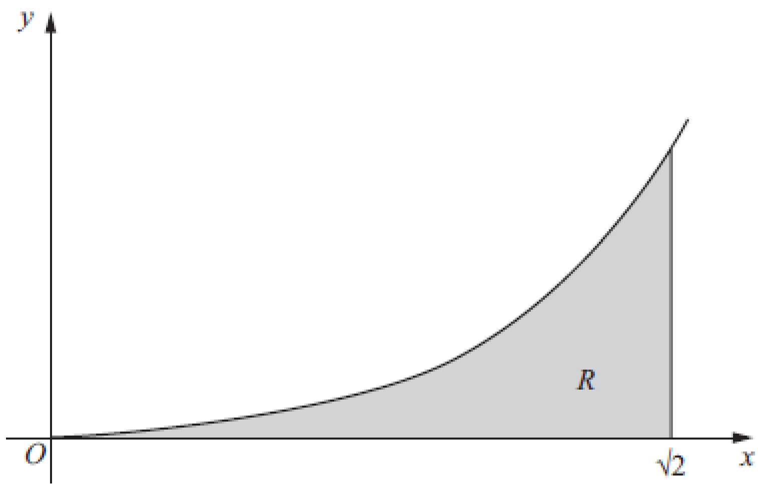

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places. **(2)**

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places. **(3)**

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \ \mathrm{d}u .$$

(4)

(d) Hence, or otherwise, find the exact area of R. **(6)**

June 11 Q4

13.

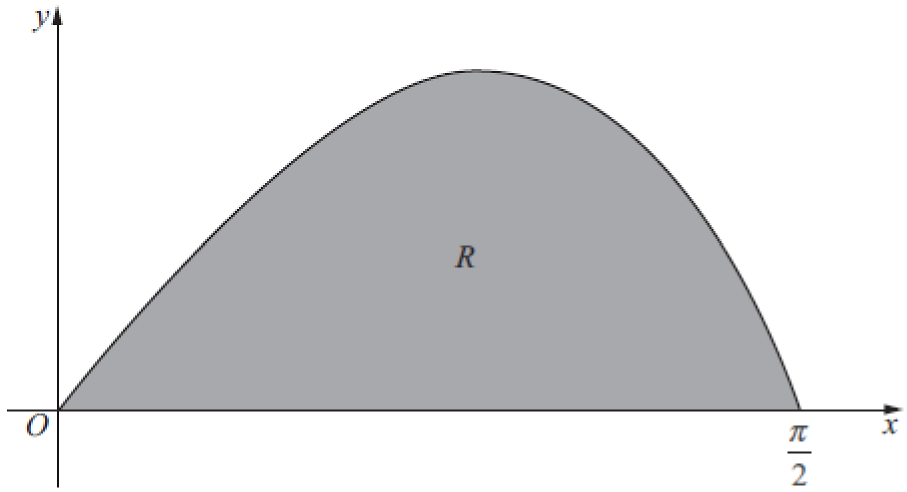

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \le x \le \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places. **(1)**

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant. **(5)**

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures. **(3)**

Jan 12 Q6