Trigonometry 2 (Addition, Double Angle & R Formulae) - Edexcel Past Exam Questions

1. (*a*) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$ **(4)**

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$ **(3)**

(*c*) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Give your answers as multiples of π. **(6)**

Jan 12 Q8

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2. (*a*) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

(2)

(*b*) Hence show that

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \sec^2\theta.$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = 4$$

giving your answers in terms of π.

(3)

June 12 Q5

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3. $$\text{f}(x) = 7\cos 2x - 24\sin 2x.$$

Given that $\text{f}(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α. **(3)**

(*b*) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \le x < 180°$, giving your answers to 1 decimal place. **(5)**

(*c*) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found. **(2)**

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x.$$ **(2)**

June 12 Q8

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4. (*a*) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 decimal places. **(4)**

(*b*) $$\text{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \le \theta \le 2\pi.$$

Calculate

(i) the maximum value of $\text{p}(\theta)$,

(ii) the value of θ at which the maximum occurs. **(4)**

Jan 13 Q4

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5. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working. **(5)**

(ii) (*a*) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$k\sin^2\theta - \sin\theta = 0$, stating the value of k. **(2)**

(*b*) Hence solve, for $0 \le \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1.$$ **(4)**

Jan 13 Q6

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6. $$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - a)$, where $R > 0$ and $0 < a < 90°$,

(*a*) find the exact value of R and the value of a to one decimal place. **(3)**

(*b*) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leq x < 360°$, giving your answers to one decimal place. **(5)**

(*c*) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leq x < 360°$. **(2)**

June 13(R) Q3

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7. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ. **(3)**

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2 \theta + 3\sec \theta = 2\tan^2 \theta$$

You must show all your working. Give your answers in terms of π. **(6)**

June 13(R) Q6

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8. (*a*) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$ **(5)**

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(5)**

June 14 Q7

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9. (*a*) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places. **(3)**

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \le \theta \le \pi$, at which this maximum value occurs. **(3)**

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \le \theta \le \pi$, at which this minimum value occurs. **(3)**

June 14 Q9

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10. $$g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta.$$

Given that $g(\theta) = R \cos (2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places. **(3)**

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1,$$

giving your answers to one decimal place. **(5)**

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k. **(2)**

June 15 Q3

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11. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z}.$$ **(5)**

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}.$$

Give your answers to 3 decimal places. **(4)**

June 15 Q8

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12. (*a*) Express $2 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places. **(3)**

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place. **(5)**

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place. **(2)**

June 16 Q3

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13. (*a*) Prove that

$$2 \cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}.$$ **(4)**

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(6)**

June 16 Q8

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14. (*a*) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places. **(3)**

(*b*) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined. **(2)**

(*c*) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(4)**

June 17 Q4

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15. (*a*) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \quad n \in \mathbb{Z}$$

(4)

(*b*) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(5)**

June 17 Q9
