Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M11

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Prove by contradiction that $\sqrt{3}$ is irrational. **(Total 4 marks)**

Question 2

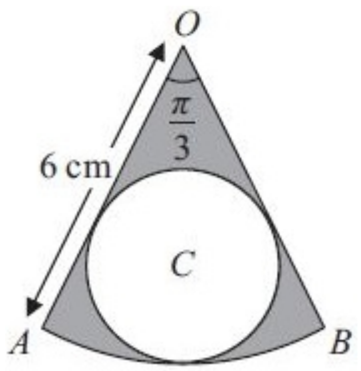

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector *OAB* of a circle centre *O*, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle *C*, inside the sector, touches the two straight edges, *OA* and *OB*, and the arc *AB* as shown.

Find

(a) the area of the sector *OAB*, **(2)**

(b) the radius of the circle *C*. **(3)**

The region outside the circle *C* and inside the sector *OAB* is shown shaded in Figure 1.

(c) Find the area of the shaded region. **(2)**

(Total 7 marks)

Question 3

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio, (2)

(b) the first term, (2)

(c) the sum to infinity, (2)

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. (4)

(Total 10 marks)

Question 4

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P. (8)

(Total 13 marks)

Question 5

Find the gradient of the curve with equation

$\ln y = 2x \ln x, \quad x > 0,\ y > 0$

at the point on the curve where $x = 2$. Give your answer as an exact value. **(7)**

(Total 7 marks)

Question 6

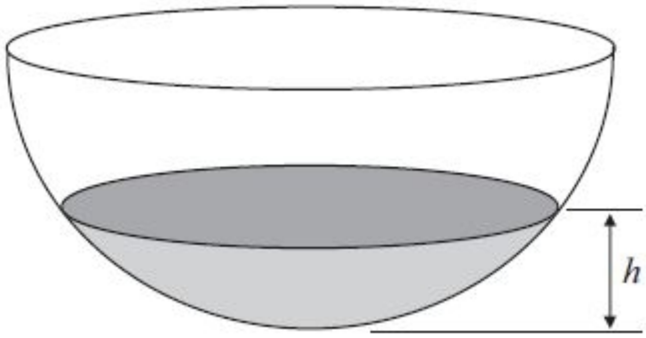

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$ **(4)**

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^{3}s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$ **(2)**

(Total 6 marks)

Question 7

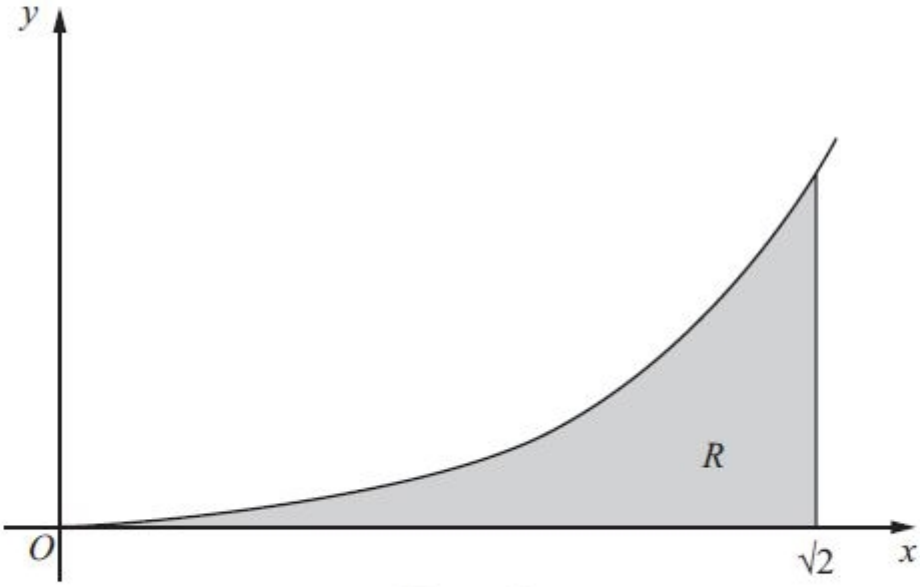

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2),\ \ x \geqslant 0.$ The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the *x*-axis and the line $x = \sqrt{2}$.

(a) Use the substitution $u = x^2 + 2$ to show that the area of *R* is

$$\frac{1}{2}\int_2^4 (u - 2)\ln u \,\mathrm{d}u$$

(4)

(b) Hence, or otherwise, find the exact area of *R*. **(6)**

(Total 10 marks)

Question 8

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°,\ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

(Total 12 marks)

Question 9

(a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. (4)

$$f(x) = e^{2x}\cos 3x$$

(b) Show that f '(x) can be written in the form

$$f'(x) = Re^{2x}\cos(3x + \alpha)$$

where R and α are the constants found in part (a). (5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point. (3)

(Total 12 marks)

Question 10

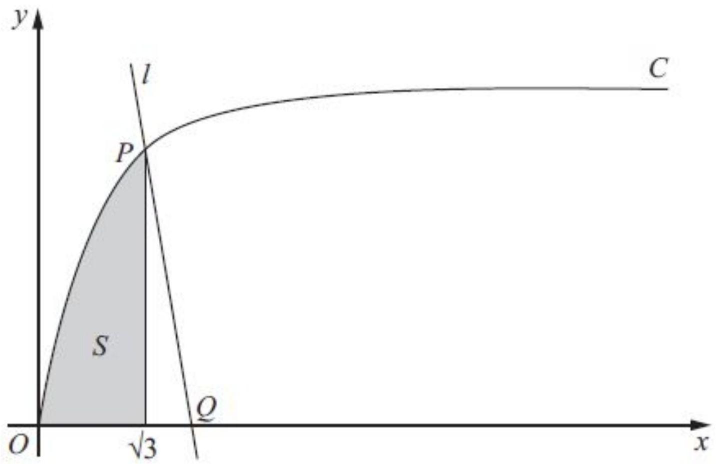

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$

(a) Find the value of θ at the point P. (2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k. (6)

(Total 8 marks)

Question 11

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p. **(1)**

(b) Show that $k = \frac{1}{4}\ln 3$. **(4)**

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$. **(6)**

(Total 11 marks)

TOTAL FOR PAPER IS 100 MARKS