Arc Length and Areas of Sectors 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question number	Scheme	Marks
(a)	$r\theta = 6\times 0.95, = 5.7$ (cm)	M1, A1 (2)
(b)	$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 0.95, = 17.1\ (\text{cm}^2)$	M1, A1 (2)
(c)	Let $AD = x$ then $\frac{x}{\sin 0.95} = \frac{6}{\sin 1.24}$ so $x = 5.16$ *	M1 A1 (2)
	OR $x = 3 / \cos 0.95$ OR so $x = 3/ \sin 0.62$ so $x = 5.16$ *	
	OR $x^2 = 6^2 + x^2 - 12x\cos 0.95$ leading to $x =$, so $x = 5.16$ *	
(d)	Perimeter = '5.7'+5.16 +6 – 5.16= "11.7" **or 6 + their 5.7**	M1A1 ft (2)
(e)	Area of triangle $ABD = \frac{1}{2}\times 6\times 5.16\times \sin 0.95 = 12.6$ or	M1 A1
	$\frac{1}{2}\times 6\times 3\times \tan 0.95 = 12.6$ (½ base x height) or $\frac{1}{2}\times 5.16\times 5.16\times \sin 1.24 = 12.6$	
	So Area of R = '17.1' – '12.6' = 4.5	M1 A1 (4) **12**

Notes	(a) **M1:** Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1:** Does not need units (b) **M1:** Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1:** Does not need units (c) **M1:** Needs complete correct trig method to achieve $x =$ May have worked in degrees, using 54.4 degrees and 71.1 degrees Using angles of triangle sum to 360degrees is not correct method so is M0 **A1:** accept answers which round to 5.16 (NB This is given answer) If the answer 5.16 is **assumed** and **verified** award M1A0 for correct work (d) **M1:** Accept answer only as implying method, or just 6 + 5.7 **A1** : can be scored even following wrong answer to part (c) (e) **M1:** needs complete method for area of triangle ABD not ABC **A1:** Accept awrt 12.6 (If area of triangle is not evaluated or is given as 12.5 (truncated) this mark may be implied by 4.5 later) **M1:** Uses area of R = area of sector – area of triangle ABD (not ABC) **A1:** Answers wrt 4.5
Alternative For part (e)	Finds area of segment **and** area of triangle BDC by correct methods **M1** Obtains 2.4585 **and** 2.0498 – accept answers wrt 2.5, 2.1 **A1** Uses area of segment + area of triangle BDC ,to obtain 4.5 (not 4.6) **M1, A1** **NB Just finding area of segment is M0**

Question 2

Question Number	Scheme		Marks
(a)	$9^2 = 4^2 + 6^2 - 2\times4\times6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \frac{4^2+6^2-9^2}{2\times4\times6}\left(=-\frac{29}{48}=-0.604..\right)$		
	$\alpha = 2.22$ *	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		**(2)**
(a) Way 2	$XY^2 = 4^2 + 6^2 - 2\times4\times6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY^2 = 81.01....$		
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366.....)$	$2\pi - 2.22$ or awrt 4.06	B1
	$\frac{1}{2}\times4^2\times$"4.06"	Correct method for major sector area.	M1
	32.5	Awrt 32.5	A1
			(3)
(b) Way2	**Circle – Minor sector**		
	$\pi\times4^2$	Correct expression for circle area	B1
	$\pi\times4^2 - \frac{1}{2}\times4^2\times2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2}\times4\times6\times\sin 2.22 (=9.56)$	Correct expression for the area of triangle XYZ	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + (part (b) answer or correct attempt at major sector)	M1
	= 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (Or just 42)	A1
			(3)
(d)	Arc length $= 4\times4.06 (=16.24)$ Or $8\pi - 4\times2.22$	M1: $4\times$*their* $(2\pi - 2.22)$ Or circumference – minor arc	M1A1ft
		A1: Correct ft expression	
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			[12]

Question 3

Question Number	Scheme	Marks
(a)	Mark (a) and (b) together.	
	Usually answered in radians: Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{1}{2}(12)^2(\text{angle})$ or both	M1
	Area $= \frac{1}{2}(23)(12)\sin 0.64$ **or** $\frac{1}{2}(12)^2(\pi - 0.64)$ $\{=$ 82.41297091... **or** 180.1146711...$\}$	A1
	Area $= \frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ $\{=$ 82.41297091... + 180.1146711...$\}$	A1
	$\{$Area = 262.527642...$\}$ = awrt 262.5 (m^2) or 262.4(m^2) or 262.6 (m^2)	A1
		(4)
(b)	$CDE = 12\times(angle)$, $=12(\pi - 0.64)\{\Rightarrow CDE = 30.01911...\}$	M1, A1
	$AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ $\{AE = 15.17376...\}$	M1
	Perimeter = 23 + 12 + 15.17376... + 30.01911...	M1
	= 80.19287... = awrt 80.2 (m)	A1
		(5)
		[9]

Notes for Question

(a)

M1: uses either area of triangle formula as given in mark scheme, or area of sector or both (may be implied by answer)

A1: one correct area expression (with **correct angle** used) $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ or see awrt 82.4 **or** awrt 180 (180 may be split as 226.2(semicircle) minus 46.1(small sector))

A1: two correct area expressions (with correct angles) **added together** (allow 2.5 as implying $\pi - 0.64$) or see awrt 82.4 + awrt 180 (or 226 - 46)

A1: answers which round to 262.5 or 262.4 or 262.6

(b)

1st M1 for attempt to use $s = r\,\theta$ (any angle)

1st A1 for $\pi - 0.64$ in the formula (or 2.5)

2nd M1: Uses correct cosine rule to obtain AE or AE^2 (this may appear in part (a))

3rd M1(**independent**): Perimeter = 23 + 12 + their AE + their CDE

2nd A1: awrt 80.2 (ignore units – even incorrect units)

Degrees

(a)

Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{\text{angle in degrees}}{360}\times\pi(12)^2$ or both for M1

Area $= \frac{1}{2}(23)(12)\sin 36.7$ **or** $\frac{(180 - 36.7)}{360}\times\pi(12)^2$ $\{=$ *awrt* 82.4... **or** 180$\}$ A1

Area $= \frac{1}{2}(23)(12)\sin 36.7 + \frac{(180 - 36.7)}{360}\times\pi(12)^2$ $\{=$ *awrt* 82.4... + 180$\}$ A1

Final mark as before

(b)

$CDE = \frac{\text{Angle in degrees}}{360}\times 24\pi, = \frac{180 - 36.7}{360}\times 24\pi\{\Rightarrow CDE = 30.01268...\}$ M1, A1

Final three marks as before

Question 4

Question Number	Scheme	Marks
(a)	**Way 1**: $10^2 = 7^2 + 13^2 - 2\times7\times13\cos\theta$ or $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$	M1
	$\cos\theta = \dfrac{59}{91}$ or $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$ or $\cos\theta = 0.6483$ or 0.8644	A1 o.e
	$(\theta = 0.8653789549\ldots) = 0.865$ * (to 3 dp)	A1* cso (3)
	Way 2: Uses $\cos\theta = \dfrac{x}{7}$, where $7^2 - x^2 = 10^2 - (13-x)^2$ and finds x $(=59/13)$	M1
	$\cos\theta = \dfrac{59}{91}$ and $(\theta = 0.8653789549\ldots) = 0.865$ * (to 3 dp) – as in Way 1	A1, A1 (3)
(b)	Area triangle $ABC = \dfrac{1}{2}\times13\times7\sin0.865$ or $\dfrac{1}{2}\times13\times7\sin49.6$ or $20\sqrt{3}$	M1
	Area sector $ABD = \dfrac{1}{2}\times7^2\times0.865$ or $\dfrac{49.6}{360}\times\pi\times7^2$	M1
	=34.6 (triangle) or 21.2 (Sector)	A1
	Area of $S = \dfrac{1}{2}\times13\times7\sin0.865 - \dfrac{1}{2}\times7^2\times0.865$ $(=13.4)$	M1 A1
	(Amount of seed =) 13.4 x 50 = 670g or 680g (need one of these two answers)	M1 A1 (7)
		Total 10

Notes for Question

(a) M1: use correct cosine **formula in any form** A1: give a value for $\cos\theta$

NB $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$ earns M1A1

A1: deduce and state the printed answer θ = 0.865

(b) M1: Uses Correct method for area of the correct triangle i.e. ABC

M1: Uses Correct method for the area of the sector

A1: This is earned for one of the correct answers. May be implied if these answers are not calculated but the final answer is correct with no errors (or shaded area is 13.4 or13.5)

M1: Their area of Triangle ABC– **Area** of Sector (may have $kr^2\theta$ but not $k\theta$)

A1: **Correct expression** or awrt 13.4 or 13.5 (may be implied by final answer)

M1: Multiply their previous answer by 50

A1: 670g or 680 g (There is an argument for rounding answer up to provide enough seed)

N.B. $(\dfrac{1}{2}\times13\times7\sin0.865 - \dfrac{1}{2}\times7^2\times0.865)\times50=670$ or 680 earns full marks

$(\dfrac{1}{2}\times13\times7\sin0.865 - \dfrac{1}{2}\times7^2\times0.865)\times50$ = **awrt** 670 or 680 just loses last mark

$(\dfrac{1}{2}\times13\times7\sin0.865 - \dfrac{1}{2}\times7^2\times0.865)\times50$ = wrong answer M1M1A0M1A1M1A0

Question 5

Question Number	Scheme		Marks
(a)	Area $BDE = \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5\ (\text{cm}^2)$	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2\times 5\times 7.5\cos DBC)$ or $\cos DBC = \frac{5^2 + 7.5^2 - 6.1^2}{2\times 5\times 7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c) (Angle $DBC = 54.04....$degrees)**		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - "0.943"$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 - "\text{their } 0.943"$ A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... - \text{their angle } DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - "0.943")$ or $AE = 5\sin(\pi - 1.4 - "0.943")$		
		$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for AB = awrt 3.49 Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for AE = awrt 3.58 **It must be clear that $\pi - 1.4 - "0.943"$ is being used for angle EBA. Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - "0.943") \times 5\sin(\pi - 1.4 - "0.943")$		
	This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB		dM1
	Allow M1 for area EAB = awrt 6.2		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1cso
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		**Total 9**

Question 6

Question Number	Scheme		Marks
(a)	Length $DEA = 7(2.1) = 14.7$	M1: 7×2.1 only A1: 14.7	M1A1
			[2]
(b)	Angle $CBD = \pi - 2.1$	**May be seen on the diagram** (allow awrt 1.0 and allow 180 – 120). Could score for sight of **Angle *CBD* = awrt 60 degrees.**	M1
	Both $7\cos(\pi - 2.1)$ **and** $7\sin(\pi - 2.1)$ or **Both** $7\cos(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\cos(\pi - 2.1))^2}$ or **Both** $7\sin(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\sin(\pi - 2.1))^2}$ **Or equivalents to these**	A correct attempt to find BC and BD. You can ignore how the candidate assigns *BC* and *CD*. $7\cos(\pi - 2.1)$ can be implied by awrt 3.5 and $7\sin(\pi - 2.1)$ can be implied by awrt 6. Note if the sin rule is used, do not allow mixing of degrees and radians unless their answer implies a correct interpretation. **Dependent on the previous method mark.**	dM1
	Note that 2.1 radians is 120 degrees (to 3sf) which if used gives angle CBD as 60 degrees. If used this gives a correct perimeter of 31.3 and could score full marks.		
	$P = 7\cos(\pi - 2.1) + 7\sin(\pi - 2.1) + 7 + 14.7$	their BC + their CD + 7 + their DEA **Dependent on <u>both</u> previous method marks**	ddM1
	$= 31.2764...$	Awrt 31.3	A1
			[4]
			Total 6

Question 7

Question Number	Scheme	Marks
(a)	In triangle OCD **complete method** used to find angle COD so:	
	Either $\cos COD = \frac{8^2+8^2-7^2}{2\times 8\times 8}$ or uses $\angle COD = 2\times \arcsin\frac{3.5}{8}$ oe so $\angle COD =$	M1
	$(\angle COD = 0.9056(331894))$ $= 0.906$ (3sf) * accept awrt 0.906	A1 * (2)
(b)	Uses $s = 8\theta$ for any θ in radians or $\frac{\theta}{360}\times 2\pi\times 8$ for any θ in degrees	M1
	$\theta = \frac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta (= awrt\ 2.24)$ and Perimeter = 23+(16 $\times\theta$)	M1
	accept awrt 40.9 (cm)	A1 (3)
(c)	Either Way 1: (Use of Area of two sectors + area of triangle)	
	Area of triangle = $\frac{1}{2}\times 8\times 8\times \sin 0.906$ (or 25.1781155 accept awrt 25.2)or $\frac{1}{2}\times 8\times 7\times \sin 1.118$ or $\frac{1}{2}\times 7\times h$ after h calculated from correct Pythagoras or trig.	M1
	Area of sector = $\frac{1}{2}8^2\times "1.117979732"$ (or 35.77535142 accept awrt 35.8)	M1
	Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 (3)
	Or Way 2: (Use of area of semicircle – area of segment)	
	Area of semi-circle = $\frac{1}{2}\times\pi\times 8\times 8$ (or 100.5)	M1
	Area of segment = $\frac{1}{2}8^2\times("0.906" - \sin"0.906")$ (or 3.807)	M1
	So area required = awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 (3) [8]

Notes

(a) **M1**: Either use correctly quoted cosine rule – may quote as $7^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\alpha \Rightarrow \alpha = ...$
Or split isosceles triangle into two right angled triangles and use arcsin or longer methods using Pythagoras and arcos (i.e. $\pi - 2\times \arccos\frac{3.5}{8}$). **There are many ways of showing this result.**

Must conclude that $\angle COD =$

A1*: (NB this is a given answer) If any errors or over-approximation is seen this is A0. It needs correct work **leading to stated answer** of 0.906 or awrt 0.906 for A1. The cosine of COD is equal to 79/128 or awrt 0.617. Use of 0.62 (2sf) does not lead to printed answer. They may give 51.9 in degrees then convert to radians. This is fine.

The minimal solution $7^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\alpha \Rightarrow \alpha = ...$ 0.906 (with no errors seen) can have M1A1 but errors rearranging result in M1A0

(b) **M1**: Uses formula for arc length with $r = 8$ and any angle i.e. $s = 8\theta$ if working in rads or $s = \frac{\theta}{360}\times 2\pi\times 8$ in degrees

(If the formula is quoted with r the 8 may be implied by the value of their $r\theta$)

M1: Uses angles on straight line (or other geometry) to find angle BOC or AOD and uses
Perimeter = 23 + arc lengths BC and AD (may make a slip – in calculation or miscopying)

A1: correct work leading to awrt 40.9 not 40.8 (do not need to see cm) This answer implies M1M1A1

(c) Way 1: **M1**: Mark is given for **correct** statement of area of triangle $\frac{1}{2}\times 8\times 8\times\sin 0.906$ (must use correct angle) or for correct answer (awrt 25.2) Accept alternative correct methods using Pythagoras and ½ base×height

M1: Mark is given for formula for area of sector $\frac{1}{2}8^2\times "1.117979732"$ with $r = 8$ and their angle BOC or AOD or $(BOC + AOD)$ **not** COD. May use $A = \frac{\theta}{360}\times\pi\times 8^2$ **if working in degrees**

A1: Correct work leading to awrt 96.7, 96.8 or 96.9 (This answer implies M1M1A1)

NB. Solution may combine the two sectors for part (b) and (c) and so might use $2\times\angle BOC$ rather than $\angle BOC$

Way 2: **M1**: Mark is given for **correct** statement of area of semicircle $\frac{1}{2}\times\pi\times 8\times 8$ or for correct answer 100.5

M1: Mark is given for formula for area of segment $\frac{1}{2}8^2\times("0.906" - \sin"0.906")$ with $r = 8$ or 3.81 **A1**: As in Way 1

Question 8

Question Number	Scheme	Marks
(a)	**Usually answered in radians: Uses** $BCD = 3.5\times(angle)$, $=3.5\times1.77=6.195$ (m) (accept awrt 6.20)	M1 A1 **(2)**
(b)	Area $= \frac{1}{2}(3.5)^2\times1.77=10.84$ (m^2)	M1 A1 **(2)**
(c)	Area of triangle $= \frac{1}{2}\times3.7\times3.5\times\sin(angle)$, $= \frac{1}{2}\times3.7\times3.5\times\sin(\frac{\pi}{2}-\frac{1.77}{2})$ (=awrt 4.1) Total area = "10.84"+2×"4.101" = 19.04	M1, A1 M1 A1cao **(4)** **[8]**

Notes

(a) M1: uses $s = 3.5\times\theta$ with θ in radians or completely correct work in degrees
A1: awrt 6.20 or just 6.2 (do not need to see units) Correct answer can imply the method.

(b) M1 for attempt to use $A = \frac{1}{2}\times3.5^2\times\theta$ (Accept θ in degrees.)
A1 for awrt 10.84 (do not need to see units) isw if correct answer is followed by 10.8. Correct answer can imply the method.

(c) M1: Uses area of triangle $\frac{1}{2}\times3.7\times3.5\times\sin(angle)$ Must be correct method for area of triangle but may be less direct.
A1: Correct expression using correct angle i.e. $\frac{\pi}{2}-\frac{1.77}{2}$ or awrt 0.69 or awrt 39 degrees (need at least 2 sf if no other work seen, but may be implied by correct final answer) If correct expression is given then isw (so e.g. isw an answer of 0.0775 implying angle set to degrees on calculator)
M1: Adds **twice** their **second calculated area** (even if rectangle or segment) to their sector area (may have been slips or errors in one or both formulae – such as missing ½ or mixture of degrees and radians or weak attempt at triangle area) so M0A0M1A0 is a possible mark distribution
A1: 19.04 cao (common answer through insufficient accuracy is 19.08 which loses final mark)
Special Case. The mark profile M1A0M1A0M1A0M1A0 can be given if the angle is misunderstood as 1.77π or as AFB for example
If "10.84"+3.5×3.7sin(angle) is used then this can gain both M marks and the A marks if correct.
But use of $3.5\times3.7\sin(angle)$ and later doubled and added to "10.84" is 1st M0, 2nd M1.